MODERN STATUS of SKALAR MESONS

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GSTU,GOMEL
NONLINEAR PHENOMENA IN COMPLEX SYSTEMS
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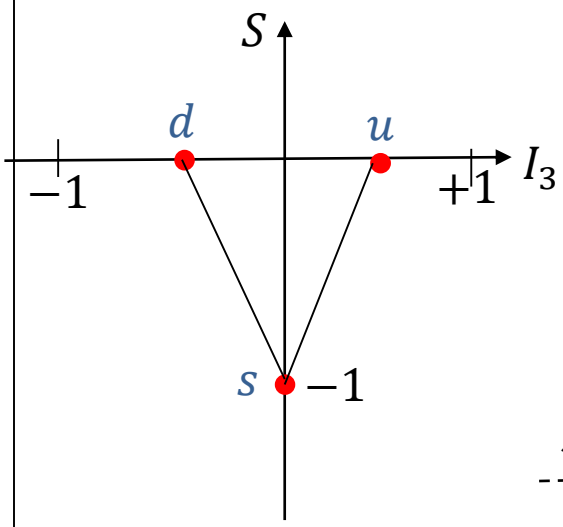
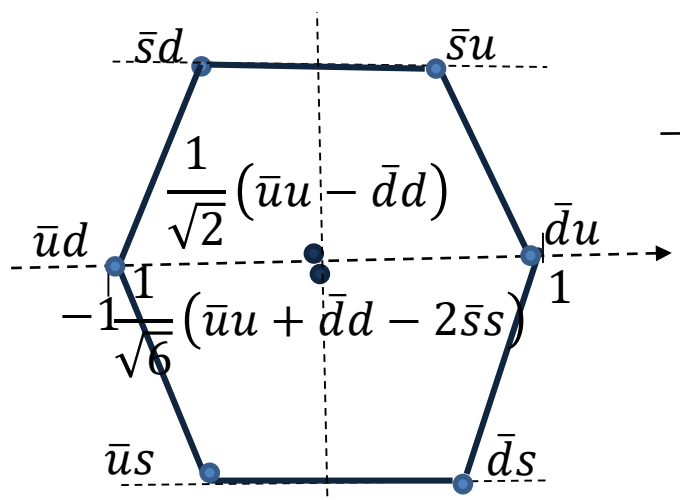
NAIV QUARK MODEL

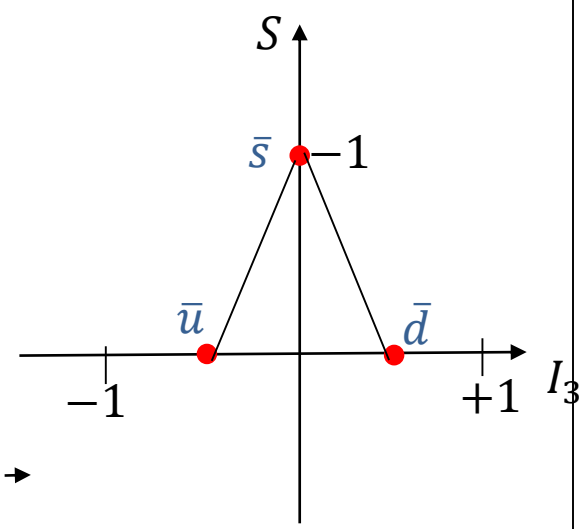
Gell-Mann, Zweig (1964):

Mesons- $q\bar{q}$-states, come from the product of 3 and 3^*representations of SU(3) group



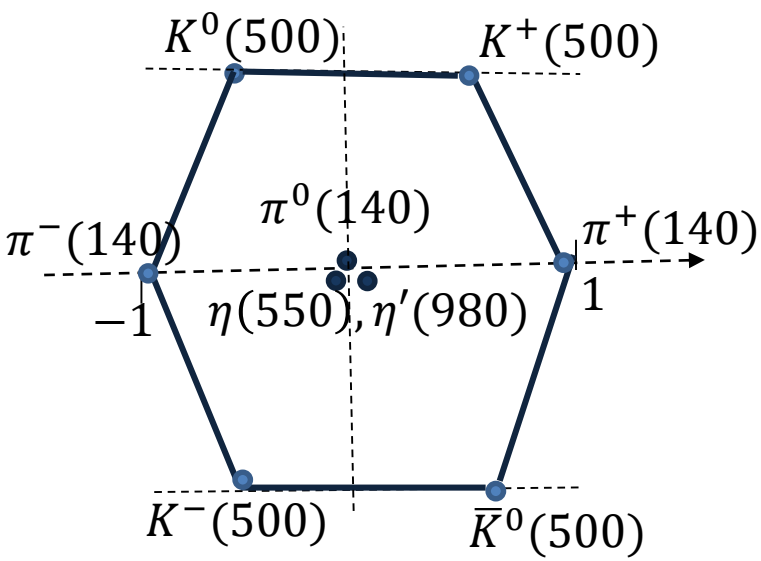
$$3 \times 3^* = \mathbf{1} + \mathbf{8}$$



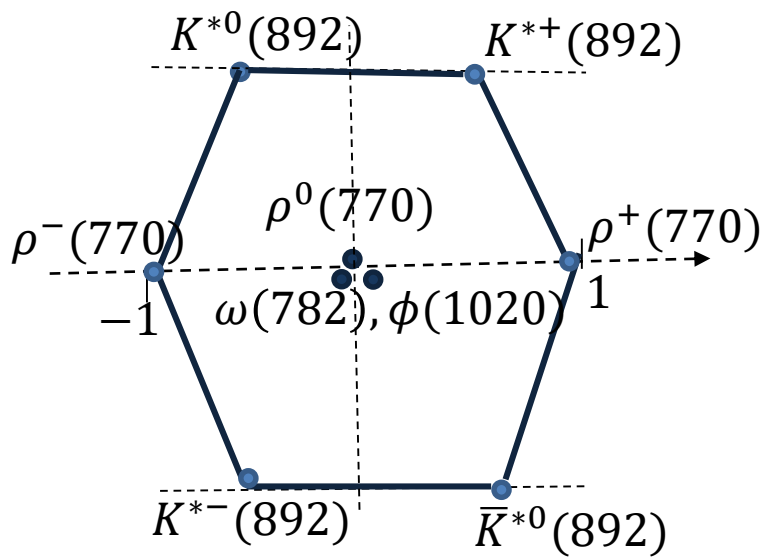


NAIV QUARK MODEL

Pseudoscalars 0^{-+}



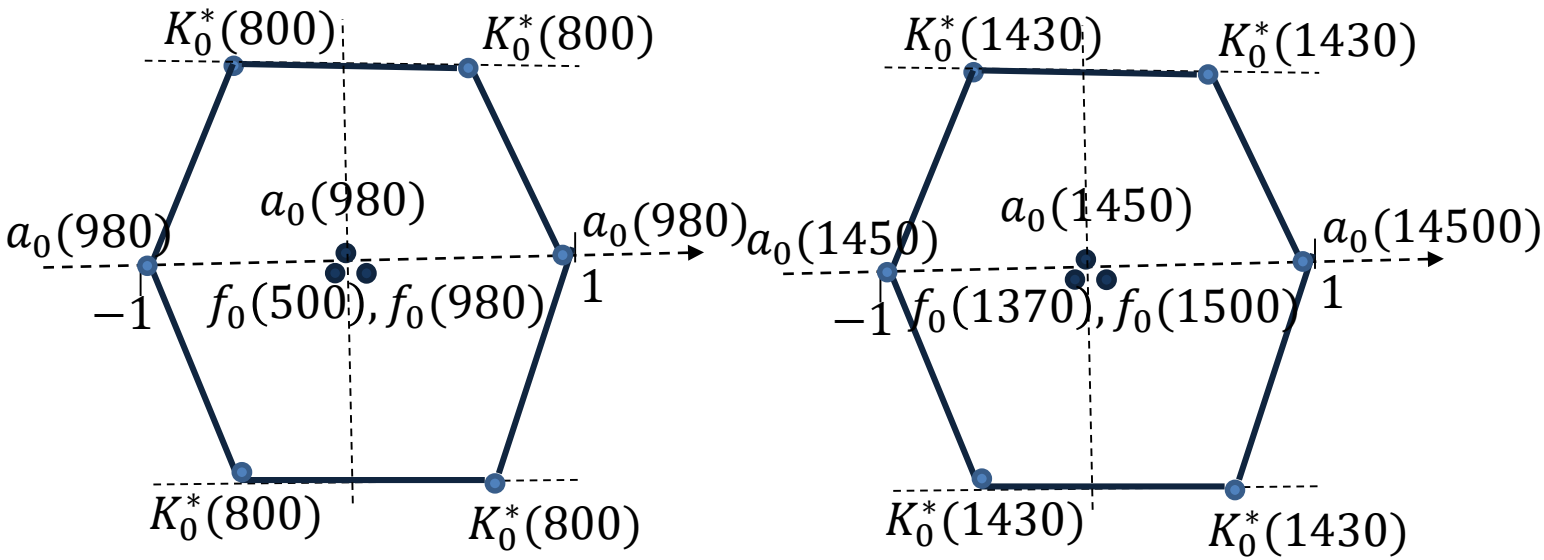
Vectors 1^{--}



SCALAR MESONS

MESON	MASS (MeV)	WIDTH(MeV)	ISOSPIN, STRANGENESS
$f_0(500), \sigma$	$400 - 550$	$400 - 700$	$I = 0, S = 0$
$K_0^*(800), \kappa$	682 ± 29	547 ± 24	$I = \frac{1}{2}, S = \pm 1$
$f_0(980)$	990 ± 20	$40 - 100$	$I = 0, S = 0$
$a_0(980)$	980 ± 20	$50 - 100$	$I = 1, S = 0$
$f_0(1370)$	$1200 - 1500$	$200 - 500$	$I = 0, S = 0$
$K_0^*(1430)$	1425 ± 50	270 ± 80	$I = \frac{1}{2}, S = \pm 1$
$a_0(1450)$	1474 ± 19	265 ± 13	$I = 1, S = 0$
$f_0(1500)$	1505 ± 6	109 ± 7	$I = 0, S = 0$
$f_0(1700)$	1720 ± 6	135 ± 8	$I = 0, S = 0$

SCALAR MESONS



“Where and what are the scalar mesons?”[*]

[*] P.Estabrooks, Phys. Rev. **D** 19, 2678 (1979)

Theoretical approaches

- Two quark models

Шабалин Е.П. ЯФ,т.40 (1984),

F.E.Close, A. Kirk, EPJ C21 (2001)

- Four quark models

R.L. Jaffe, Phys. Rev. D 15 (1977),

N.N. Achasov, Nucl.Phys. A728 (2003)…

- Gluebol , hybrids.

V. Vento EPJ A24 (2005),

E.S. Swanson Phys.Rep.429 (2006)

TWO QUARK MODEL

- **The hadronic interactions will be described in the** *Quark Confinement Model (QCM).*

[G.V.Efimov and M.A. Ivanov, *"The Quark Confinement Model of Hadron"*, IOP Publishing, 1993]. **This model is based on the following assumptions**

Lagrangian of the interaction between hadrons and quarks is obtaind

$$L_M = \frac{g_M}{\sqrt{2}} M^i \overline{q}^a_m \Gamma_\mu \lambda^{mn} q^a_n$$

$q^a_j = \begin{pmatrix} u^a \\ d^a \\ s^a \end{pmatrix}$ **-are the quark fields,**

M^i- Euclidean fields connected with the fields of physical particles, λ and Γ are the Gell-Mann and Dirac matrixes, g_M are quark-meson coupling.

TWO QUARK MODEL

$$\mathcal{L}_I^s = \frac{g_s}{\sqrt{2}} s(x)\overline{q}(x)\left(I - i\frac{\mathrm{H}}{\Lambda}\overleftrightarrow{\partial}\right)\lambda_s q(x)$$

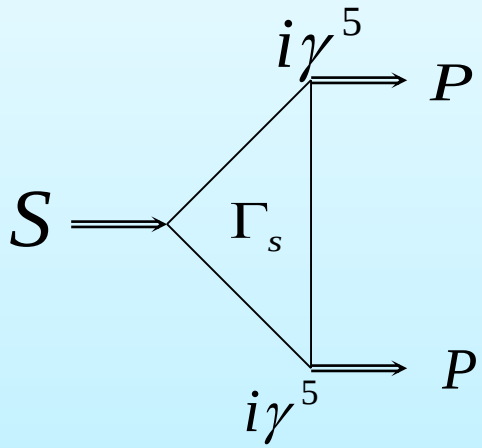
$$\overleftrightarrow{\partial} \equiv \overleftarrow{\partial} - \overrightarrow{\partial}$$

$$\lambda_s = \begin{cases} diag(1,-1,0) \Rightarrow a_0 \\ diag(\cos\delta_s, \cos\delta_s, -\sqrt{2}\sin\delta_s) \Rightarrow \varepsilon \\ diag(-\sin\delta_s, -\sin\delta_s, -\sqrt{2}\cos\delta_s) \Rightarrow f_0 \end{cases}$$

parameters: H, δ_s

Evaluation of one-loop diagrams

G.V.Efimov and M.A. Ivanov, *"The Quark Confinement Model of Hadron"*, IOP Publishing, 1993.



$$I_{SPP} = \int d\sigma_\lambda \int \frac{d^4k}{4\pi^2 i} Tr\left\{ \Gamma_s \frac{1}{\Lambda v_\lambda - \hat{k}} \gamma^5 \frac{1}{\Lambda v_\lambda - (\hat{k} + \hat{q}_1)} \gamma^5 \frac{1}{\Lambda v_\lambda - (\hat{k} + \hat{p})} \right\}$$

$$\int d\sigma_\lambda \frac{v_\lambda}{v_\lambda^2 + X} = a(X), \qquad \int d\sigma_\lambda \frac{1}{v_\lambda^2 + X} = b(X).$$

Evaluation of one-loop diagrams

$$\Gamma_s \equiv I - H\hat{P}$$

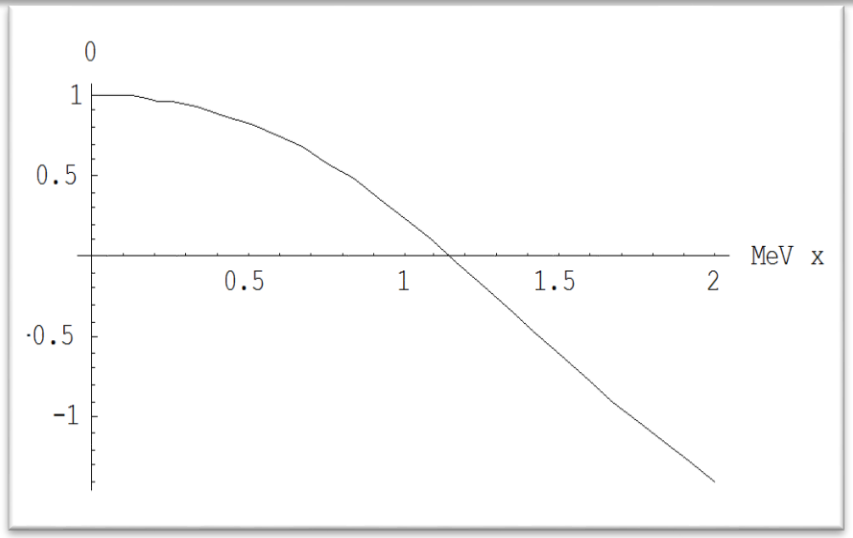
$$I_{SPP}(m_s^2) = I_0(m_s^2) - 4H I_1(m_s^2)$$

$$I_0(4\Lambda^2 x) = \int_0^\infty du a(u) - x\int_0^1 du a(-ux)[\frac{1}{2}\ln\frac{1+\sqrt{1-u}}{1-\sqrt{1-u}} - \sqrt{1-u}]$$

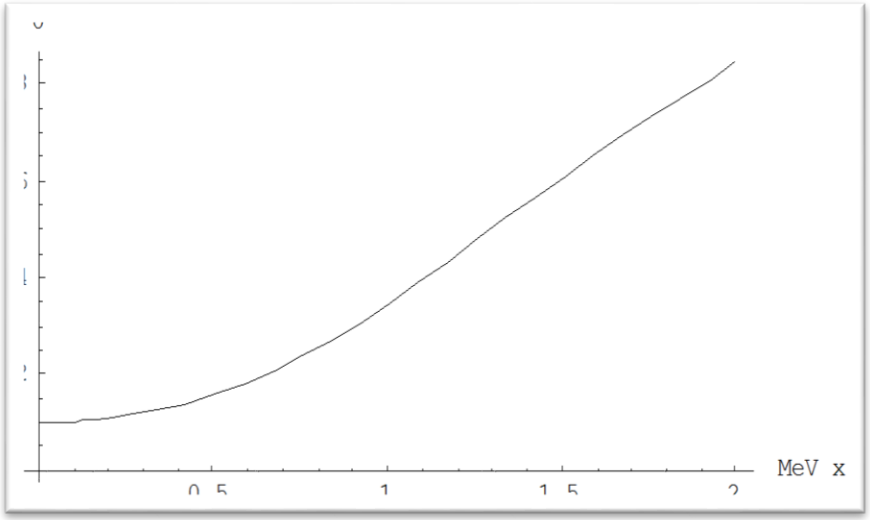
$$I_1(4\Lambda^2 x) = \int_0^\infty du u b(u) + x\int_0^1 du u b(-ux)[\frac{1}{2}\ln\frac{1+\sqrt{1-u}}{1-\sqrt{1-u}} - \sqrt{1-u}]$$

Mass dependence of S→PP

$$\Gamma_s = I$$

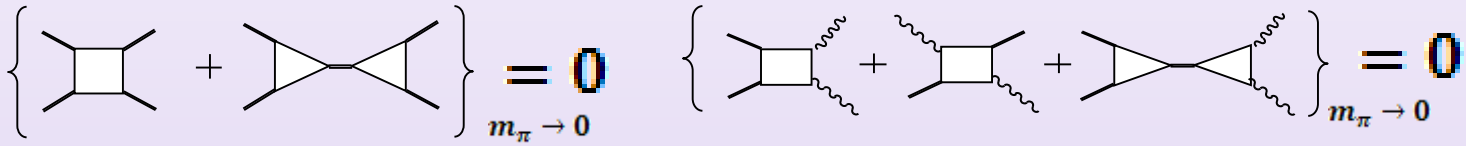


$$\Gamma_s = I - \frac{H}{\Lambda}\hat{p}$$



FIT

1.



$$\begin{cases} \int_0^\infty dub(u) = 2\Lambda^2[\int_0^\infty dua(u) - 4H\int_0^\infty duub(u)]h_\varepsilon(H)D_\varepsilon(0) \\ 5b(0) = -2\Lambda_s^2\cos\delta_s(5\cos\delta_s - \sqrt{2}\sin\delta_s)[\int_0^\infty dua(u) - 4H\int_0^\infty duub(u)]a(0)h_\varepsilon(H)D_\varepsilon(0) \end{cases}$$

$$R = -\frac{5\cos\delta_s[\int_0^\infty dua(u) - 4H\int_0^\infty duub(u)]}{(5\cos\delta_s - \sqrt{2}\sin\delta_s)a(0)} = 1$$

2. $\Gamma_{\exp}(f_0 \to \pi\pi)$

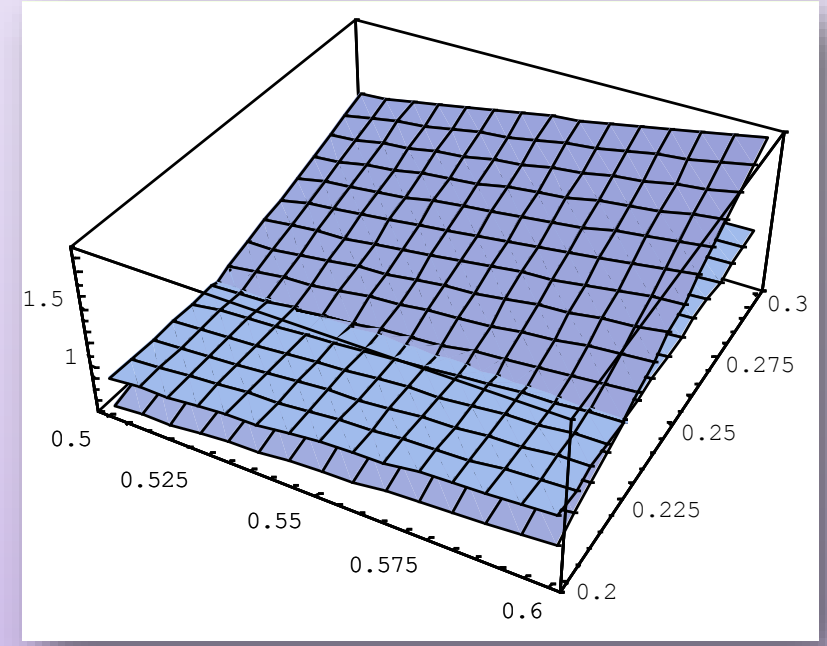
$$\Gamma(f_0 \to \pi\pi) = \frac{3}{16\pi}\sqrt{1 - \frac{4m_\pi^2}{m_{f_0}^2}}\, g_{f_0\pi\pi}^2(m_{f_0}^2, 0, 0)\frac{1}{m_{f_0}}$$

$$g_{SP_1P_2}(m_s^2, m_{P_1}^2, m_{P_2}^2) = Sp\lambda_s\{\lambda_{P_1}, \lambda_{P_2}\}\Lambda\frac{\sqrt{h_{P_1}h_{P_2}h_S(H)}}{6}I_{SPP}(m_s, m_{P_1}, m_{P_2})$$

parameters

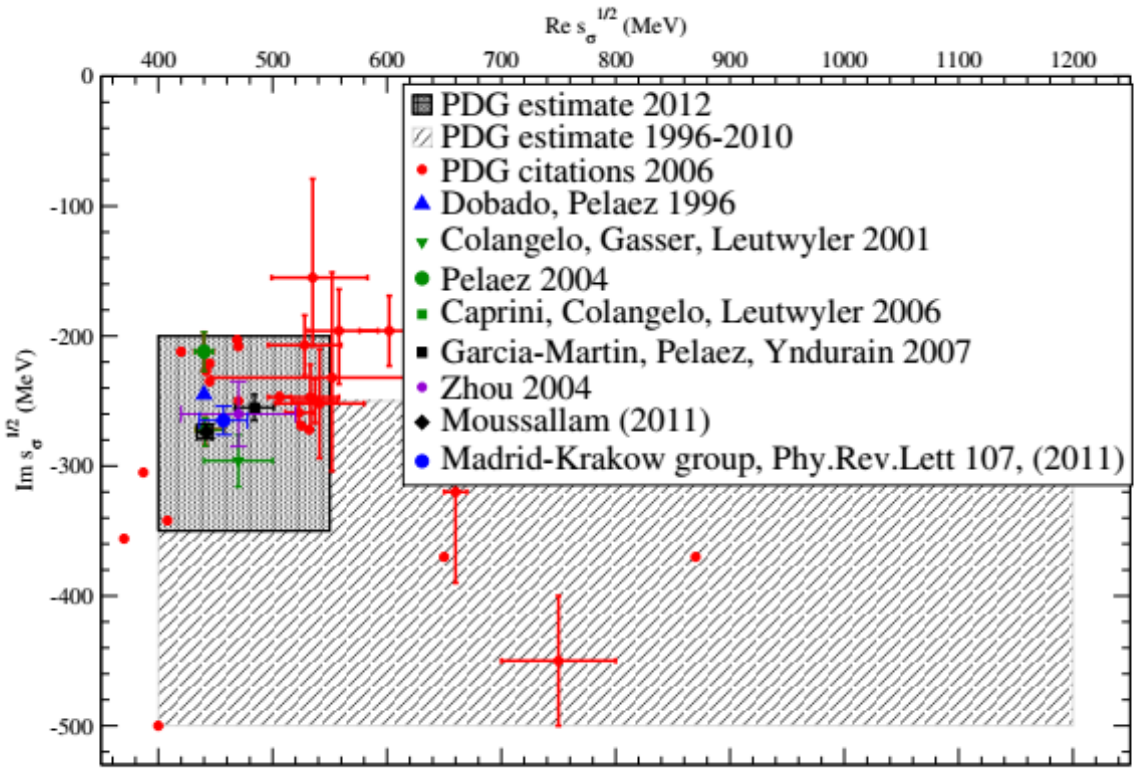
$$g = \frac{\Gamma_{QCM}(f_0 \to \pi\pi)}{\Gamma_{\exp}(f_0 \to \pi\pi)}$$

$$R = -\frac{5\cos\delta_s[\int_0^\infty du a(u) - 4H\int_0^\infty du u b(u)]}{(5\cos\delta_s - \sqrt{2}\sin\delta_s)a(0)} = 1$$

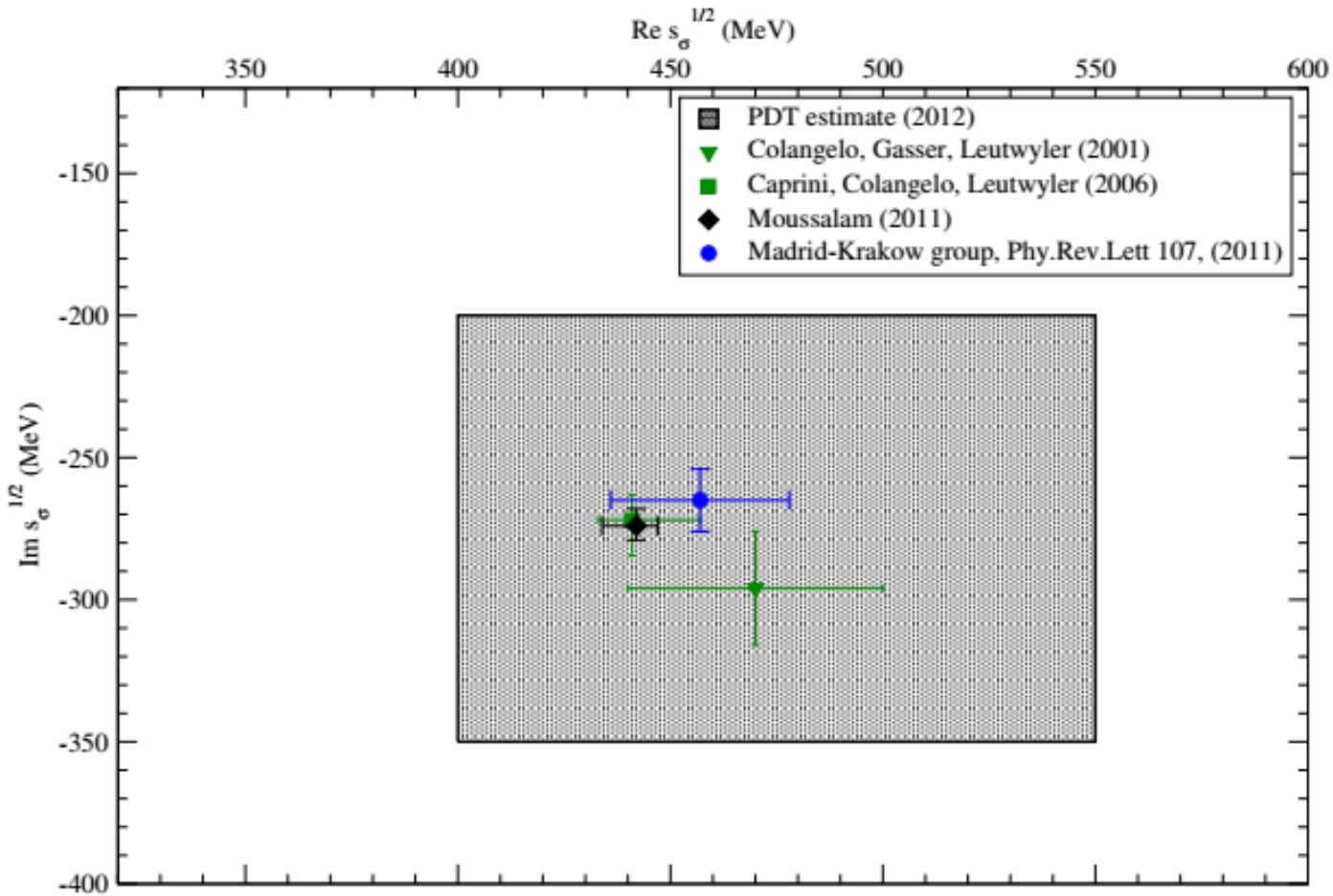


$H = 0.54, \qquad \delta_s = 17^\circ$

MASS OF LIHGTEST SCALAR



MASS OF LIHGTEST SCALAR



MASS OF LIHGTEST SCALAR

$$M_{\pi\pi}(s,t,u) = \delta^{ab}\delta^{cd}A(s,t,u) + \delta^{ac}\delta^{bd}A(t,u,s) + \delta^{ad}\delta^{bc}A(u,s,t)$$

a, b, c, d *- isotopic indexes*

$$A(s,t,u) = A(s,u,t)$$

$$A(s,t,u) = I^{\pi\pi}_{box}(s,t,u) + S^{\pi\pi}(s,t,u) + V^{\pi\pi}(s,t,u)$$

$$I^{\pi\pi}_{box}(s,t,u) = I_{box}(s,t,u,m_\pi^2,m_\pi^2,m_\pi^2,m_\pi^2,\Lambda_n,\Lambda_n,\Lambda_n,\Lambda_n)$$

$$S^{\pi\pi}(s,t,u) = F^2_{S\pi\pi}(s)\left(\frac{\cos^2\delta_s}{\Pi_s(s)-\Pi_s(m_1^2)} + \frac{\sin^2\delta_s}{\Pi_s(s)-\Pi_s(m_2^2)}\right) + \\ + F^2_{S\pi\pi}(t)\left(\frac{\cos^2\delta_s}{\Pi_s(t)-\Pi_s(m_1^2)} + \frac{\sin^2\delta_s}{\Pi_s(t)-\Pi_s(m_2^2)}\right)$$

$$F_{S\pi\pi}(x) = F_{S\pi\pi}(x, m_\pi^2, m_\pi^2)$$

m_1 *-mass of* $f_0(500)$, m_2 *-mass of* $f_0(980)$

MASS OF LIHGTEST SCALAR

$$V^{\pi\pi}(s,t,u) = \frac{1}{\Pi_1(m_\rho^2)}\left[\left(F^-(s)\right)^2(t-u) + \left(F^-(t)\right)^2(s-u)\right]$$

The amplitudes for the three possible channels $(I = 0, 1, 2)$

$$T^0(s,t,u) = 3A(s,t,u) + A(t,s,u) + A(u,t,s)$$

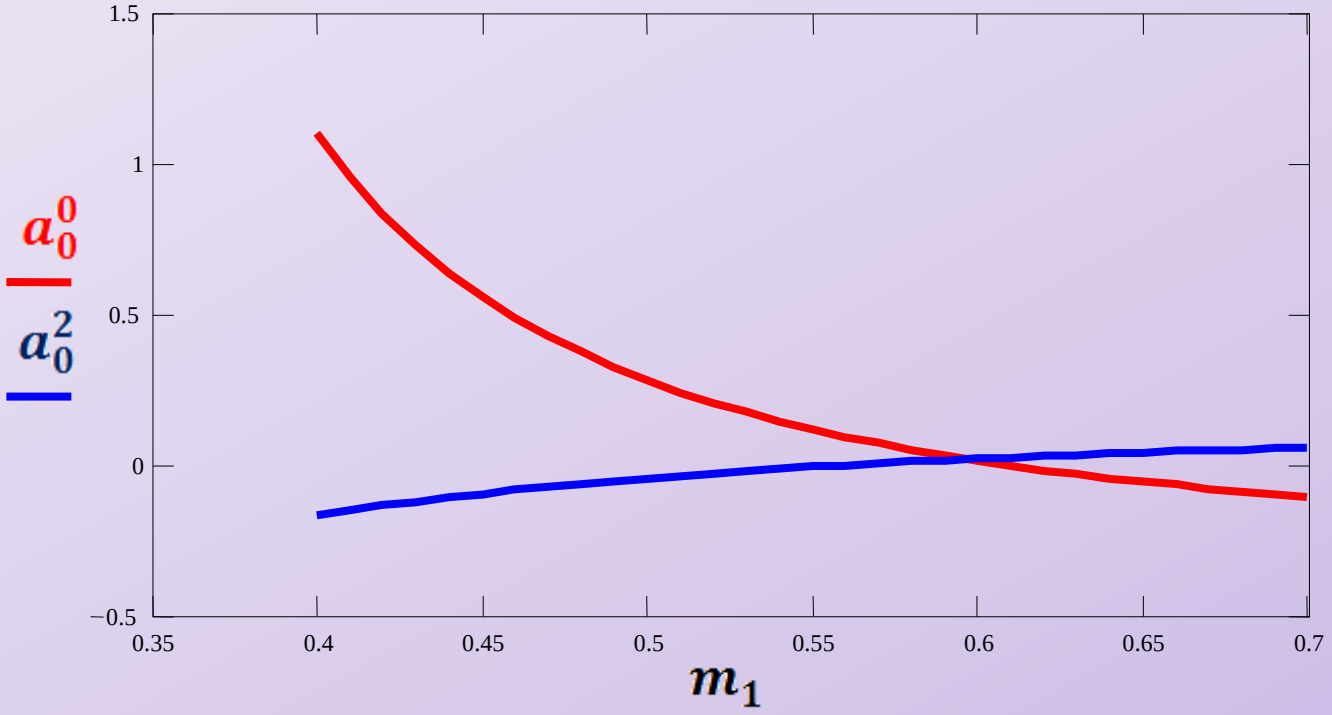
$$T^1(s,t,u) = A(t,s,u) - A(u,s,t)$$

$$T^2(s,t,u) = A(t,s,u) + A(u,s,t)$$

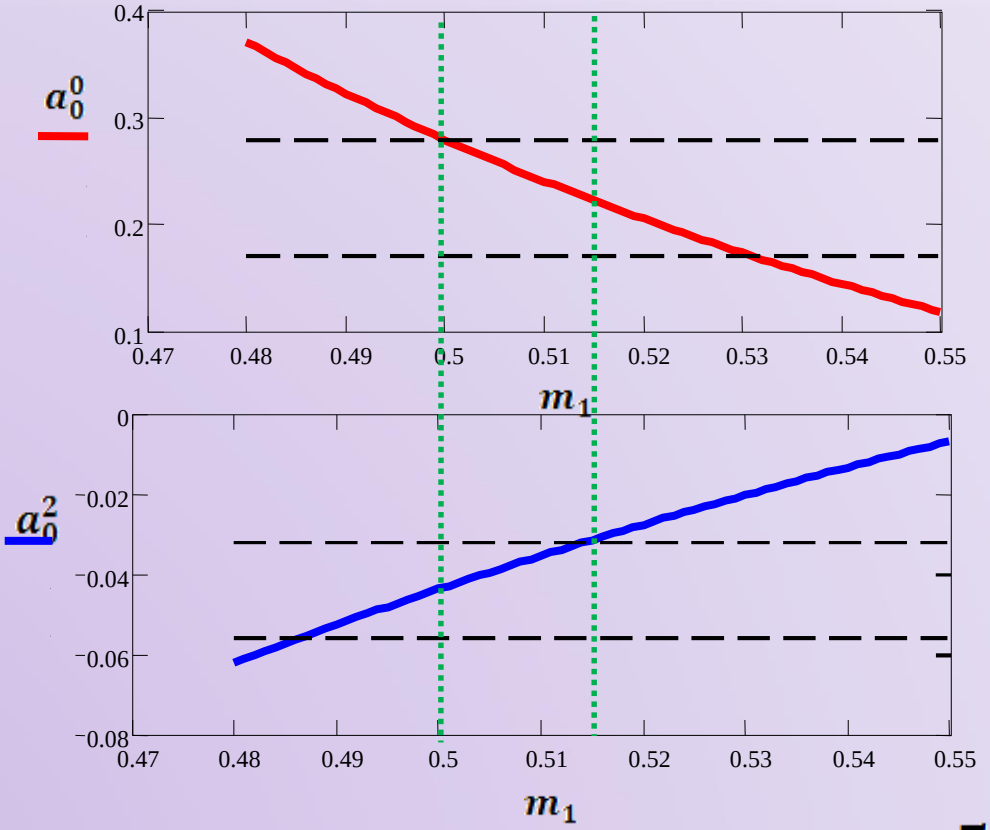
The scattering length

$$a^I = \frac{1}{32\pi}T^I(4m_\pi^2, 0, 0)$$

MASS OF LIHGTEST SCALAR



MASS OF LIHGTEST SCALAR



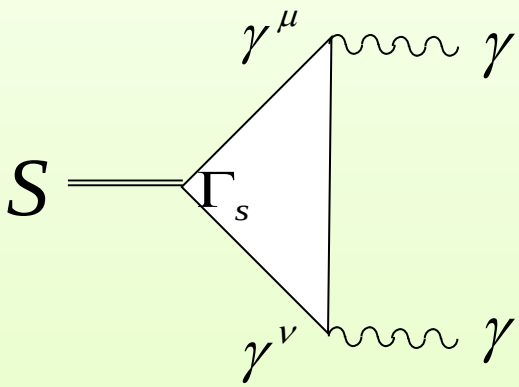
$m_1 = 500MeV$

$a_0^0 = 0.28, a_0^2 = -0.044$

$m_1 = 515MeV$

$a_0^0 = 0.22, a_0^2 = -0.032$

Radiative decay



$$M(S \to \gamma\gamma) = [g^{\mu\nu} q_1 q_2 - q_1^{\mu} q_2^{\nu}] g_{s\gamma\gamma}(m_s^2)$$

$$g_{s\gamma\gamma}(m_s^2) = \alpha \sqrt{6h_s(H)} \frac{1}{\Lambda} SpQ^2 \lambda_s [F_{SVV}^1(m_s^2) + HF_{SVV}^2(m_s^2)]$$

$$\alpha = \frac{e^2}{4\pi}, \quad Q = \frac{1}{3} diag(2,-1,-1)$$

$$F_{SVV}^1(x) = \frac{x}{4}\int_0^1 du a(-u\frac{x}{4})(1+u)\ln\frac{1+\sqrt{1-u}}{1-\sqrt{1-u}}; \quad F_{SVV}^2(x) = \frac{x}{4}\int_0^1 du b(-u\frac{x}{4}) u \ln\frac{1+\sqrt{1-u}}{1-\sqrt{1-u}}$$

$$\Gamma(S \to \gamma\gamma) = \frac{m_s^2}{64\pi} g_{s\gamma\gamma}^2(m_s^2)$$

NUMERICAL RESULTS

MODE	THEORY	EXPERIMENT
$f_0(980) \to \pi\pi$	$22\ MeV$	26 ± 5 MeV
$a_0(980) \to \eta\pi$	$56\ MeV$	54 ± 7 MeV
$f_0(550) \to \pi\pi$	$357\ MeV$	365 ± 2 MeV
$f_0(980) \to \gamma\gamma$	$0{,}368\ KeV$	$0{,}39^{+0{,}08}_{-0{,}11} KeV$
$a_0(980) \to \gamma\gamma$	$0{,}43\ KeV$	$0{,}30 \pm 0{,}10\ KeV$
$f_0(550) \to \gamma\gamma$	$0{,}33\ KeV$	

MULTY QUARK MODEL

- Scalars as diquark- antidiquark states

$$f_0(600)(\sigma) \Rightarrow (ud)(\bar{u}\bar{d})$$

$$K_0^*(800),(\kappa) \Rightarrow (su)(\bar{u}\bar{d});\ (sd)(\bar{u}\bar{d}) + c.c$$

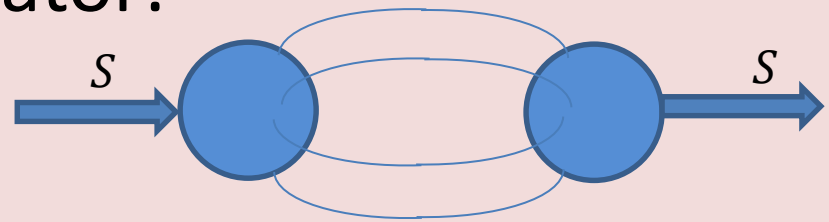
$$f_0(980) \Rightarrow \frac{(su)(\bar{s}\bar{u}) + (sd)(\bar{s}\bar{d})}{\sqrt{2}}$$

$$a_0(980) \Rightarrow (su)(\bar{s}\bar{d});\ \frac{(su)(\bar{s}\bar{u}) - (sd)(\bar{s}\bar{d})}{\sqrt{2}};$$

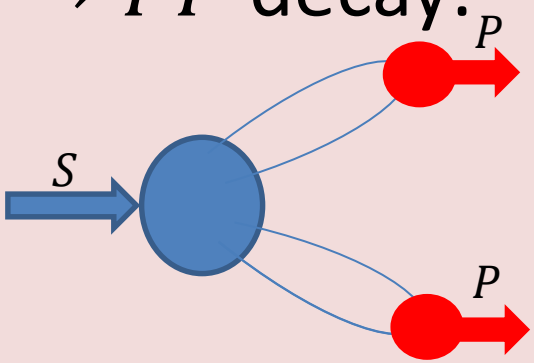
$$(sd)(\bar{s}\bar{u})$$

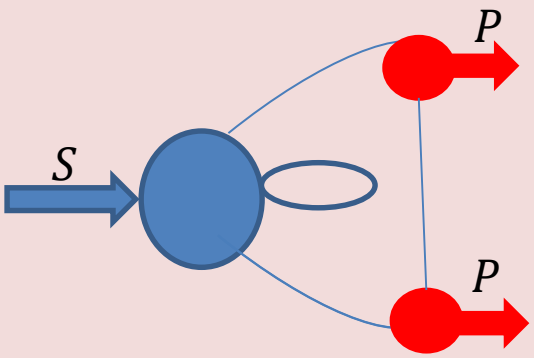
MULTY QUARK MODEL

- Mass operator:



- $S \to PP$ decay:





Relativistic constituent quark model with infrared confinement

Michail A. Ivanov et all (2010)

$$L_{int} = g_M M(x) J_M(x)$$

g_M is determined from the compositeness condition

$$Z_M^{\frac{1}{2}} = \langle M_{bare} | M_{dressed} \rangle = 0$$

$$Z_M = 1 - \frac{3 g_M^2}{4\pi^2} \widetilde{\Pi}'(m_M^2) = 0$$

For two-quark states

$$J_M(x) = \int dx_1 \int dx_2 \, \delta\left(x - \frac{1}{2}(x_1 + x_2)\right) \Phi_M\left((x_1 - x_2)^2\right) \bar{q}_a(x_1) \Gamma_M q_a(x_2)$$

$$\Phi_M(-l^2) = \exp\left(\frac{l^2}{\Lambda_M^2}\right)$$

Relativistic constituent quark model with infrared confinement

❑For four-quark states

$$J_M(x) = \int dx_1 \int dx_2 \int dx_3 \int dx_4\, \delta\left(x - \sum_{i=1}^{4} \varpi_i x_i\right) \Phi_M\left(\sum_{i<j} (x_i - x_j)^2\right) \cdot$$

$$\cdot \frac{1}{\sqrt{2}} \varepsilon_{abc} \varepsilon_{dec} \left\{ \left[q_a(x_4) C \Gamma_{M_1} q_b(x_1) \right] \left[\bar{q}_d(x_3) \Gamma_{M_2} C \bar{q}_d(x_2) \right] + \Gamma_{M_1} \leftrightarrow \Gamma_{M_2} \right\}$$

$$\varpi_i = \frac{m_{q_i}}{\sum m_{q_i}}$$

$$C = \gamma^0 \gamma^2 \qquad C = C^\dagger = C^{-1} = -C^T$$

$$C \Gamma^T C^{-1} = \pm \Gamma; \begin{matrix} + \text{ for } \mathrm{S, P}, A \\ - \text{for } \mathrm{V}, T \end{matrix}$$

Relativistic constituent quark model with infrared confinement

Mass operator

$$\Pi(x-y) = i\int dx_1 \cdots \int dx_4\, \delta\left(x - \sum x_i \varpi_i\right) \Phi\left(\sum_{i<j} \left(x_i - x_j\right)^2\right)$$

$$\int dy_1 \cdots \int dx_y\, \delta\left(y - \sum y_i \varpi_i\right) \Phi\left(\sum_{i<j} \left(y_i - y_j\right)^2\right) \langle 0 | T\{J_q(x_1, \cdots, x_4) J_q(y_1, \cdots, y_4)\} | 0 \rangle$$

Jucobi coordinates:

$$x_i = x + \sum_{j=1}^{3} w_{ij} \rho_j^x \qquad y_i = y + \sum_{j=1}^{3} w_{ij} \rho_j^y$$

$$\Pi(x-y) = i\int d^3\vec{\rho}_x \Phi(\vec{\rho}_x^2) \int d^3\vec{\rho}_y \Phi(\vec{\rho}_y^2) \langle 0 | T\{J_q(x_1, \cdots, x_4) J_q(y_1, \cdots, y_4)\} | 0 \rangle$$

Relativistic constituent quark model with infrared confinement

$$\widetilde{\Pi}(p,p') = \int dx e^{-ipx} \int dy\, e^{-p'y} \Pi(x-y)$$

$$\widetilde{\Pi}(p,p') = (2\pi)^4 \delta^{(4)}(p-p') \widetilde{\Pi}(p^2)$$

$$\widetilde{\Pi}(p^2) = 12 \prod_{i=1}^{3} \left[\frac{d^4k}{(2\pi)^4 i} \right] \widetilde{\Phi}(-\vec{\omega}^2) \cdot$$

$$\cdot\, Tr\{S_1(k_1 - w_1 p) S_3(k_3 + w_3 p)\} \cdot Tr\{S_2(k_2 - w_2 p) S_4(k_1 + k_2 - k_3 + w_4 p)\}$$

$$\vec{\omega}^2 = \frac{1}{2}[k_1^2 + k_2^2 + k_3^2 + k_1 k_2 - k_1 k_3 - k_2 k_3]$$

Relativistic constituent quark model with infrared confinement

Free local fermion propagator for the constituent quark:

$$S_q(k) = \frac{1}{m_q - \hat{k} - i\epsilon}$$

$$\frac{1}{m^2 - p^2} = \int_0^\infty d\alpha \, exp[-\alpha(m^2 - p^2)]$$

$$\infty \to \frac{1}{\lambda^2}:$$

$$\int_0^{\frac{1}{\lambda^2}} d\alpha \, exp[-\alpha(m^2 - p^2)] = \frac{1 - exp\left[-\frac{m^2 - p^2}{\lambda^2}\right]}{m^2 - p^2}$$

Relativistic constituent quark model with infrared confinement

For every loop integral:

$$\Pi = \int_0^\infty d^n\alpha F(\alpha_1, \alpha_2, \cdots, \alpha_n) = \int_0^{\frac{1}{\lambda^2}} dt\, t^{n-1} \int_0^1 d^n\alpha \delta\left(1 - \sum \alpha_n\right) \alpha F(t\alpha_1, t\alpha_2, \cdots, t\alpha_n)$$

Model parameters:

Quark masses: $m_{u,d} = 0{,}235\ GeV; m_s = 0{,}424\ GeV$

Cut-off parameter $\lambda = 0{,}181$