

Quarks in the Stochastic Vacuum

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Introduction

Di Giacomo A. et al Field correlators in QCD. Theory and applications
// Physics Reports. 2002. V. 372, №4. P. 319–368.

Stochastic vacuum + colour particles (quarks) = confinement.

In order to determine the particle state we average over the vacuum degrees of freedom.

St. V. \longleftrightarrow nonperturbative QCD processes as Gaussian (all field correlators higher than second order are neglected).

The method is proposed in the works of V. Kuvshinov and colleagues. Crucial point is in the consideration of the stochastic vacuum as an environment (in terms of quantum information theory) for a system of colour charges.

Kuvshinov V.I., Buividovich P.V. White Mixed States in QCD Stochastic Vacuum // Nonlinear Phenomena in Complex Systems. 2005. V. 8, №3. P. 313–316.

Kuvshinov V.I., Buividovich P.V. Decoherence of quark colour states in QCD vacuum // Acta Physica Polonica B Proc. Suppl. 2008. V. 1, №3. P. 579–582.

Further development:

- Various initial states: entangled, squeezed, superpositions, mixed etc.;
- Quantitative description with the help of quantum optics' characteristics purity, fidelity, von Neumann entropy;
- Deconfinement phase analysis (QGP);
- Intermediate cases.

Colour Superposition

Kuvshinov V.I., Bagashov E.G. Evolution of Colour Superposition in the Stochastic QCD Vacuum // Nonlinear Phenomena in Complex Systems. 2013. V. 16, №3. P. 242–246.

Initial quark state:

$$|\phi_{in}\rangle = \alpha|1\rangle + \beta|2\rangle + \gamma|3\rangle, \quad (1)$$

$$|\alpha|^2 + |\beta|^2 + |\gamma|^2 = 1. \quad (2)$$

$$\hat{\rho}_{in} = |\phi_{in}\rangle\langle\phi_{in}|, \quad (3)$$

$$\hat{\rho}_{in} = \begin{pmatrix} |\alpha|^2 & \alpha\beta^* & \alpha\gamma^* \\ \alpha^*\beta & |\beta|^2 & \beta\gamma^* \\ \alpha^*\gamma & \beta^*\gamma & |\gamma|^2 \end{pmatrix}. \quad (4)$$

Quark propagates along the trajectory M from x to y . The state in point y :

$$|\phi_f\rangle = \mathcal{P} \exp \left(i \int_M dx^\mu \hat{A}_\mu \right) |\phi_{in}\rangle, \quad (5)$$

where \mathcal{P} is the path ordering operator.

Let $\hat{\rho}_1$ be one of the solutions for a given implementation of vacuum variables.

Propagation of $\hat{\rho}_1$ along M is described as

$$\partial_\mu \hat{\rho}_1 = i[\hat{A}_\mu, \hat{\rho}_1]. \quad (6)$$

Density matrix might be presented as

$$\hat{\rho}_1 = N_c^{-1} \hat{I} + \rho_1^a \hat{T}_a, \quad (7)$$

where \hat{I} is the unit operator and \hat{T}_a ($a = 1, \dots, (N_c^2 - 1)$) are $SU(N_c)$ algebra generators normalized as $\text{Tr}(\hat{T}_a \hat{T}_b) = \delta_{ab}$, N_c is the number of colours.

The trajectories M should be closed, i.e. $x=y$. Thus instead of arbitrary M we'd have a closed loop L . Physically this corresponds to the creation and annihilation of particle-antiparticle pair.

Substituting (7) into (6) yields:

$$\hat{\rho}_f = \langle\langle \hat{\rho}_{1f} \rangle\rangle = N_c^{-1} \hat{I} + (\hat{\rho}_{in} - N_c^{-1} \hat{I}) W_{adj}(L), \quad (8)$$

where $\langle\langle \dots \rangle\rangle$ denotes the averaging over vacuum implementations, and $W_{adj}(L)$ is Wilson loop in the adjoint representation.

(Def. for Wilson loop: $W = \text{Tr} \left(\mathcal{P} \exp i \int dx^\mu \hat{A}_\mu \right)$).

Wilson loop decay is the confinement criterion. In the case of rectangular loop with dimensions T and R we'd get:

$$W_{adj}(L) = \exp(-\sigma_{adj}RT). \quad (9)$$

Here σ_{adj} is the QCD string tension in the adjoint representation. Obviously, in asymptotics of big T and R we get

$$\hat{\rho}(L : RT \rightarrow \infty) = N_c^{-1} \hat{I}. \quad (10)$$

Thus the interaction of initial superposition (4) with the stochastic vacuum leads to the emergence of a state with equal probabilities for different colours. In the initial basis this might be written as

$$\begin{pmatrix} |\alpha|^2 & \alpha\beta^* & \alpha\gamma^* \\ \alpha^*\beta & |\beta|^2 & \beta\gamma^* \\ \alpha^*\gamma & \beta^*\gamma & |\gamma|^2 \end{pmatrix} \rightarrow \begin{pmatrix} N_c^{-1} & 0 & 0 \\ 0 & N_c^{-1} & 0 \\ 0 & 0 & N_c^{-1} \end{pmatrix} \quad (11)$$

Multiparticle Case

Kuvshinov V.I., Bagashov E.G. Confinement of color states in a stochastic vacuum of quantum chromodynamics // Theoretical and Mathematical Physics. 2015. V. 184, №3. P. 475–482.

Kuvshinov V.I., Bagashov E.G. Stochastic Vacuum of Quantum Chromodynamics as an Environment for Color Particles // Physics of Particles and Nuclei Letters. 2016. V. 13, №3, P. 295–297.

$$\hat{\rho}(L) = N_c^{-N_p} \hat{I} + (\hat{\rho}_{in} - N_c^{-N_p} \hat{I}) W_{adj}^{N_p}(L). \quad (12)$$

$$\hat{\rho}(L : RT \rightarrow \infty) = N_c^{-N_p} \hat{I}. \quad (13)$$

Quantitative Characteristics

Purity P is defined as $P = \text{Tr}(\hat{\rho}^2)$. For the final state (8) we get

$$P = N_c^{-1} + (1 - N_c^{-1})W_{adj}^2(L). \quad (14)$$

In the asymptotical case of $RT \rightarrow \infty$ we get $P = N_c^{-1}$. This is the fully mixed state.

In the initial state $P = \text{Tr}(\hat{\rho}_{in}^2) = \text{Tr}(\hat{\rho}_{in}) = 1$, i.e. it is pure. For $RT \rightarrow 0$ and $W_{adj}(L) \rightarrow 1$, i.e. in absence of interaction it remains pure: $P = 1$.

Consider **von Neumann entropy**.

It might be used to quantitatively describe the loss of quantum information in the system during its departure from pure state. By definition:

$$S = -\text{Tr} (\hat{\rho} \ln \hat{\rho}). \quad (15)$$

In the initial state $S = 0$ due to the idempotency of the density matrix. In the final state under the condition $RT \rightarrow \infty$ (see (10)) we get:

$$S = -\text{Tr} (N_c^{-1} \hat{I} \ln (N_c^{-1} \hat{I})) = \ln N_c. \quad (16)$$

In intermediate cases (15) might be rewritten as

$$S = (1 - N_c^{-1}) \left(1 - \ln \left(\frac{\ln W_{adj}(L)}{N_c} \right) \right) + O(\ln W_{adj}(L)). \quad (17)$$

The proposed information measure might be written as

$$I = 1 - \frac{S}{N_p \ln N_c}. \quad (18)$$

So the overall range of this measure is $[0, 1]$: in case of zero entropy it is equal to **1** and in case of maximum entropy it is equal to **0**. The latter case corresponds to the asymptotically big values of RT (the confinement region).

Purity and entropy of different states of multiparticle system before the interaction with vacuum

State:	pure sep.	mixed sep.	pure ent.	mixed ent.
P	1	$\frac{1}{N_c^{N_p}} \leq P < 1$	1	$\frac{1}{N_c^{N_p}} < P < 1$
S	0	$0 < S \leq N_p \ln N_c$	0	$0 < S < N_p \ln N_c$

Purity and entropy of different states of multiparticle system after the interaction with vacuum

State:	pure sep.	mixed sep.	pure ent.	mixed ent.
P (purity)	$\frac{1}{N_c^{N_p}}$	$\frac{1}{N_c^{N_p}}$	$\frac{1}{N_c^{N_p}}$	$\frac{1}{N_c^{N_p}}$
S (entropy)	$N_p \ln N_c$	$N_p \ln N_c$	$N_p \ln N_c$	$N_p \ln N_c$

Conclusions

Thus, the initial pure state have turned into a mixed one (fully mixed in the asymptotics of big distances and/or time intervals - density matrix is proportional to unity).

As a result of this process (which might be treated as decoherence), the information about the colour state of the initial quark is lost.

Evolution of two- and three-particle systems is considered, as well as the generalization on the case of arbitrary number of particles.

The process might be described using some quantum characteristics: purity, fidelity, von Neumann entropy.

Thus the interaction of colour particles with stochastic vacuum treated as an environment might lead to confinement and related phenomena.

Thank you!