HADRONIC DECAYS OF KAONS.

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NONLINEAR PHENOMENA IN COMPLEX SYSTEMS

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QUARK MODEL

- **The hadronic interactions will be described in the** *Quark Confinement Model (QCM).*

[G.V.Efimov and M.A. Ivanov, *"The Quark Confinement Model of Hadron"*, IOP Publishing, 1993]. **This model is based on the following assumptions**

Lagrangian of the interaction between hadrons and quarks is obtaind

$$L_M = \frac{g_M}{\sqrt{2}} M^i \overline{q}^a_m \Gamma_\mu \lambda^{mn} q^a_n$$

$q^a_j = \begin{pmatrix} u^a \\ d^a \\ s^a \end{pmatrix}$ **-are the quark fields,**

M^i- Euclidean fields connected with the fields of physical particles, λ and Γ are the Gell-Mann and Dirac matrixes, g_M are quark-meson coupling.

QUARK MODEL

- **The quark confinement is provided by nontrivial gluon vacuum background. The confinement ansatz in the case of one-loop quark diagrams:**

$$\int d\sigma_{VAC}\, Tr|M(x_1)S(x_1,x_2|B_{VAC})\ldots M(x_n)S(x_n,x_1|B_{VAC})| \rightarrow$$

$$\int d\sigma_{VAC}\, Tr|M(x_1)S_v(x_1-x_2)\ldots M(x_n)S_v(x_n-x_1)|,$$

- **where**

$$S_v(x_1-x_2)=\int \frac{d^4p}{i(2\pi)^4}e^{-ip(x_1-x_2)}\frac{1}{\sigma\Lambda_q-\hat{p}}$$

QUARK MODEL

The measure $d\sigma_v$ is defined by $\int \frac{d\sigma_v}{v-\hat{z}} = G(\hat{z}) = a(-z^2) + \hat{z}\, b(-z^2)$

- The confinement function G(z) is independent of quark color and flavor and represents an entire function decreasing in the Euclidean region faster then any power of z.
- The choice of G(z) or $a(-z^2)$ and $b(-z^2)$ is one of the model assumptions. We use $a(-z^2)$ and $b(-z^2)$ in a form

$$a(u) = 2e^{-u^2-0.5u},$$

$$b(u) = 2e^{-u^2+0.2u}.$$

- The parameters Λ are

$$\Lambda_u = \Lambda_d = \Lambda_n = 0,460\ GeV$$

$$\Lambda_s = 0,508\ GeV$$

QUARK MODEL

g_M -The coupling constants for meson-quark interaction are defined from so-called compositeness condition

$$Z_M = 1 + \frac{3g_M^2}{4\pi^2}\widetilde{\Pi}'_M(m_M) = 0$$

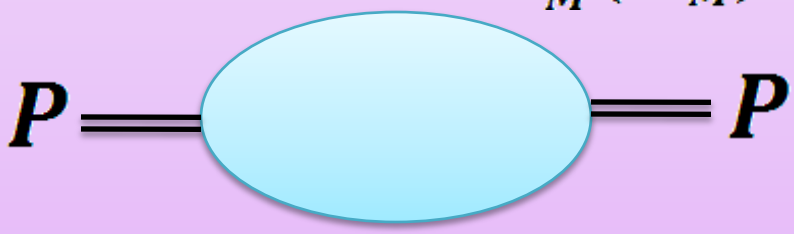
It is convenient to use

$$h_M = \frac{3g_M^2}{4\pi^2} = -\frac{1}{\widetilde{\Pi}'_M(m_M)}$$

MESON-QUARK INTERACTION CONSTANTS

Pseudo scalar meson constant

$$h_M = \frac{3g_M^2}{4\pi^2} = \frac{1}{\widetilde{\Pi}'_M(m_M)}$$



$$\Pi_M(p^2) = \int \frac{d^4k}{4\pi^2 i} \int d\sigma_\lambda \, Sp\{\Gamma_\mu S_\lambda(\hat{k}+\hat{p})\Gamma_\mu S_\lambda(\hat{k})\}$$

$$\widetilde{\Pi}'_M(p^2) \sim F_{PP}(x,\Lambda)$$

$$F_{PP}(x,\Lambda) = \int_0^\infty dub(u) + \frac{x}{4\Lambda^2}\int_0^1 dub\left(-\frac{x}{4\Lambda^2}u\right)\frac{1-\frac{u}{2}}{\sqrt{1-u}}$$

KAONS

$$H = \mathcal{M} - \frac{i}{2}\Gamma = \begin{pmatrix} \mathcal{M}_0 & \mathcal{M}_{12} \\ \mathcal{M}_{12}^* & \mathcal{M}_0 \end{pmatrix} - \frac{i}{2}\begin{pmatrix} \Gamma_0 & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_0 \end{pmatrix}$$

$$CP = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$|K_\pm\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle \pm |\tilde{K}^0\rangle\right)$$

$$|K_L\rangle = \frac{|K_-\rangle + \bar{\varepsilon}|K_+\rangle}{\sqrt{1 + |\bar{\varepsilon}|^2}} \qquad |K_S\rangle = \frac{|K_+\rangle + \bar{\varepsilon}|K_-\rangle}{\sqrt{1 + |\bar{\varepsilon}|^2}}$$

CP PARAMETERS

Parameter $\bar{\varepsilon}$ - measure of CP violation.

$$\bar{\varepsilon} = \frac{1+i}{2}\frac{Im\mathcal{M}_{12}}{2Re\mathcal{M}_{12}} - \frac{1-i}{2}\frac{Im\Gamma_{12}}{2Re\Gamma_{12}}$$

$$\mathcal{A}_I e^{i\delta_I} = \langle \pi\pi(0)|H_W|K^0\rangle \qquad \mathcal{A}_I^* e^{i\delta_I} = \langle \pi\pi(0)|H_W|\widetilde{K}^0\rangle$$

Where $I = 0, 2$ -isospin of final ππ state, δ_I - "strong" phases.

To quantify the direct CP violation, we introduce the parameter ε'

$$\varepsilon' = \frac{\langle \pi\pi(0)|H_W|K_S\rangle\langle \pi\pi(2)|H_W|K_L\rangle - \langle \pi\pi(0)|H_W|K_L\rangle\langle \pi\pi(2)|H_W|K_S\rangle}{\sqrt{2}\langle \pi\pi(0)|H_W|K_S\rangle^2}$$

$$\varepsilon' \approx i\frac{e^{i(\delta_2-\delta_0)}}{\sqrt{2}} Im\frac{\mathcal{A}_2}{\mathcal{A}_0} \approx i\frac{e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\frac{Im\mathcal{A}_2 - \omega Im\mathcal{A}_0}{Re\mathcal{A}_0}$$,where $\omega = Re\mathcal{A}_2/Re\mathcal{A}_0\,;\ \omega \ll 1$

Another parameter that is independent of the phase agreement is the parameter characterizing the CP violation related to the mixing effects.

$$\varepsilon = \frac{\langle \pi\pi(0)|H_W|K_L\rangle}{\langle \pi\pi(0)|H_W|K_S\rangle} \approx \bar{\varepsilon} + i\frac{Im\mathcal{A}_0}{Re\mathcal{A}_0}$$

CP PARAMETERS

In order to relate the parameters entered to the experimental data, we determine the following ratios of the amplitudes:

$$\eta_{00} = \frac{\langle \pi^0\pi^0 | H_W | K_L \rangle}{\langle \pi^0\pi^0 | H_W | K_S \rangle} \qquad \eta_{+-} = \frac{\langle \pi^+\pi^- | H_W | K_L \rangle}{\langle \pi^+\pi^- | H_W | K_S \rangle}$$

Neglecting small terms, we obtain:

$$\eta_{00} \approx \varepsilon - 2\varepsilon' \qquad \eta_{+-} \approx \varepsilon + \varepsilon'$$

$$|\varepsilon|^2 \approx |\eta_{00}|^2 \approx |\eta_{+-}|^2 \qquad \mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) \approx \frac{1}{6}\left(1 - \frac{|\eta_{00}|^2}{|\eta_{+-}|^2}\right)$$

$$|\eta_{00}|^2 = \frac{\Gamma(K_L \to \pi^0\pi^0)}{\Gamma(K_S \to \pi^0\pi^0)} \qquad |\eta_{+-}|^2 = \frac{\Gamma(K_L \to \pi^+\pi^-)}{\Gamma(K_S \to \pi^+\pi^-)}$$

EFFECTIVE HAMILTONIAN

$$H_{eff}^{\Delta S=1} = \frac{G_F}{2\sqrt{2}} V_{ud} V_{us} \sum_{i=1}^{6} c_i O_i$$

$$O_1 = (\bar{d} O_L^\mu s)(\bar{u} O_L^\mu u) - (\bar{d} O_L^\mu u)(\bar{u} O_L^\mu s) \qquad \Delta I = 1/2$$

$$O_2 = (\bar{d} O_L^\mu u)(\bar{u} O_L^\mu s) + (\bar{d} O_L^\mu s)(\bar{u} O_L^\mu u) + 2(\bar{d} O_L^\mu s)(\bar{d} O_L^\mu d) + \\ + 2(\bar{d} O_L^\mu s)(\bar{s} O_L^\mu s) \qquad \Delta I = 1/2$$

$$O_3 = (\bar{d} O_L^\mu u)(\bar{u} O_L^\mu s) + (\bar{d} O_L^\mu s)(\bar{u} O_L^\mu u) - (\bar{d} O_L^\mu s)(\bar{s} O_L^\mu s) \qquad \Delta I = 1/2$$

$$O_4 = (\bar{d} O_L^\mu u)(\bar{u} O_L^\mu s) + (\bar{d} O_L^\mu s)(\bar{u} O_L^\mu u) - (\bar{d} O_L^\mu s)(\bar{d} O_L^\mu d) \qquad \Delta I = 3/2$$

$$O_5 = (\bar{d} O_L^\mu \lambda^a s) \sum_{q=u,d,s} (\bar{q} O_R^\mu \lambda^a q) \qquad \Delta I = 1/2$$

$$O_6 = (\bar{d} O_L^\mu s) \sum_{q=u,d,s} (\bar{q} O_R^\mu q) \qquad \Delta I = 1/2$$

$$O_L^\mu = \gamma^\mu (1 - \gamma^5)\,; \quad O_R^\mu = \gamma^\mu (1 + \gamma^5)$$

EFFECTIVE HAMILTONIAN COEFFICIENTS

	Input values	c_1	c_2	c_3	c_4	c_5
Donogue J., et al	$\alpha_s = 1$ $m_t = 40\ GeV$	-2,38	0,1	0,084	0,42	-0,047
Minkowski P.	$\alpha_s(m_W) = 0,1$ $\mu = 3GeV$	-3,04	0,32	0,22	1,08	-0,13
Guberina B., et al	$\alpha_s = 1$ $\Lambda = 0,1GeV$ $\lvert\varepsilon\rvert = 1,1$	-5,11	0,03	0,04	0,2	-0,17
QCM	$\mu = 0,25GeV$ $\alpha_s = 0,45$	-1,97	0,12	0,093	0,47	-0,036

EFFECTIVE HAMILTONIAN

$$H_{eff}^{\Delta s=2} = \frac{G_F^2}{16\pi^2}(V_{ud})^2(V_{ud})^2 m_c^2\, \eta\, O^{\Delta s=2}$$

η - QCD parameter, m_C - C quark mass

$$O^{\Delta s=2} = (\bar{s}O_L^{\mu}d)(\bar{s}O_L^{\mu}d)$$

$K \rightarrow \pi\pi$

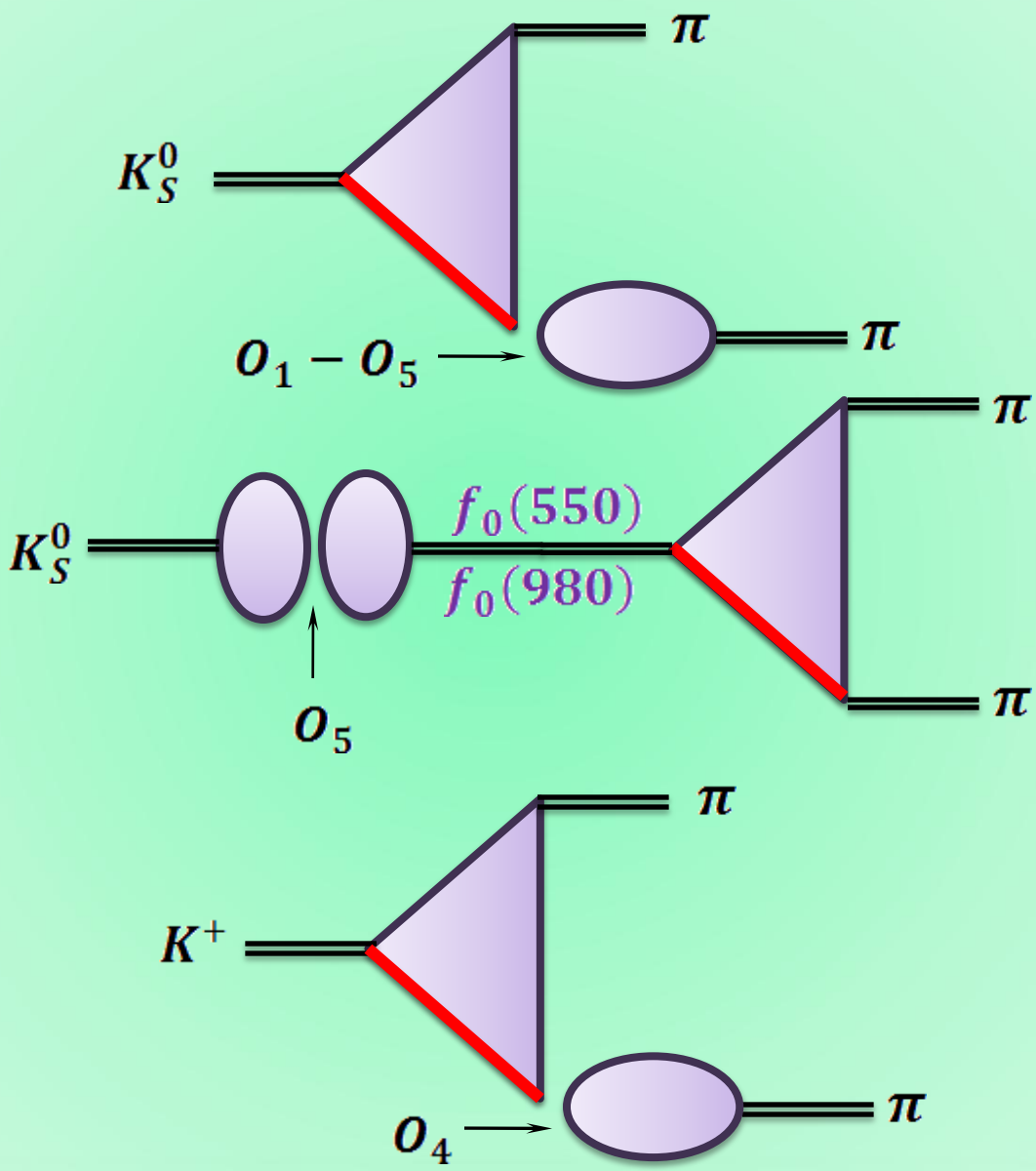
$$K_S^0 \rightarrow \pi^+\pi^- \qquad \Delta I = \frac{1}{2}$$

$$\Gamma\left(K_S^0 \rightarrow \pi^+\pi^-\right) = (5,058 \pm 0,006) \cdot 10^{-15} GeV$$

$$K^+ \rightarrow \pi^+\pi^0 \qquad \Delta I = \frac{3}{2}$$

$$\Gamma\left(K^+ \rightarrow \pi^+\pi^0\right) = (1,099 \pm 0,006) \cdot 10^{-17} GeV$$

$K \to \pi\pi$



$K \longrightarrow \pi\pi$

$$\mathcal{A}_0 = \mathcal{M}\left(K_S^0 \to \pi^+\pi^-\right) = \frac{G_F}{2\sqrt{2}} V_{ud} V_{us} \Big\{ (c_1 + 2c_2 + 2c_3 + 2c_4) T^1_{K_s^0\pi^+\pi^-} +$$

$$+ \left(c_5 + \frac{3}{16} c_6\right) \left(T^5_{K_s^0\pi^+\pi^-} + T^5_{Kf_0} D_{f_0}(m_K^2) g_{f_0\pi\pi}(m_K^2)\right) \Big\}$$

$$\mathcal{A}_2 = \mathcal{M}\left(K^+ \to \pi^+\pi^0\right) = \frac{G_F}{2\sqrt{2}} V_{ud} V_{us} \cdot c_4 \cdot T^4_{K^+\pi^+\pi^0}$$

$$T^i_{K_s^0 12} = \int dy dx_1 dx_2 dx_3 e^{i(p_1x_1 + p_2x_2 + p_3x_3)} \left\langle 0 \middle| T\left(\mathcal{L}_K(x_1)\mathcal{L}_\pi(x_2)\mathcal{L}_\pi(x_3) O^i(y)\right) \middle| 0 \right\rangle$$

$$T^i_{Kf_0} = \int dy dx_1 dx_2 \left\langle 0 \middle| T\left(\mathcal{L}_K(x_1)\mathcal{L}_{f_0}(x_2) O^i(y)\right) \middle| 0 \right\rangle$$

$g_{f_0\pi\pi}(m_K^2)$ - $f_0 \to \pi^+\pi^-$ decay form factor, with $m_{f_0}^2 = m_K^2$

$K \to \pi\pi$

	QCM GeV	EXP GeV
$K_S^0 \to \pi^+\pi^-$	$5,35 \cdot 10^{-15}$	$(5,058 \pm 0,006) \cdot 10^{-15}$
$K^+ \to \pi^+\pi^0$	$1,24 \cdot 10^{-17}$	$(1,099 \pm 0,006) \cdot 10^{-17}$

$K \to \pi\pi$

$\mathcal{M}(K_S^0 \to \pi^+\pi^-)$	$\frac{\mathcal{M}_{O_1-O_4}(K_S^0 \to \pi^+\pi^-)}{\mathcal{M}(K_S^0 \to \pi^+\pi^-)}$	$\frac{\mathcal{M}_{O_5}(K_S^0 \to \pi^+\pi^-)}{\mathcal{M}(K_S^0 \to \pi^+\pi^-)}$	$\frac{\mathcal{M}_{O_5}^{f_0}(K_S^0 \to \pi^+\pi^-)}{\mathcal{M}(K_S^0 \to \pi^+\pi^-)}$
$3,82 \cdot 10^{-8}$ GeV	$0,228$	$0,016$	$0,756$

CP PARAMETERS

Because $Im\mathcal{A}_2 = 0$

$$\varepsilon' \approx i\frac{e^{i(\delta_2-\delta_0)}}{\sqrt{2}}\frac{Im\mathcal{A}_0}{Re\mathcal{A}_0}$$

separating the real and imaginary parts of the propagator of the scalar meson.

$$ReD_s(p^2) = \frac{\Pi_s(m_s^2) - \Pi_s(p^2)}{\left(\Pi_s(m_s^2) - \Pi_s(p^2)\right)^2 + m_s^2\Gamma_s^2}\ ; \qquad ImD_s(p^2) = \frac{m_s\Gamma_s}{\left(\Pi_s(m_s^2) - \Pi_s(p^2)\right)^2 + m_s^2\Gamma_s^2}$$

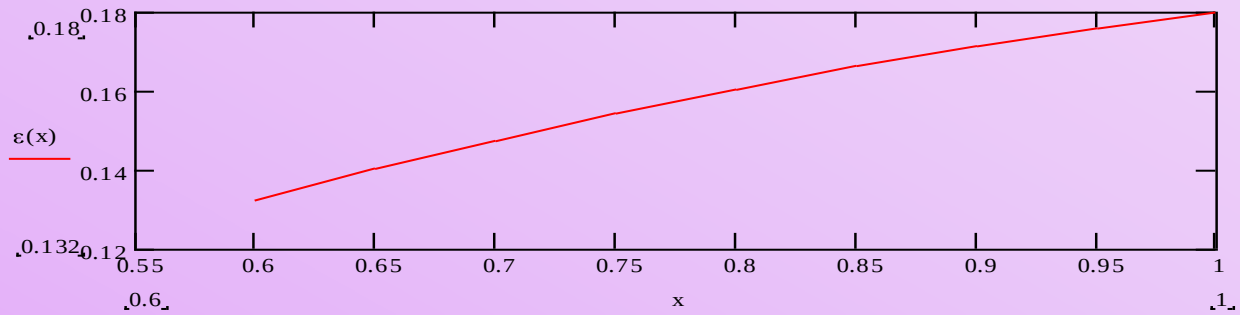
$$|\varepsilon'| \approx \frac{\left(c_5 + \frac{3}{16}c_6\right)T^5_{Kf_0}ImD_s(m_K^2)g_{f_0\pi\pi}(m_K^2)}{\sqrt{2}\{T^1 + T^5\}}$$

$$T^1 \equiv (c_1 + 2c_2 + 2c_3 + 2c_4)T^1_{K\pi\pi}$$

$$T^5 \equiv \left(c_5 + \frac{3}{16}c_6\right)\left(T^5_{K\pi\pi} + T^5_{Kf_0}D_{f_0}(m_K^2)g_{f_0\pi\pi}(m_K^2)\right)$$

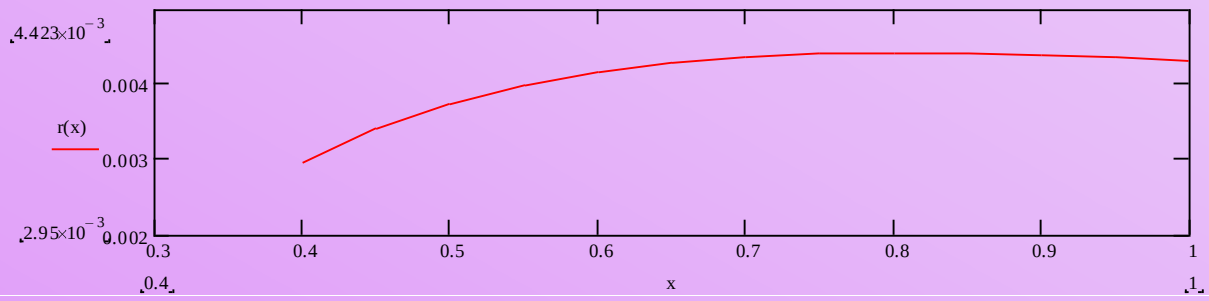
CP PARAMETERS

The dependence of $|\varepsilon'|$ on $\Gamma(f_0)$ is given



$$|\varepsilon'| = 0,095 \div 0,18$$

The dependence of $\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right)$ on $\Gamma(f_0)$ is given



$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (2,95 \div 4,42) \cdot 10^{-3}$$

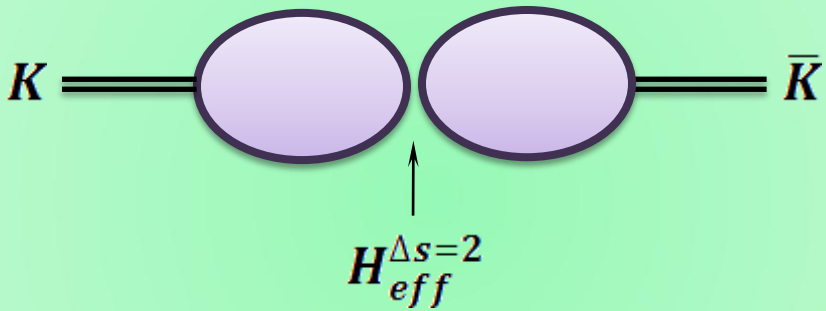
$\mathbb{K} \to \bar{\mathbb{K}}$

$$\Delta m_{LS} = \frac{Re\left(\mathcal{M}(K^0 \to \bar{K}^0)\right)}{2m_K}$$

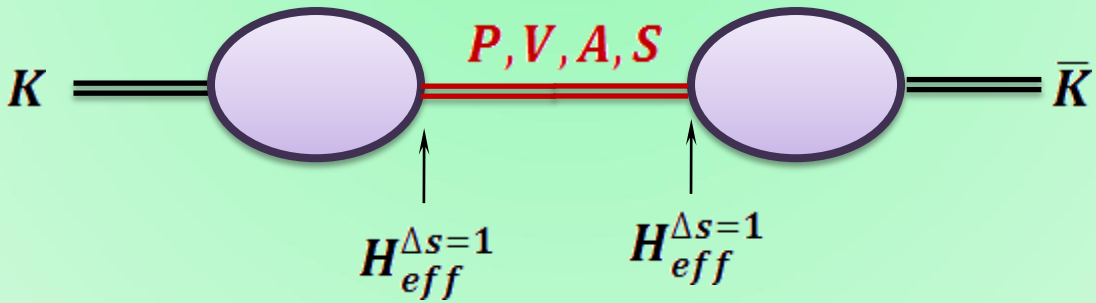
$$\Delta m_{LS} = \Delta m^{SD} + \Delta m^{LD}$$

Δm^{SD} - short distance contribution $\left(H_{eff}^{\Delta s=2}\right)$, Δm^{LD} - long distance contribution

SD:



LD:



$\mathbb{K} \to \bar{\mathbb{K}}$

$$\mathcal{M}_{SD}(K^0 \to \bar{K}^0) = \frac{G_F^2}{16\pi^2} V_{ud}^2 V_{us}^2 m_c^2 \eta \, T_{K^0\bar{K}^0}^{\Delta s=2}$$

$$T_{K^0\bar{K}^0}^{\Delta s=2} = \int dx_1 dx_2 dy e^{ip(x_1+x_2)} \langle 0 | T\{L_{K^0}(x_1) L_{\bar{K}^0}(x_2) O^{\Delta s=2}(y)\} | 0 \rangle$$

$$T_{K^0\bar{K}^0}^{\Delta s=2} = \frac{8}{3} m_K^2 f_K^2$$

$$\eta = 0,78 \quad \mu = 0,25; \; \alpha = 0,45$$

$$\mathcal{M}_{LD}(K^0 \to \bar{K}^0) = \sum_{M=P,S,A} D_M(m_K^2) \left[\mathcal{M}(K^0 \to M)\right]^2$$

$$D_M(p^2) = \frac{1}{\Pi_M(p^2) - \Pi_M(m_{P,S}^2) + i m_M \Gamma_M}$$

$\mathbb{K} \to \bar{\mathbb{K}}$

$$\mathcal{M}_{12}(K^0 \to \bar{K}^0) = \mathcal{M}_{SD}(K^0 \to \bar{K}^0) + \mathcal{M}_{LD}(K^0 \to \bar{K}^0)$$

$$Re\mathcal{M}_{12} = \mathcal{M}_{SD} + Re\mathcal{M}_{LD} =$$

$$= \mathcal{M}_{SD} + \sum_{M=P,S,A} [\mathcal{M}(K^0 \to M)]^2 \frac{\Pi_M(p^2) - \Pi_M(m_M^2)}{[\Pi_M(p^2) - \Pi_M(m_M^2)]^2 + m_M^2 \Gamma_M^2}$$

$$Im\mathcal{M}_{12} = Im\mathcal{M}_{LD} = \sum_{M=P,S,A} [\mathcal{M}(K^0 \to M)]^2 \frac{\mathrm{m}_M \Gamma_M}{[\Pi_M(p^2) - \Pi_M(m_M^2)]^2 + m_M^2 \Gamma_M^2}$$

$$C = Im\mathcal{M}_{12}$$

	π	η	η'	f_0 (550)	a_0	f_0 (980)	a_1	f_1 (1280)	f_1 (1420)
C	$7,9 \cdot 10^{-10}$	$4,2 \cdot 10^{-6}$	$1,4 \cdot 10^{-6}$	$6,1 \cdot 10^{-2}$	$3,3 \cdot 10^{-3}$	$2,5 \cdot 10^{-3}$	$-3,2 \cdot 10^{-2}$	$-1,3 \cdot 10^{-2}$	$-2,6 \cdot 10^{-3}$

$\mathbb{K} \rightarrow \overline{\mathbb{K}}$ Numerical results

$$\Delta m^{SD} = 2,04 \cdot 10^{-15} GeV$$

$$\Delta m^{LD} = 1,21 \cdot 10^{-15} GeV$$

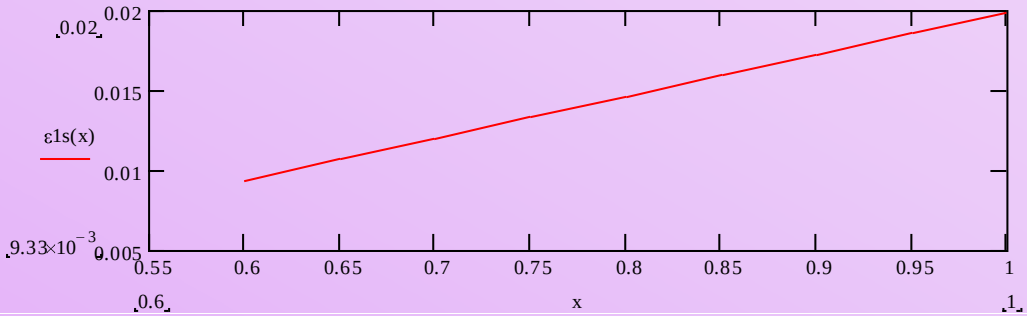
$$\Delta m_{LS} = 3,25 \cdot 10^{-15} GeV$$

$$\Delta m_{LS}^{exp} = (3,484 \pm 0,006) \cdot 10^{-15} GeV$$

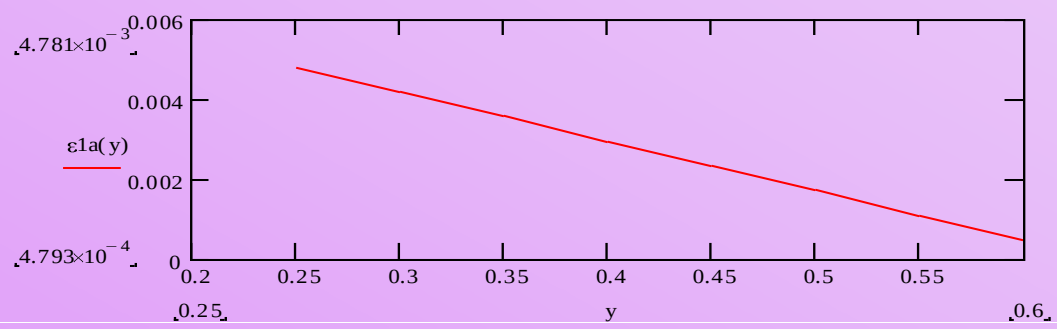
$$D = \frac{\Delta m^{LD}}{\Delta m_{LS}} \approx 0,37$$

CP PARAMETERS

The dependence of $|\bar{\varepsilon}|$ on $\Gamma(f_0)$ is given



The dependence of $|\bar{\varepsilon}|$ on $\Gamma(a_1)$ is given



$$|\bar{\varepsilon}| = 4,79 \cdot 10^{-4} \div 0,02$$