

Nonlinear Effects in Generation of Radiation by Relativistic Electron Beams in One-Dimensional Periodic Structures

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Nonlinear processes can significantly influence the operation of different types of electromagnetic radiation sources driven by intense relativistic electron beams. This paper considers the transfer from a chaotic to a single-frequency regime of generation by the example of radiation sources, driven by the relativistic beams moving in one-dimensional periodic structures. It is shown that by shifting the operating point of generation close to the upper end of a passband wave and using the adjustable feedback reflector for a backward wave, one can suppress the parasitic generation waves and obtain a single-mode oscillation. As a result, the radiation power significantly increases compared to the case of a chaotic regime of generation. The experimental results presented in this paper demonstrate such an increase of the radiation power in relativistic backward wave oscillators.

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1. Introduction

At present, a lot of works are devoted to the study of nonlinear effects and chaotic processes occurring in interactions of relativistic electron beams with a periodic structure (Volume Free Electron Lasers (VFEL), backward wave oscillators (BWO), etc.) [1–5]. The effect of a significant increase in the radiation power in the Compton regime near the region of degeneracy of the roots of dispersion equations describing the radiative instability of a beam was first reported in [6, 7]. The new law, formulated in [6, 7], states that the increment of radiative instability of a particle beam passing through a three-dimensional spatially-periodic structure depends on the number of additional waves appearing due to diffraction. This law underlies the invention of a brand-new generator of electromagnetic radiation—Volume Free Electron Laser (VFEL) [2]. In a one-dimensional diffraction case, the increment occurs in the vicinity of the so-called

" π point", i.e., the point near which the group velocity of the wave tends to zero, and the amplitudes of the forward and backward waves appear to be comparable. Later experiments confirmed a significant increase in the radiation power [8, 9].

In this report, we shall consider the nonlinear effects accompanied by the excitation of hybrid electromagnetic waves by the example of BWO operation. It is currently a widely held approach that with appearing self-modulation, the stable, steady-state, single-frequency generation regime is gradually transformed into a chaotic one. The chaotic regime is finally reached when the electron beam current exceeds a certain value [10–14]. In many cases, self-modulation manifests itself as a parasitic effect hampering high-power single-frequency generation. However, nonstationary processes and complex dynamical regimes are of practical interest, for example, in generation of wideband radiation.

Earlier theoretical and experimental studies of relativistic BWOs with resonance reflectors have demonstrated the possibility of selective excitation of the operating TM_{01} mode in oversized periodic structures where

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$D/\lambda \sim 1.5$ [15–18] and $D/\lambda \sim 3$ [19]. The implemented configurations of BWOs were excited by the electron beam of energy ranging from 0.5 to 1.2 MeV, the generation efficiency of electromagnetic radiation reached 32% for the wavelength ranges of 3 cm and 10 cm. High efficiency in such generators is achieved by applying a large guiding magnetic field, in some experiments its value being as large as 4.5 T [18].

This paper considers the selection of TM_{01} wave lasing in a relativistic BWO with the electron beam energy up to 200 keV and the maximum diameter of the periodic structure equal to 0.86λ . The value of the magnetic field for guiding the electron beam along the periodic structure does not exceed 1.0 T. We considered here the chaotic regimes of generation in a high power microwave relativistic BWO generator with a one-dimensional periodic structure. These generation regimes are determined by simultaneous generation of electromagnetic radiation by asymmetric HEM -type (HE and EH) waves. The boundary conditions at a metal surface of an axisymmetric periodic structure can only be satisfied when both longitudinal components of the electromagnetic field of the asymmetric mode have nonzero values, i.e., $E_z \neq 0$ and $B_z \neq 0$. Such modes are called the hybrid electric and magnetic modes, or HEM modes for short. They are classified as HE modes—the modes having a predominant longitudinal electric field E_z and EH modes—those having a predominant longitudinal magnetic field B_z .

The optimal conditions of interaction of the electron beam with the (-1) st harmonic of the backward wave as well as with the fundamental harmonic of the wave is achieved by selecting the operating point of the TM_{01} wave generation in the π -mode region with due account of the energy distribution of the electron beam over the longitudinal velocities of electrons. The transition to optimal generation conditions is marked by a significant rise of the radiation power in the case of single-mode, single-frequency generation.

2. Mode selection

Let us consider the generation of electromagnetic radiation in a cylindrical waveguide of a relativistic BWO. For selecting the TM_{01} mode in a one-dimensional periodic structure, we shall shift the operating point of generation close to the upper edge of the passband of the TM_{01} wave. As seen from the equation for the diffraction quality factor Q , the Q -factor in this region is rather large due to both a small group velocity and large end reflections [19, 20]:

$$Q_{diff} = \frac{\omega L}{v_{gr}(1 - |R|)} \quad (1)$$

where ω , v_{gr} , R , and L are the angular frequency, the group velocity, the reflection coefficient, and the total length of the structure, respectively.

The corresponding phase velocity for a synchronous spatial harmonic must be close or less than the beam velocity. For the sake of concreteness, we shall further study the case of practical importance—the motion of a weakly relativistic 200 keV electron beam in a one-dimensional periodic structure [slow wave structure (SWS)] to obtain generation in the frequency range $2.5 \div 3.0$ GHz.

The starting current for the regime of BWO generation at that point is expressed by the formula [19]:

$$I_{start} = \frac{I_A \beta_z^3 \gamma^3}{\pi N^2 C(r_b) Q_{diff} \eta_{lin}} \quad (2)$$

where N is the number of periods in the structure, β_z is the longitudinal electron velocity normalized to the speed of light, $C(r_b)$ is the coupling coefficient, η_{lin} is the linear efficiency of the operating wave. The linear efficiency of the asymmetric EH_{11} mode is one-half as large as the linear efficiency of the axially symmetric mode [19]. It is assumed here that the coupling coefficient of the EH_{11} mode must be less than that of the TM_{01} mode.

It should be mentioned that when the BWO operates on the TM_{01} mode, a more simple

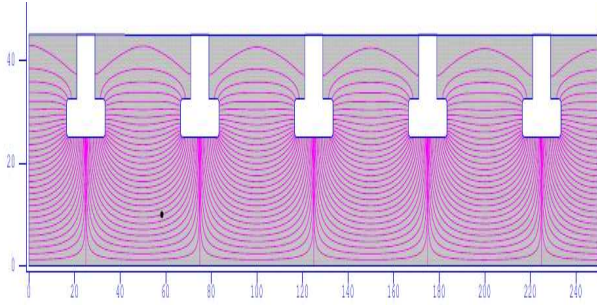


FIG. 1. Electric field distribution for π -type oscillation of operating TM_{01} wave and slow wave structure period 45 mm. (in color)

approximate formula can be used [20]:

$$I_{start} = \frac{U_0}{8.2R_c N^3} \quad (3)$$

where N is the number of wavelengths in the length of the structure, eU_0 is the kinetic energy of the electrons in the beam, and R_c is the coupling impedance of the electron beam with the mode of the electromagnetic wave.

So, if the operating current of the generator is close to the optimal value $I_{operating} = (3 - 5)I_{start}$, the TM_{01} mode will dominate over the EH modes. Conventionally, the chaotic regime of BWO operation is considered to occur when $I_{operating} \gg I_{start}$ (by a factor of nine or ten); a steady, single-frequency generation is considered to occur when $I_{operating} = (3 \div 5)I_{start}$ [10–13].

3. Numerical simulation

Different electromagnetic codes are used in numerical simulations of the operation of BWO-based microwave sources. In this paper, we used a relativistic electromagnetic code based on the particle-in-cell method. The trajectory of each macroparticle was found by solving the following set of the equations of motion in cylindrical

coordinates [21]:

$$\begin{aligned} \ddot{r} &= \frac{e}{m\gamma} \left[E_r - \dot{r}\delta + \frac{1}{c} (r\dot{\varphi}H_z - \dot{z}H_\varphi) \right] + r\dot{\varphi}^2 \\ \ddot{\varphi} &= \frac{1}{r} \frac{e}{m\gamma} \left[E_\varphi - r\dot{\varphi}\delta + \frac{1}{c} \dot{r}B_z \right] - 2\frac{\dot{r}\dot{\varphi}}{r} \\ \ddot{z} &= \frac{e}{m\gamma} \left[E_z - \dot{z}\delta + \frac{1}{c} (\dot{r}H_\varphi) \right]. \end{aligned} \quad (4)$$

Here $\gamma = \sqrt{1 - \frac{v^2}{c^2}}$, $v^2 = \dot{r}^2 + (r\dot{\varphi})^2 + \dot{z}^2$, and $\delta = \frac{1}{c^2}(\dot{r}E_r + r\dot{\varphi}E_\varphi + \dot{z}E_z)$; e and m are the macroparticle charge and mass, respectively.

The values of the components $\vec{E}$ and $\vec{H}$ of the electromagnetic field, induced by the electron beam in a periodic SWS, which appear on the right-hand side of the expression for the Lorentz force are found by using the linear numerical weighting scheme [22–24].

The boundary conditions for the fields on the SWS surface are $E_\varphi = 0$ and $E_z = 0$. The boundary conditions for the fields on the right open end of the waveguide are the conditions of radiation into free space:

$$\frac{E_r}{H_\varphi} = 1, \quad \frac{E_\varphi}{H_r} = -1.$$

As a result of the mechanism of charge and current distribution on the discrete mesh, the continuity equations are not satisfied exactly, which leads to the instability of numerical solution. This instability can be eliminated by solving the Laplace equation:

$$\Delta\Phi = -4\pi\rho + \text{div } \vec{E}, \quad \vec{E} \rightarrow \vec{E} - \nabla\Phi. \quad (5)$$

For further study we chose a one-dimensional periodic structure with the following parameters: internal diameter 50 mm, external diameter 90 mm, width of corrugation 17 mm (see Fig. 1).

Figures (2) and (3) represent the corresponding dispersion diagrams for TM_{01} versus different values of the structure periods and the diagrams of the hybrid electromagnetic modes of the chosen periodic structure.

The dispersion diagram defines the dependence of the electromagnetic wave

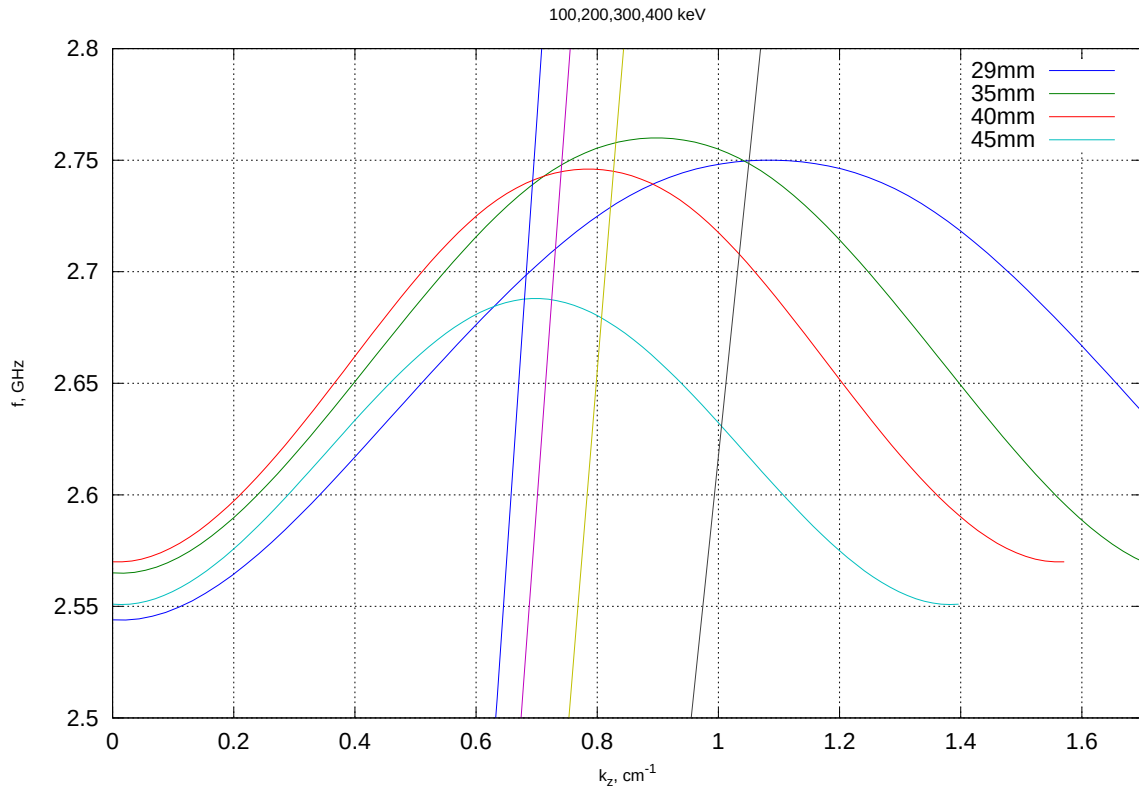


FIG. 2. Calculated dispersive curves of TM_{01} mode for the set of structure periods and electron beam lines 100 ÷ 400 keV. (in color)

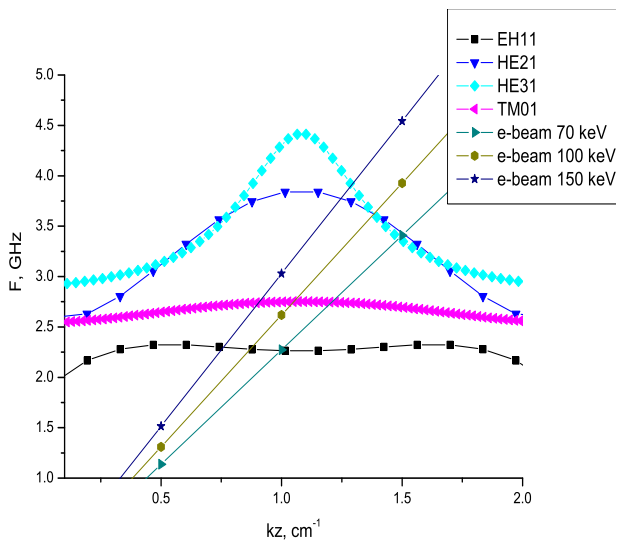


FIG. 3. Calculated dispersion curves of hybrid electromagnetic waves and TM_{01} wave for structure period of 29 mm. (in color)

frequency on its axial wave number. The operating point and the operation frequency range of a relativistic high power microwave BWO generator are determined by the intersection of the electron beam line, $\omega = k_z v_z$, with the dispersion curve of the operating mode, which corresponds with the condition of Cherenkov synchronism. Two antipodal generation regimes of BWO operation were specified in the numerical simulations and were then investigated experimentally in detail.

In what follows we shall consider the results of particle-in-cell simulations of the equations (see e.g. [20, 21]) describing the nonlinear behavior of BWOs and take account of the interaction with hybrid EH waves. An electron beam of energy 100 keV and current 1.5 kA was transported through a periodic structure by a finite, uniform magnetic field of 10 kG. We analyzed two cases: structures with a period of 29 mm (operating

near the π point) and 40 mm (operating far from the π point). The simulation results are shown in Figs. 4 and 5. The finite magnitude of the axial magnetic field leads to a nonzero value of the perpendicular velocity of the electrons and an effective excitation of hybrid waves. Under these conditions, the steady, single-frequency generation regime turns into chaotic.

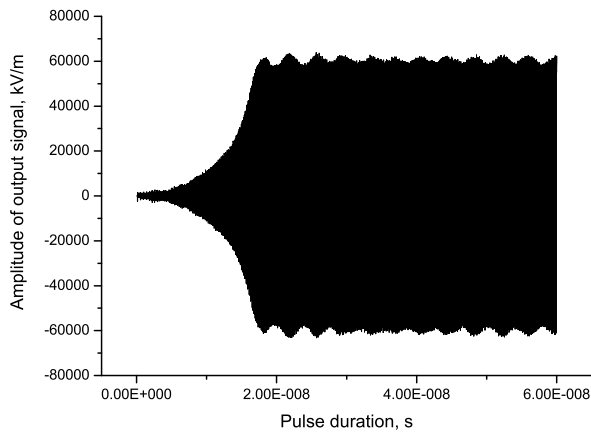


FIG. 4. Intensity of electromagnetic field generated by BWO for $d=29$ mm

The transient time required to establish a steady, single-mode operation of the relativistic BWO generator amounted to ~ 20 nanoseconds. The authors of [25] performed the experiments for the regime $\lambda < D$ and came to a similar conclusion in a one-dimensional case. Similar calculations using the electromagnetic PIC code XOOPIC confirmed the results obtained for the case of a steady-state BWO operation regime.

4. Experimental results

This section presents the experimental study of a nonlinear behavior of the developed BWO. High voltage Marx generator produce 200 kV, 160 ns pulses that feed a relativistic electron beam (REB) diode. The generated relativistic electron beam, having the peak amplitude of 1.7 kA, was guided through a SWS by an axial magnetic field of 10 kG. The electron beam generated in a magnetically insulated coaxial

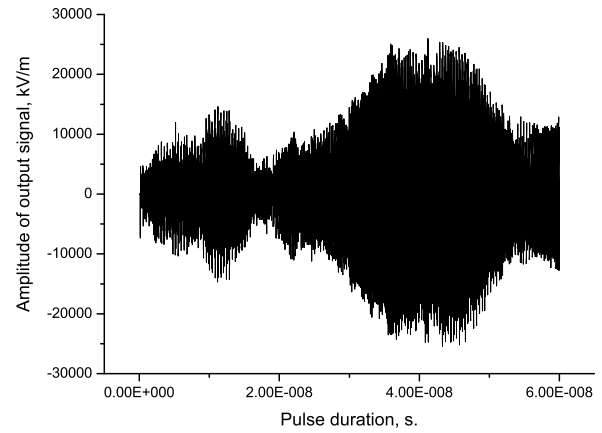


FIG. 5. Intensity of electromagnetic field generated by BWO for $d=40$ mm

vacuum diode was injected into the SWS via a cutoff waveguide section, which ensures the reflection of the backward electromagnetic wave. A smooth cylindrical waveguide insert was placed between the cutoff section and the SWS. It provides the optimal phase shift between the fundamental harmonic of the forward wave and the (-1)-st harmonic of the backward wave. The BWO periodic structure had a maximum diameter of 0.86λ and consisted of 10 periods.

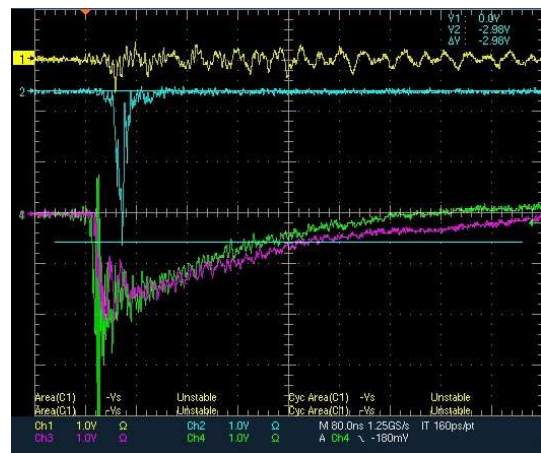


FIG. 6. Typical signal from a TDS5 oscilloscope: diode voltage - green curve, diode current - magenta curve, blue curve - microwave signal. (in color)

We studied experimentally the generation regimes of a relativistic BWO with a set

of microwave resonators having the periods of 40 mm, 35 mm, and 29 mm and fixed corrugation parameters. The characteristic energy distribution of the relativistic electron beam over the longitudinal velocities was established to be $150 \div 70$ keV, since the feeding voltage dropped throughout the pulse duration. A typical view of the current and voltage oscillograms and microwave signals is shown in Figs. 6 and 7.

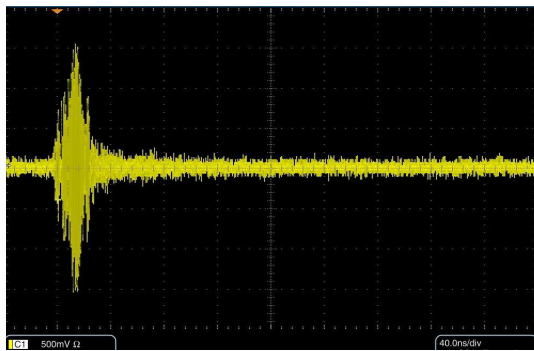


FIG. 7. Typical microwave signal registered by a TDS7 oscilloscope with 7 GHz analog passband. (in color)

Let us recall that we simulated the cases where the SWS periods were equal to 29 mm and 40 mm. The typical Fourier spectra calculated from the experimentally obtained signals for the two regimes of generation are shown in Figs. 8 and 9.

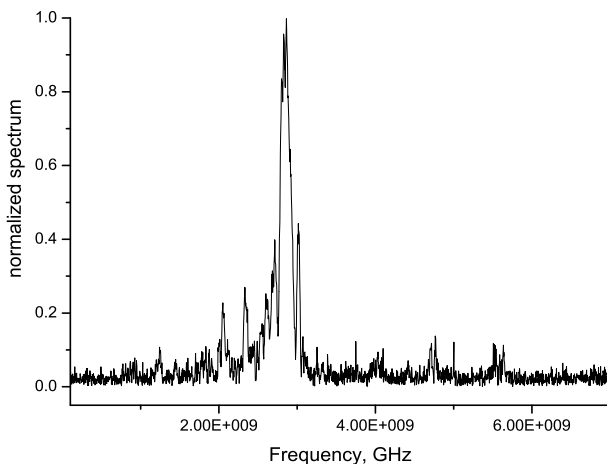


FIG. 8. Typical single mode radiation spectrum obtained by fast Fourier transform of the signal.

The stable, single-frequency generation was observed in experiments with a 29 mm SWS. The optimization of the single-mode regime of BWO operation was achieved by using an adjustable feedback for a backward wave. Single-

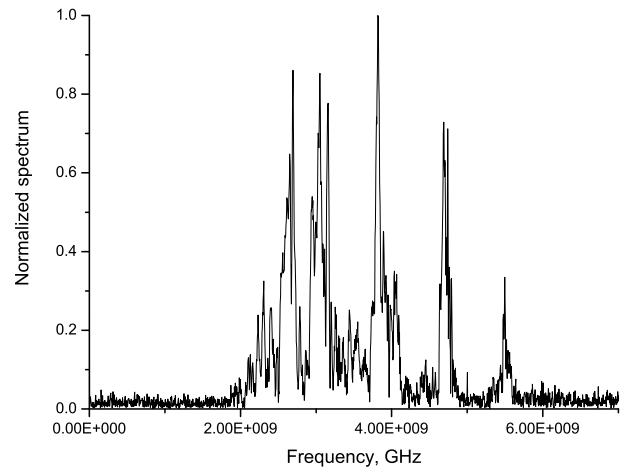


FIG. 9. Typical chaotic radiation spectrum obtained by fast Fourier transform of the signal.

mode (single-frequency) generation of microwave pulses by a relativistic BWO was experimentally achieved under the conditions when the operating point of generation was chosen near the upper edge of the passband of TM_{01} -type waves (a π -oscillation, called the π point).

The optimal conditions of interaction of the electron beam with both the (-1)st harmonic of the backward wave and the fundamental harmonic of the forward wave were achieved when the operating point of generation was chosen in the region of π -type oscillation of a TM_{01} wave taking into account the implemented energy distribution of the electron beam.

The chaotic regimes of generation considered here were determined by the accompanying excitation and generation of electromagnetic radiation by asymmetric HEM waves. The numerical simulations and the analysis of experimentally obtained radiation spectra led us to the conclusion that the chaotic generation regimes in a relativistic BWO with a one-dimensional periodic structure are determined by a simultaneous generation on TM_{01} , EH_{11} , HE_{21} ,

and HE_{31} oscillations. These regimes occurred under the conditions when the operating current was greater by a factor of 10 than the start current for the operating TM_{01} wave and the operating point of generation was away from the π point region.

The maximum radiation power was achieved at a fixed length of the waveguide insert between the beginning of the SWS and the cutoff waveguide section reflecting the backward wave of the BWO. The radiation power resonantly depends on the length of the insert relatively to the wavelength of the TM_{01} mode and achieves maximum experimental values in the range $(2/3 \div 3/4)\lambda_\omega$ where λ_ω is the wavelength in the waveguide.

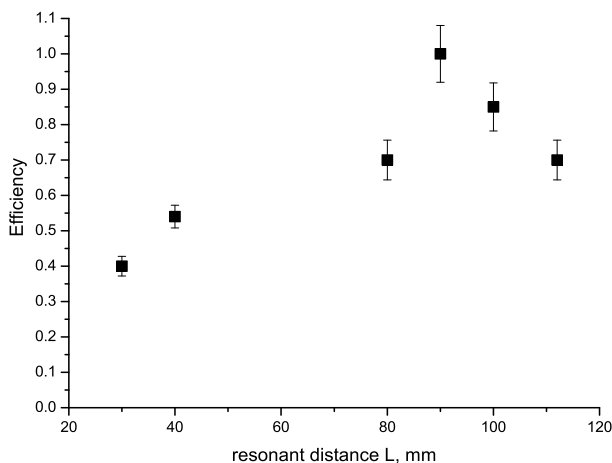


FIG. 10. Experimentally obtained generation efficiency versus the distance from the backward wave reflector.

The investigations infer that the radiation power appreciably depends on the regime of microwave generation in a BWO. In the regime of single-mode excitation, the radiation power is by a factor of five greater than that observed in the regime of chaotic multi-wave excitation. In the experiments performed at 2.8 GHz, the measured power level was 35 MW for the regime of single-mode excitation and the obtained efficiency was 10%.

5. Conclusion

The effect of a fivefold increase in the radiation power near the π point was proved theoretically and verified experimentally by the example of a relativistic BWO with a one-dimensional periodic structure and the electron beam energy up to 200 keV. The paper demonstrated the possibility of selective excitation of the operating TM_{01} wave in the case when the maximum diameter of the periodic structure is less than the operating wavelength.

The chaotic regimes of generation considered in this paper are determined by the accompanying excitation and generation on the EH_{11} , HE_{21} , and HE_{31} types of oscillations. In the experiments performed at 2.8 GHz, the measured power level was 35 MW for the regime of single-mode excitation and the obtained efficiency was 10%.

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