Redshift from a binary that moves in gravitational field of a supermassive Schwarzschild black hole

Stanislav Komarov, Alexander Gorbatsievich, Alexander Tarasenko

Belarusian State University,
Physics Faculty, Department of Theoretical Physics and Astrophysics

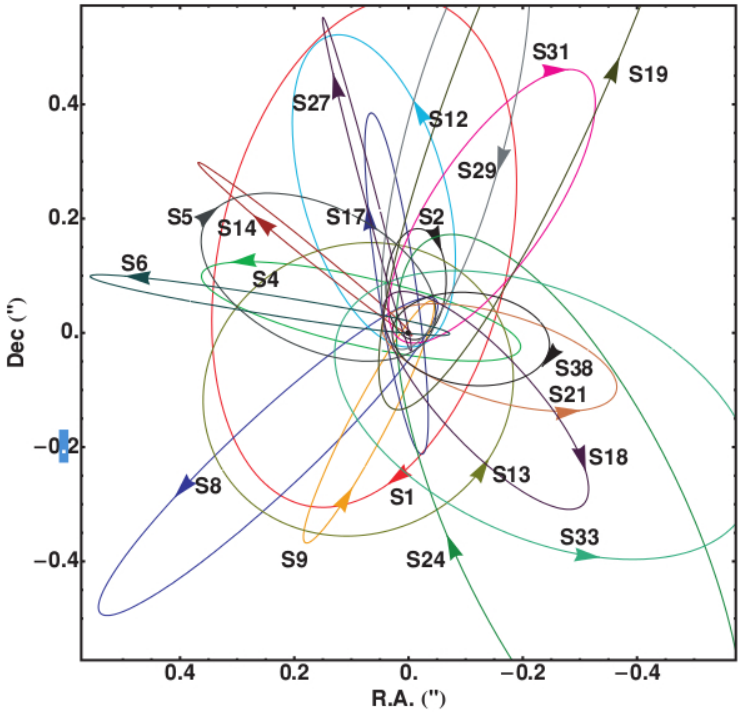
Jipnr Sosny, 16 May, 2017

Introduction.
Observations of the Galactic center

- Observational evidences have shown that a supermassive black hole with a mass $M \approx 10^6 M_\odot$ exists in the center of our galaxy (Sgr A^*). Also there are a large population of compact binary systems in this region.

Introduction.
Observations of the Galactic center

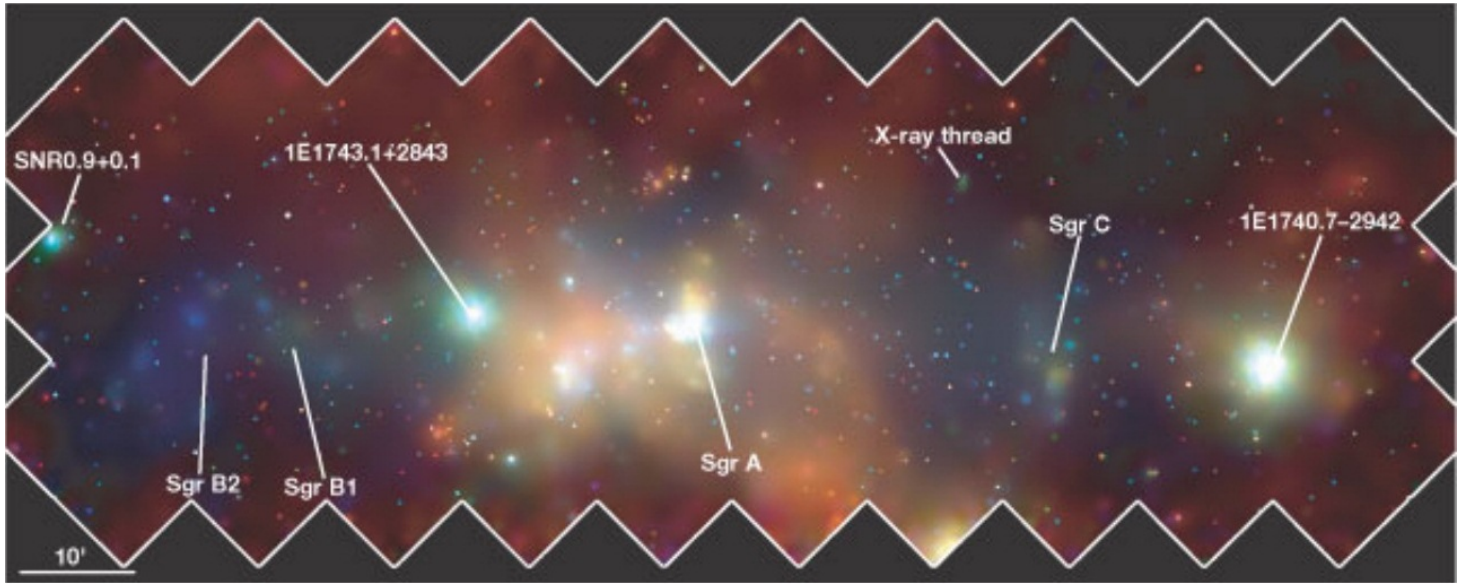
- Observational evidences have shown that a supermassive black hole with a mass $M \approx 10^6 M_\odot$ exists in the center of our galaxy (Sgr A^*). Also there are a large population of compact binary systems in this region.
- The results of the observations of the stars in the innermost arcsecond (the so-called S-stars) of our galaxy give as possibilities for testing theories of gravity (the pericenter of the star S2 lies within the 0,01").



$1''$ to correspond $0,04\,\mathrm{pc} \approx 400000 \cdot M$.

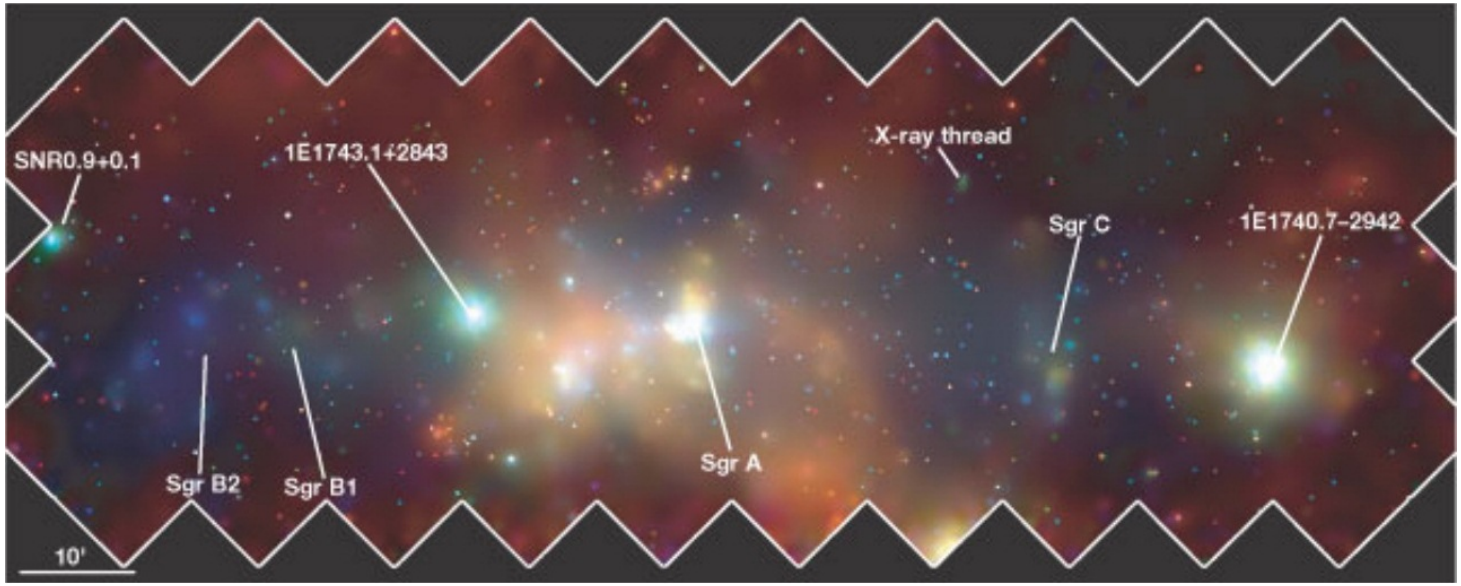
Introduction.
Observations of the Galactic center

- The astrophysical observations gives also evidences for the existence of the X-ray pulsars in the closed vicinity of the Galactic Center. This gives us possibilities to study gravity in the strong field regime.



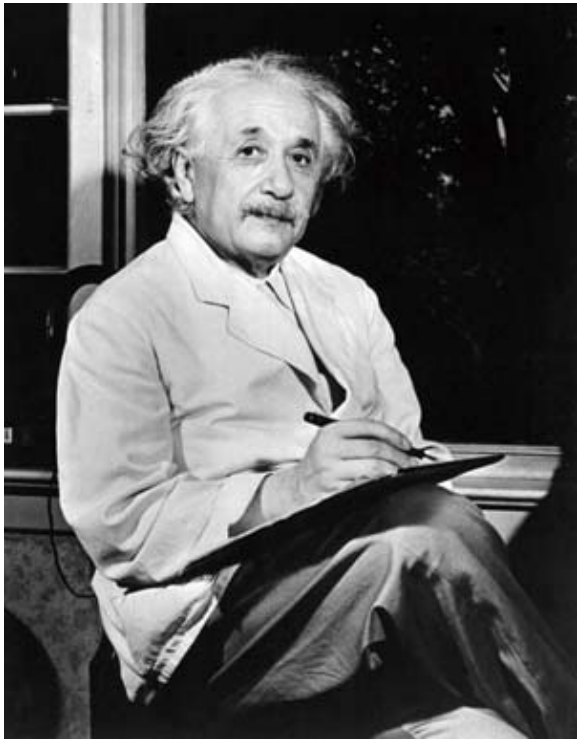
Introduction.
Observations of the Galactic center

- The astrophysical observations gives also evidences for the existence of the X-ray pulsars in the closed vicinity of the Galactic Center. This gives us possibilities to study gravity in the strong field regime.



- Do to the observational difficulties it is possible to restrict the radiation only for a certain sources. But the future technologies, such as MAXIM can give us more possibilities to studying of these objects.

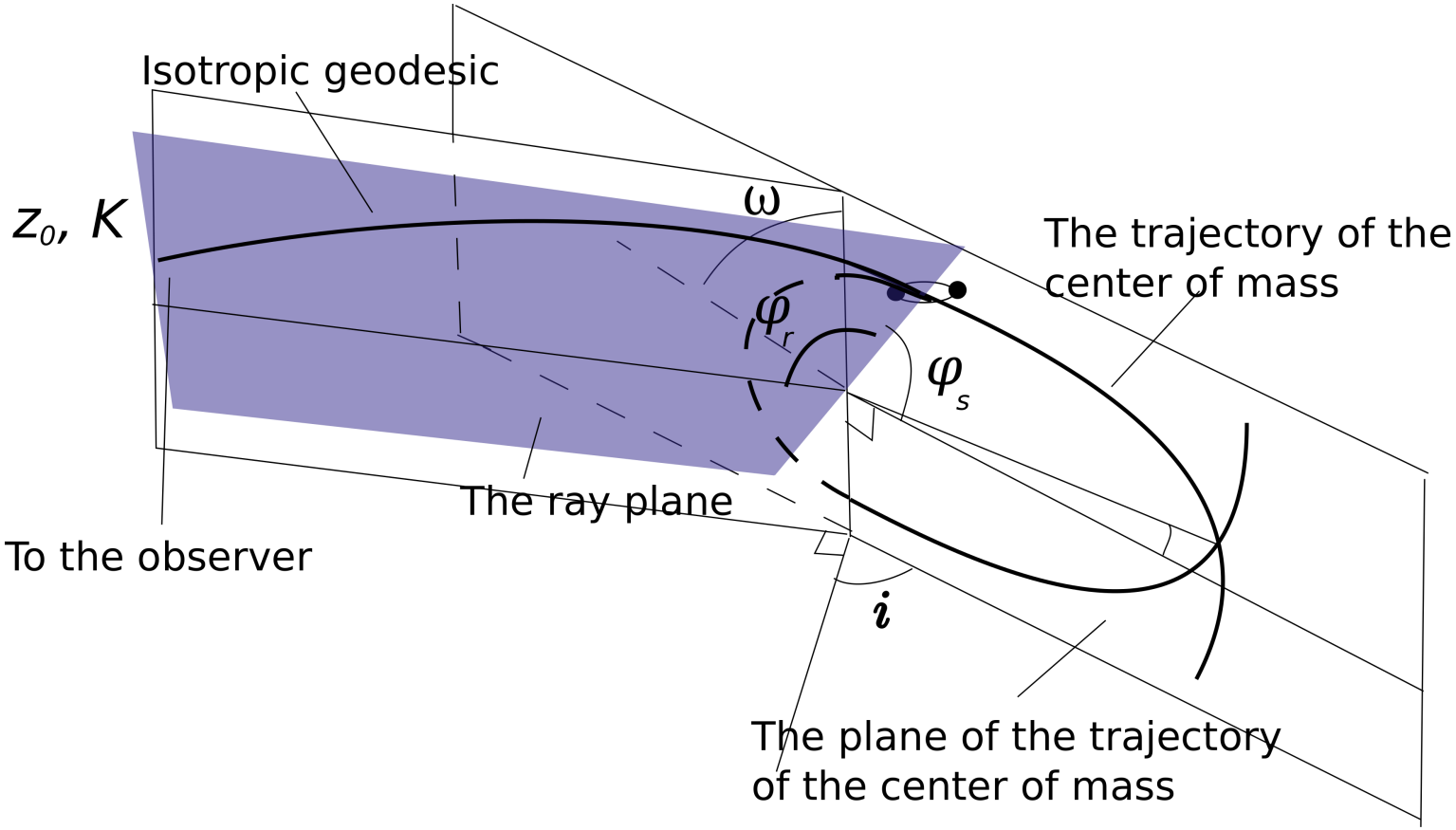
Introduction.
General Theory of Relativity



$$R_{ij} - \frac{1}{2} R g_{ij} = \frac{8\pi G}{c^4} T_{ij} \Rightarrow$$
$$T^{ji}{}_{;i} = 0 \text{ — equations of motion.} \tag{1}$$

The formulation of the problem

The purpose of the present research is to work out methods to study time of arrival of the pulses or redshift of the emitted light by source in binary system as a function of parameters of the motion of the binary pulsar.



The equations of motion

Equations of motion of the center of mass

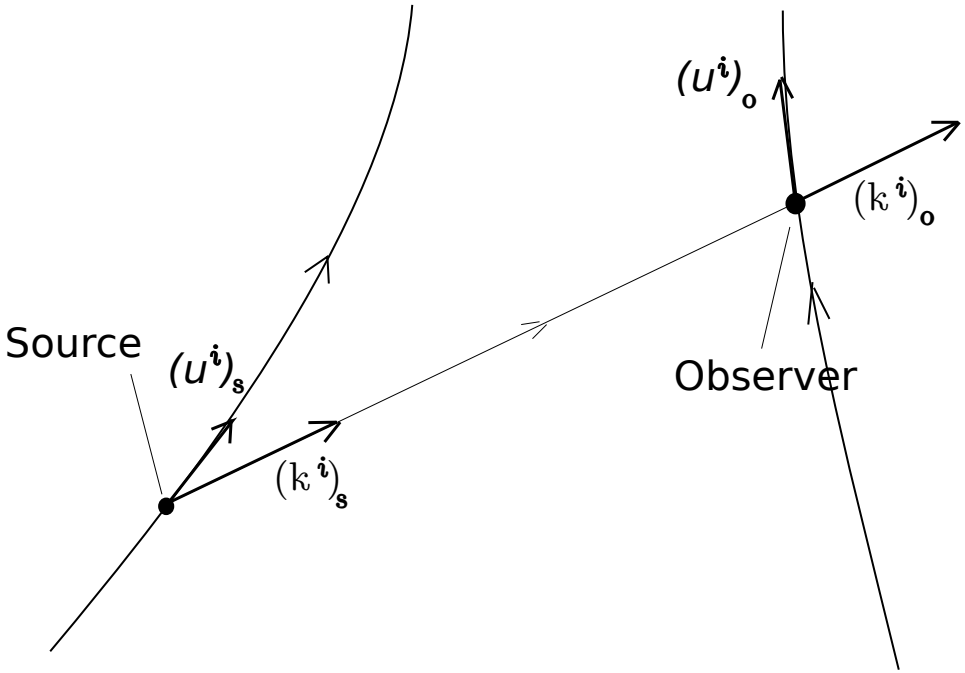
$$(m_1+m_2)\frac{dV^{(\kappa)}}{dT} = (m_1+m_2)(2\varepsilon^{(\kappa)}{}_{(\mu)(\nu)}V^{(\mu)}\omega^{(\nu)} - 2cR^{(\kappa)}{}_{(\nu)(\mu)(4)}X^{(\mu)}V^{(\nu)} + 2D^{(\kappa)}{}_{(\nu)}X^{(\nu)} - W^{(\kappa)}) - 2mcR^{(\kappa)}{}_{(\nu)(\mu)(4)}x^{(\mu)}v^{(\nu)}\,. \tag{2a}$$

Equations of the relative motion of the components of the binary

$$\frac{dv^{(\kappa)}}{dT} = \left(\frac{G(m_1+m_2)}{r}\right)_{,(\kappa)} - 2\varepsilon^{(\kappa)}{}_{(\alpha)(\tau)}\omega^{(\alpha)}v^{(\tau)} - \frac{2c(m_2-m_1)}{(m_1+m_2)}R^{(\kappa)}{}_{(\nu)(\mu)(4)}x^{(\mu)}v^{(\nu)} + 2D^{(\kappa)}{}_{(\nu)}x^{(\mu)} + \frac{2}{3}c(R_{(\alpha)(\mu)(\kappa)(4)} + R_{(\alpha)(\kappa)(\mu)(4)})(X^{(\mu)}v^{(\alpha)} + x^{(\mu)}V^{(\alpha)})\,. \tag{2b}$$

The equations (2a) and (2b) are given in article:
Gorbatsievich, A. The equations of motion of compact binaries in the neighbourhood of supermassive black hole / A. Gorbatsievich, A. Bobrik // The Sun, The Stars, The Universe and General Relativity: International Conference in Honor of Ya. B. Zeldovich's 95th Anniversary. — Vol. 1205. — AIP, 2010. — Pp. 87-96.

Time of arrival of the pulses and the redshift of the received light



$$z = \frac{\delta\lambda}{\lambda} = \frac{(k^i u_i)_s}{(k^i u_i)_o} - 1.$$

$$t_{TOA}^{(N)} = t_{TOA}^{(N-1)} + T_p(z+1) = t_{TOA}^{(N-1)} + T_p \frac{(k^i u_i)_s}{(k^i u_i)_o}\,.$$

Here T_p — is the pulsar period in the reference frame of the pulsar, z is the redshift, $t_{TOA}^{(j)}$ — is the time of arrival of the j th pulse.

Calculation of the redshift. Single source in a Schwarzschild spacetime

Consider a single source that moves in the field of a Schwarzschild black hole. For the isotropic geodesic we have:

$$\frac{1}{r_r(\phi_r)} = \frac{1}{P} - \frac{Qk^2}{2PM}\,\mathrm{cn}^2\left(\left[\frac{\phi_r}{2}\sqrt{\frac{Q}{P}} + \mathrm{F}\left(\arccos\left(\sqrt{\frac{2M}{Qk^2}}\right), k\right)\right], k\right);$$

$$\text{where } k = \sqrt{\frac{Q - P + 6M}{2Q}},\ Q = \sqrt{P^2 + 4MP - 12M^2}\,.$$

To determine the closest approach distance of the ray it is necessary to solve the following non-linear equation:

$$r_r(\phi_r, P) = r \Rightarrow P(\phi_r, r)\,.$$

Calculation of the redshift. Single source in a Schwarzschild spacetime

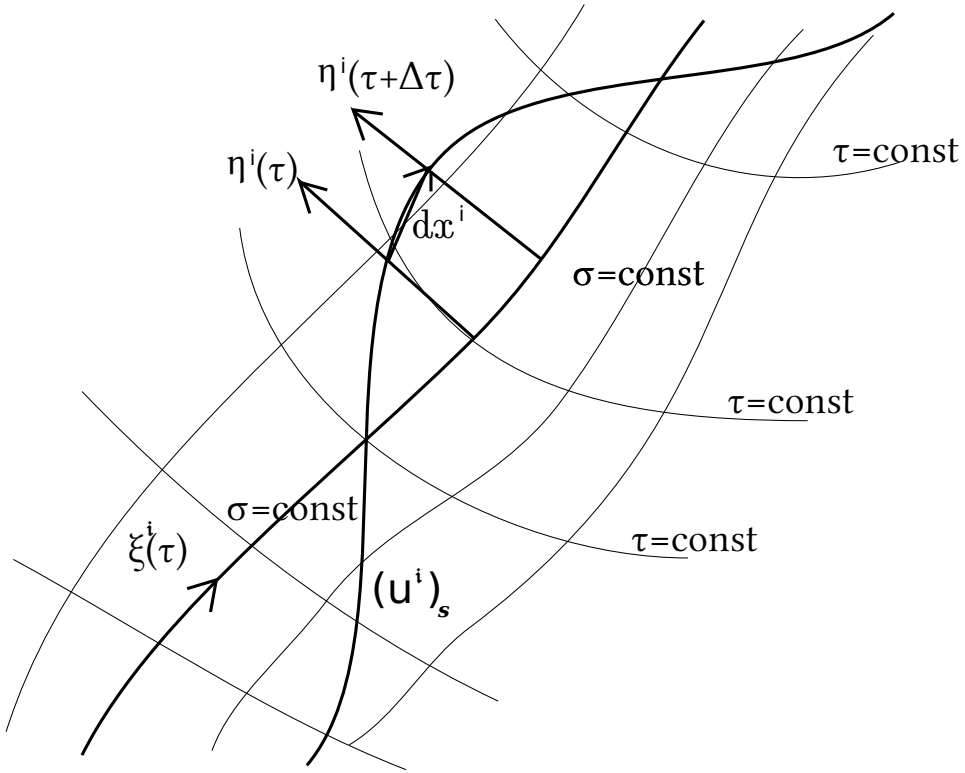
Then, the redshift z_0 of the received light can be expressed as:

$$z_0 = \pm\sqrt{\left(\frac{E^2 r}{r-2M} - (1 + L^2/r^2)\right)\left(\frac{r}{r-2M} - P^3/(r^2(P-2M))\right)} + \frac{L\sqrt{P^3}}{r^2\sqrt{P-2M}}\cos\psi + \frac{E}{1-2M/r} - 1. \tag{3}$$

It is possible with hight accuracy consider a worldline of the center of mass of the binary system as geodesic. Therefore, redshift of the source (non-real) that is located at the center of mass has the form of (3).

Calculation of the redshift. Decomposition of the redshift

Consider a source in a binary system. We can decompose redshift into series of v and ϱ :



$$1 + z(\tau) = -\frac{k_i(x^l)dx^i}{\omega_0\sqrt{-g_{km}(x^l)dx^k dx^m}} = 1 + z_0(\tau + \tau_{ret}) - \frac{d}{d\tau}(k_i\eta^i\varrho)$$
$$- \frac{1}{2}\frac{d}{d\tau}(k_{m;s}\eta^m\eta^s\varrho^2) - \frac{1}{2}R_{lijm}u^i_C u^j_C\eta^l\eta^m\sigma_P^2 + O(\varrho^3, \varrho^2 v, v^2).$$

Calculation of the redshit. Time of the retardation

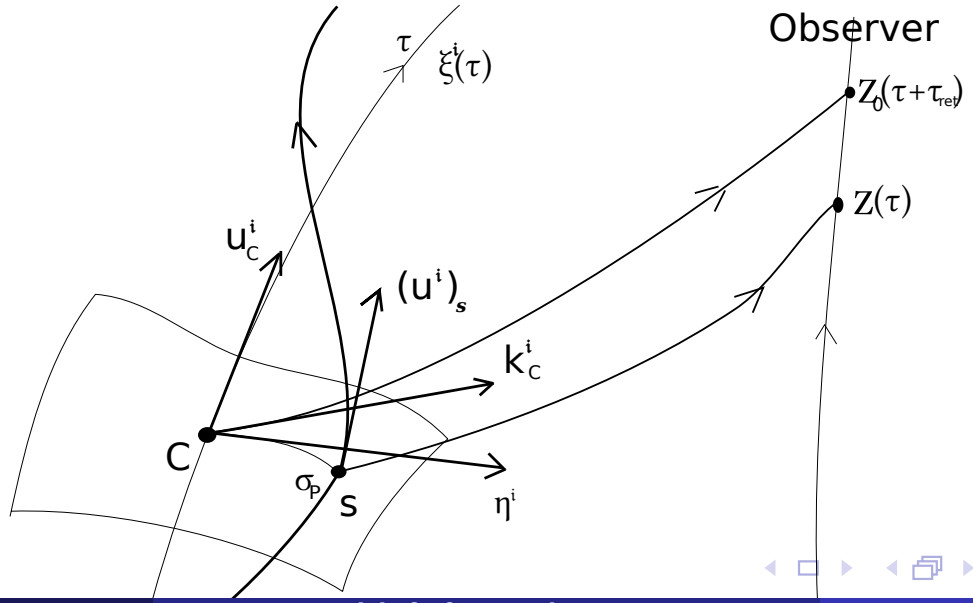
$$k_i dx^i = const, \quad \text{along an isotropic geodesic.}$$

Here dx^i is the coordinate distance between two close geodesics. Applying this equation to the points C and P yields:

$$k_i \frac{dx^i}{d\sigma} d\sigma = -(k_i u^i)_o dt = \omega_0 dt \Rightarrow \omega_0 t_{ret} = \int_0^{\varrho} k_i \frac{dx^i}{d\sigma} d\sigma \approx k_i \eta^i \varrho + \frac{1}{2} k_{i;j} \eta^i \eta^j \varrho^2 .$$

τ_{ret} can be expressed as:

$$\omega_0 \tau_{ret} = \omega_0 \int_0^{t_{ret}} \frac{dt}{z+1} \approx \frac{k_i \eta^i}{z_0+1} \varrho + \frac{1}{2} \frac{k_{i;j} \eta^i \eta^j}{z_0+1} \varrho^2 - \frac{1}{2} \frac{(k_i \eta^i)^2}{(z_0+1)^3} \frac{dz_0}{d\tau} \varrho .$$



Expression for the redshift

$$1+z=(1+z_0)\left[1-\frac{d}{d\tau}(n_i\eta^i\varrho)-\frac{1}{2}\frac{d}{d\tau}(n_{i;j}\eta^i\eta^j\varrho^2)+\frac{1}{2}(n_i\eta^i)^2\frac{d}{d\tau}\left(\frac{1}{z_0+1}\frac{dz_0}{d\tau}\right)\varrho^2-\right.$$
$$\left.\frac{1}{2}R_{lijm}u^i_C u^j_C\eta^l\eta^m\varrho^2\right]+O(\varrho^3,\varrho^2 v,v^2)\,. \tag{4}$$

Here $n^i=\dfrac{k^i}{\omega_0(z_0+1)}$ and $n_i\eta^i\varrho=n_{(\alpha)}X^{(\alpha)}$. Also we use the equations $(k_i/\omega_0)_{;j}-(k_j/\omega_0)_{;i}=0\,.$

$$t=\int\limits_0^{\tau_S}(z(\tau')+1)d\tau'.$$

Where

$$\frac{d\tau}{d\tau_S}=1-R_{lijm}u^i_C u^j_C\eta^l\eta^m\varrho^2+O(\varrho^3,\varrho^2 v,v^2)\,.$$

Alexander Gorbatsievich, Stanislav Komarov and Alexander Tarasenko. Optical appearance of a compact binary system in the neighbourhood of supermassive black hole. — arXiv:1702.08381 (2017)

Example of the calculation of the redshift

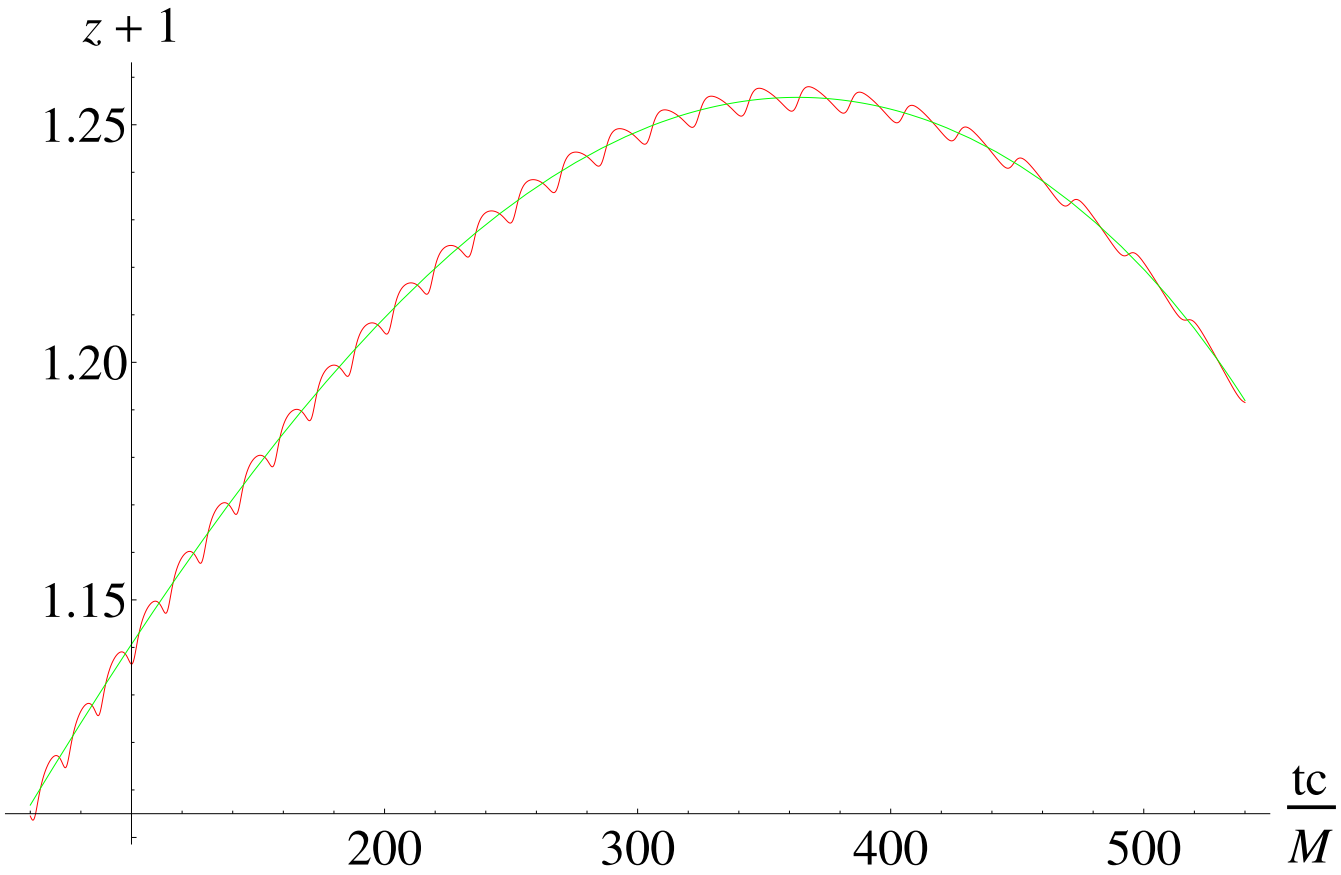


FIG : The redshift as a function of the proper time of the source. The chosen parameters of the motion are: angular momentum rer unit mass, $L = 5,152M$; energy per unit mass, $E = 0,9643$; orbital inclination, $i = 1$ rad; mass of the source, $m_1 = 8,89 \cdot 10^{-7} M$; mass of the companion star, $m_2 = 4,45 \cdot 10^{-7} M$; initial relative distances, $x^i = \{0; 0,02M; 0\}$; initial relative velocities, $v^i/Mc = \{0,0083; 0; 0,0057\}$.

Estimation of the derivatives of the redshift z_0

$$\frac{\mathrm{d}z_0(\tau)}{\mathrm{d}\tau} = B_{ij}u^i u^j = \tilde{B}_{ij}u^i u^j + 2B_{sj}N^s k_i u^i u^j + k_i k_j B_{sm} N^s N^m u^i u^j .$$

Here $\tilde{B}_{ij} = h^s{}_i h^m{}_j B_{sm}$ — transversal part of the tensor B_{ij}, $h^s{}_i = g^s{}_i + N^s k_i + k^s N_i$, N^i — some isotropic vector that satisfies the relation $N^i k_i = -1$.
After calculating of these values we have:

$$\frac{\mathrm{d}z_0}{\mathrm{d}\tau} = O\left(\frac{1}{r^2}\right). \tag{5}$$

$$\frac{\mathrm{d}^2 z_0}{\mathrm{d}\tau^2} = O\left(\frac{1}{r^3}\frac{\mathrm{d}r}{\mathrm{d}\tau}, \frac{LD}{r^5}\right). \tag{6}$$

Tarasenko A. Reconstruction of a compact object motion in the vicinity of a black hole by its electromagnetic radiation / A. Tarasenko // Physical review D — 2010 — Vol. 81 — P. 123005

Raychaudhuri,s equation

$$\tilde{B}_{ij} = \frac{1}{2}\theta + \sigma_{ij} + \omega_{ij} \,. \tag{7}$$

$$\frac{\mathrm{d}\theta}{\mathrm{d}\lambda} = -\frac{1}{2}\theta^2 - \sigma^{ij}\sigma_{ij} + \omega^{ij}\omega_{ij} - R_{ij}k^i k^j \,. \tag{8}$$

Here $\theta = B^i{}_{;i}$, $\sigma_{ij} = \tilde{B}_{(ij)} - \frac{1}{2}\theta h_{ij}$, $\omega_{ij} = \tilde{B}_{[ij]}$.

$$\tilde{B}_{ij} \approx \frac{1}{2}\theta h_{ij};$$

$$\theta \approx \frac{2}{r}.$$

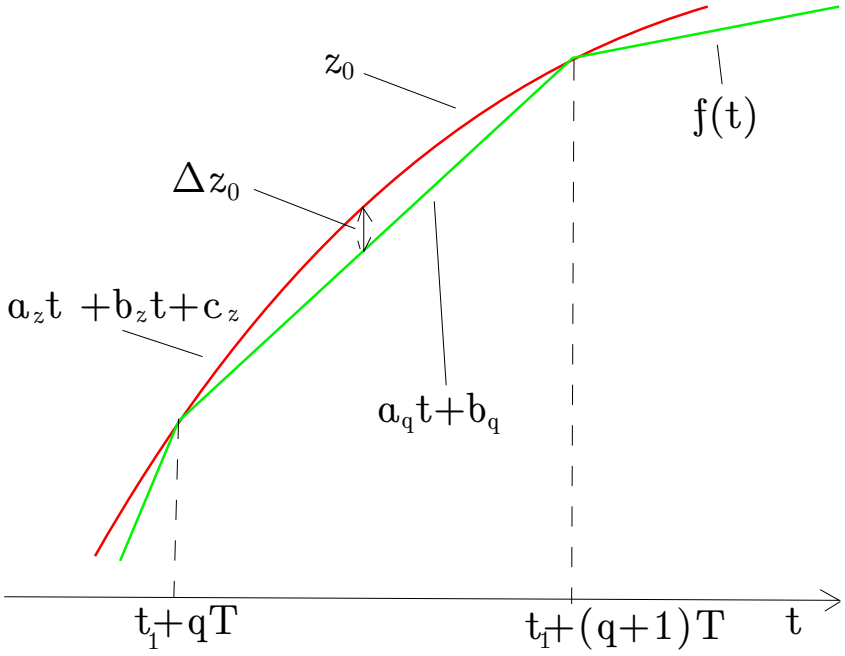


FIG : $\Delta z_0 = -a_z \frac{T^2}{4}$.

$$f(t) = a_q t + b_q, \quad t \in (t_1 + qT,\, t_1 + (q+1)T), \quad \forall q\,;$$

$$a_q(t_1 + (q+1)T) + b_q = a_{q+1}(t_1 + (q+1)T) + b_{q+1}; \quad q \in [0,\, n-1];$$

$$\int\limits_{t_1+qT}^{t_1+(q+1)T} \frac{\mathrm{d}t}{a_q t + b_q} = \int\limits_{t_1+qT}^{t_1+(q+1)T} \frac{\mathrm{d}t}{1 + z_t(t)}.$$

$$\Delta z_0 =\lesssim max\left(\frac{T^2}{r^3}(\mathrm{d}r/\mathrm{d}\tau),\ \frac{T^2 LD}{r^5}\right).$$

Thank you for you attention!