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The effective contour in the inverse Mellin transform for the structure function F_3

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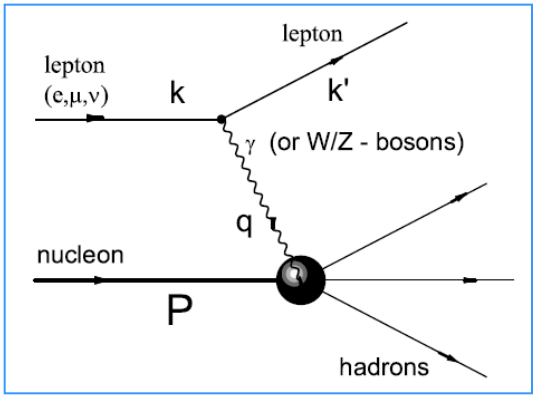
ABSTRACT

We consider the inverse Mellin transformation method for evaluation Q^2-dependence of the xF_3 structure function based on the efficient contour corresponding saddle point proposed by Kosower. We demonstrate the applicability of the integration contour calculated for Q_0^2 for evaluation of the xF_3 structure function at any experimental value of Q^2, including the target mass corrections and the higher term of the contour expansion.

Outline



- **Introduction and Motivation**
- **Overview of theoretical framework**
- **Theoretical result and Numerical estimations**
- **Conclusions**

Introduction

- The Inverse Mellin transformation method is
- one of the main methods, on which is based a
- comparision of QCD on of the of violation of scaling
- with experimental data on the structure functions
- and fragmtntations functions.
- The volume of experimental data which is included
- In the QCD analis quikly increases and their accu
- racy improves.

- Numerical processing of this experimental material
- recuires an effective application of the inverse Mellin
- transformation method . In tiis paper we study the
- Effectiveness of this method for the structure funcnion, which is the most simply and conveniently
- obyect QCD terms, a nonsinglet structure function does not contain contributions from sea quarks or gluons.
- The evalution of the structure-function nonsinglet
- moments has a simple form and, in the strong-interaction
- constant and by the values of the nonsinglet anomalous
- demension.

The Transformation of Mellin

- $\tilde{f}(z) = \int_0^1 f(x) * x^{z-1}\, dx$

- $f(x) = \frac{1}{2*\pi i} \int_{a-i\infty}^{a+i\infty} \tilde{f}(z)\, e^{u*z} dz, \quad u = -lnx$

- $f(x) = \frac{1}{2*\pi i} \int \tilde{f}(z)\, e^{u*z} dz$

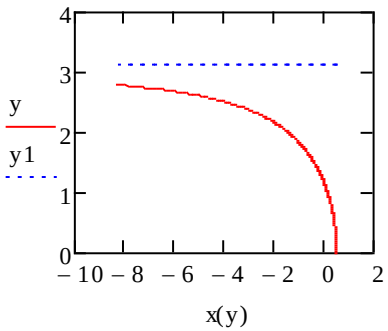
Method of Kosover

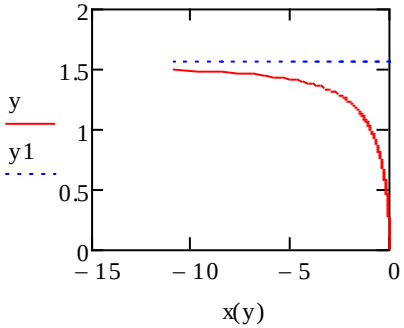
- F(z)=$e^{z*u}\tilde{f}(z)\,, z(y) = x(y) + iy$
- Im(F(z(y))=0, F'(z0)=0

Contour of Kosover

- $f\ (x) = x^{1/2}$, $\tilde{f}(z)=\frac{1}{z+\frac{1}{2}}$
- F(z)=$e^{z*u} * \frac{1}{z+\frac{1}{2}}$, u=-ln(x)
  - $z = -\frac{1}{2}$+yctg(uy)+iy

The Shape of Kosover





Overview of basic relations

- $f(x,Q^2)=\frac{1}{2\pi i}\int \tilde{f}(z,Q^2)e^{z*lnu}\mathrm{dz}$
- $f(x,Q^2)=\frac{1}{\pi}\int Re(-iF(z,Q^2)\mathrm{dz})$
- $F(z,Q^2)=\tilde{f}(z,Q^2)\,e^{z*lnu}$

$$F(z(t)) = F(c_0) - \frac{F''(c_0)}{2}t^2 + \frac{1}{6}\left[-iF^{(3)}(c_0) + 3iF''(c_0)x''(0)\right]t^3 + \cdots$$

$$z(t) = c_0 + it + \frac{F^{(3)}(c_0)}{6F''(c_0)}t^2$$

$$f(x,Q^2) = \frac{1}{\pi}\int_0^\infty \mathbf{Re}\left[\left(1 - i\frac{F^{(3)}(c_0)}{3F''(c_0)}t\right)F(z(t),Q^2)\right]dt$$

$$t \;=\; c_2\sqrt{u},$$

$$f(x,Q^2) = \frac{c_2}{2\pi}\int_0^\infty \frac{du}{\sqrt{u}}e^{-u}\mathbf{Re}\left[e^u\left(1 - ic_2c_3\sqrt{u} - 4ic_6c_2^3\,u^{3/2}\right)F(z(c_2\sqrt{u}),Q^2)\right]$$

$$z(c_2\sqrt{u}) = c_0 + ic_2\,\sqrt{u} + \frac{c_3}{2}c_2^2\,u + c_6c_2^4\,u^2\,,$$

$$c_3 = \frac{F^{(3)}(c_0)}{3F''(c_0)},\; c_4 = \frac{c_3F^{(4)}(c_0)}{12F''(c_0)},\; c_5 = \frac{F^{(5)}(c_0)}{120F''(c_0)},\text{ and } c_6 = \left(c_4 - c_5 - \frac{3}{8}c_3^3\right).$$

$$\int_0^\infty \frac{du}{\sqrt{u}}e^{-u}\,h(u) \simeq \sum_{j=1}^{n} w_j h(u_j)$$

$xF_3(x,Q_0)$

$$xF_3(x,Q_0^2) = A\, x^{\alpha}(1-x)^{\beta}(1+\gamma x)$$

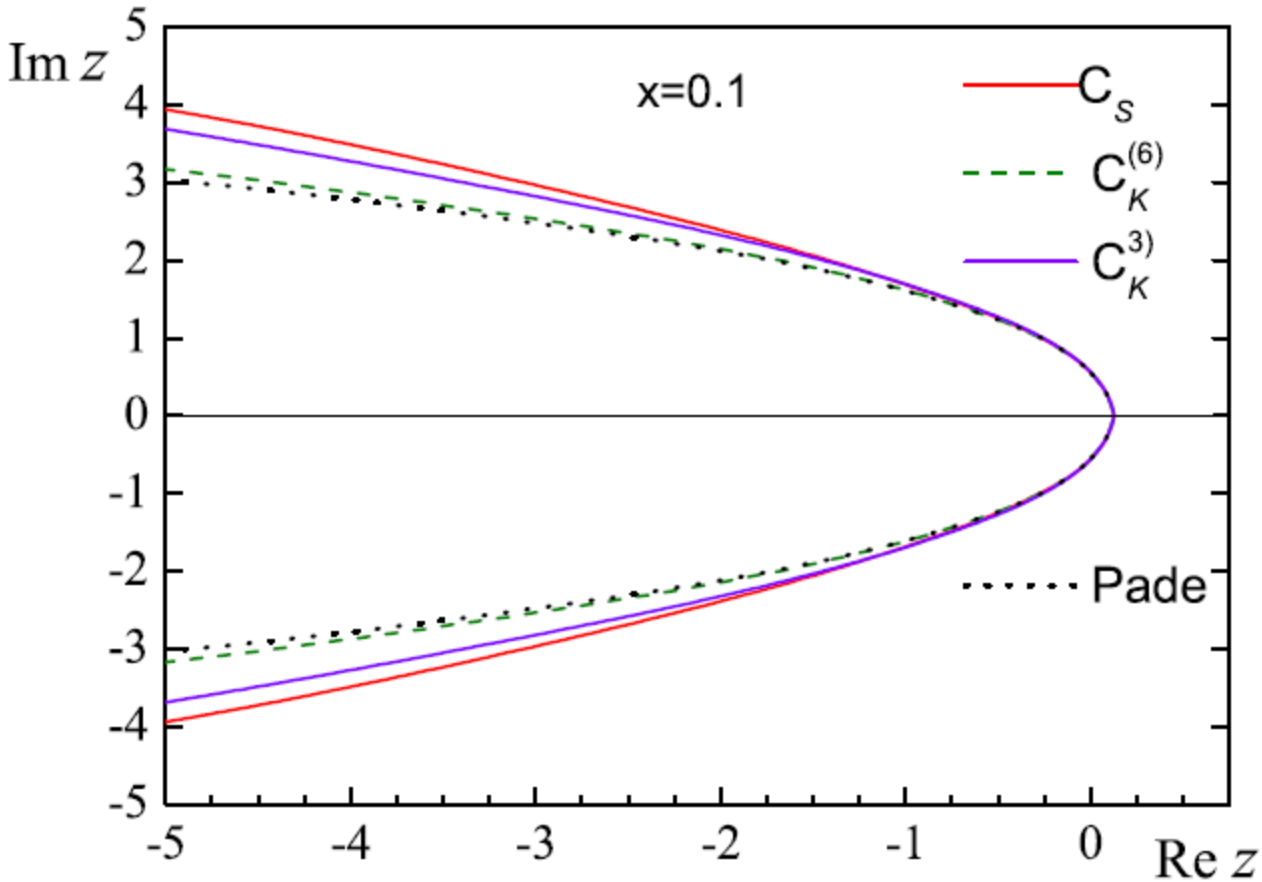
$$M_3(z,Q_0^2) = \int_0^1 dx\, x^{z-1} xF_3(x,Q_0^2)$$

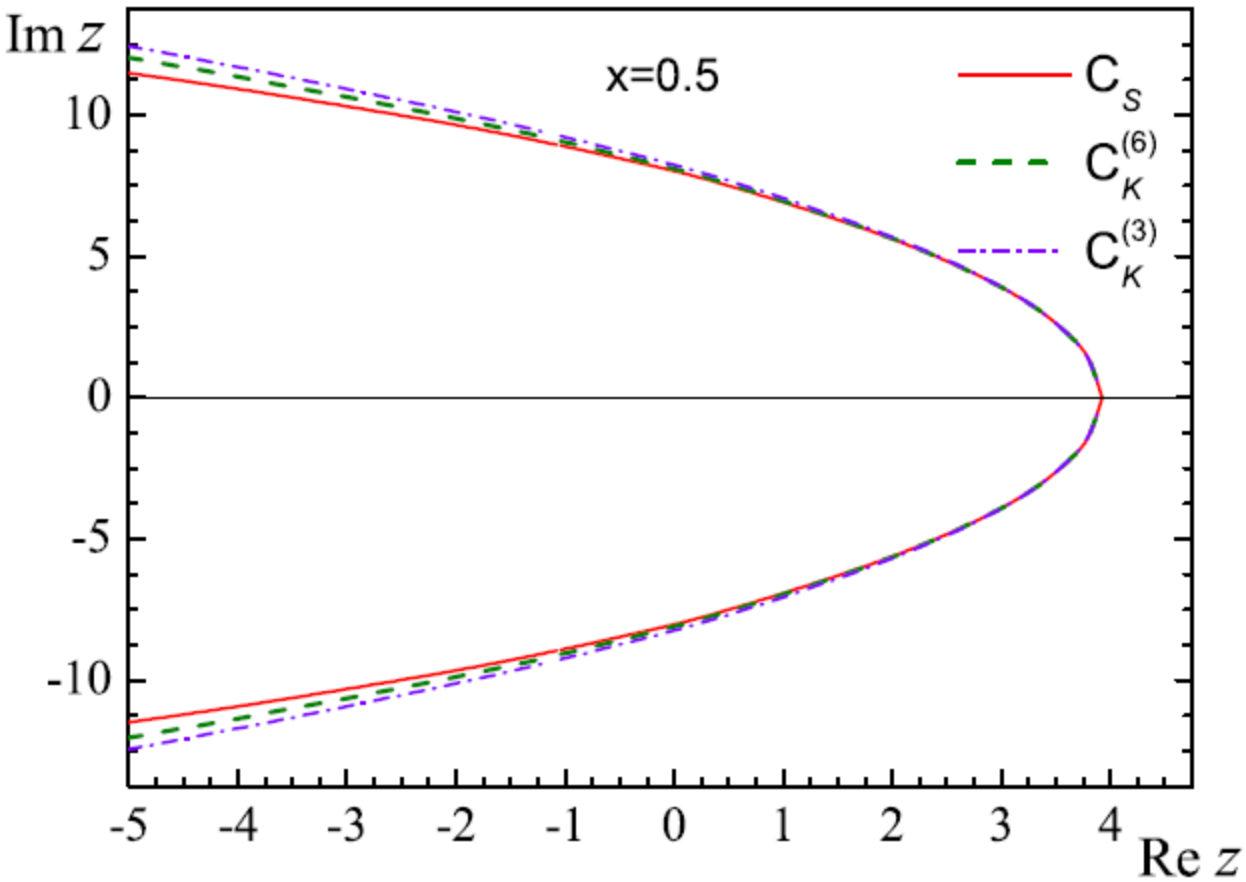
$$= A\left[\frac{\Gamma(\beta+1)\,\Gamma\,(\alpha+z)}{\Gamma(\alpha+\beta+z+1)} + \gamma\frac{\Gamma(\beta+1)\,\Gamma(\alpha+z+1)}{\Gamma(\alpha+\beta+z+2)}\right]$$

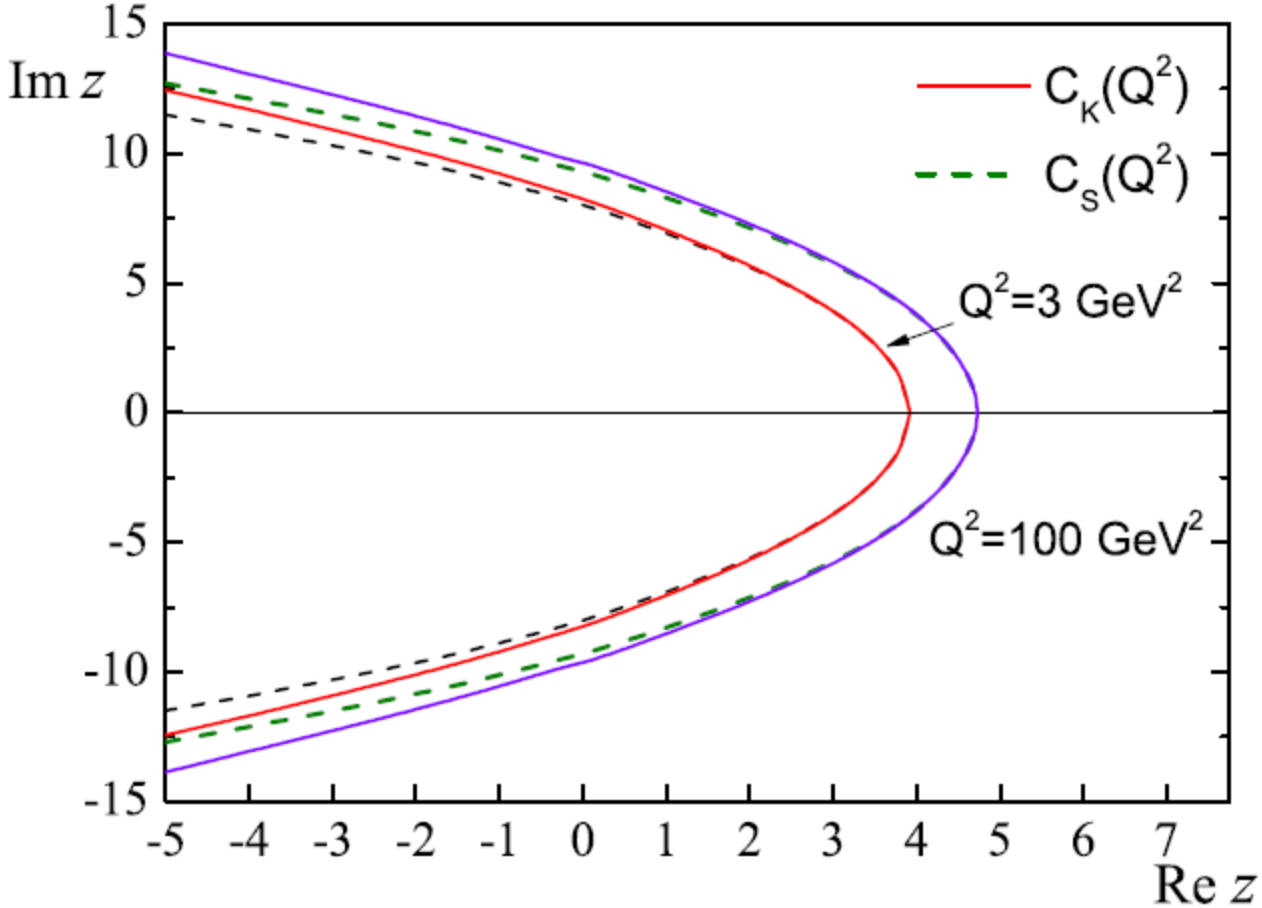
$xF_3(x,Q^2)$

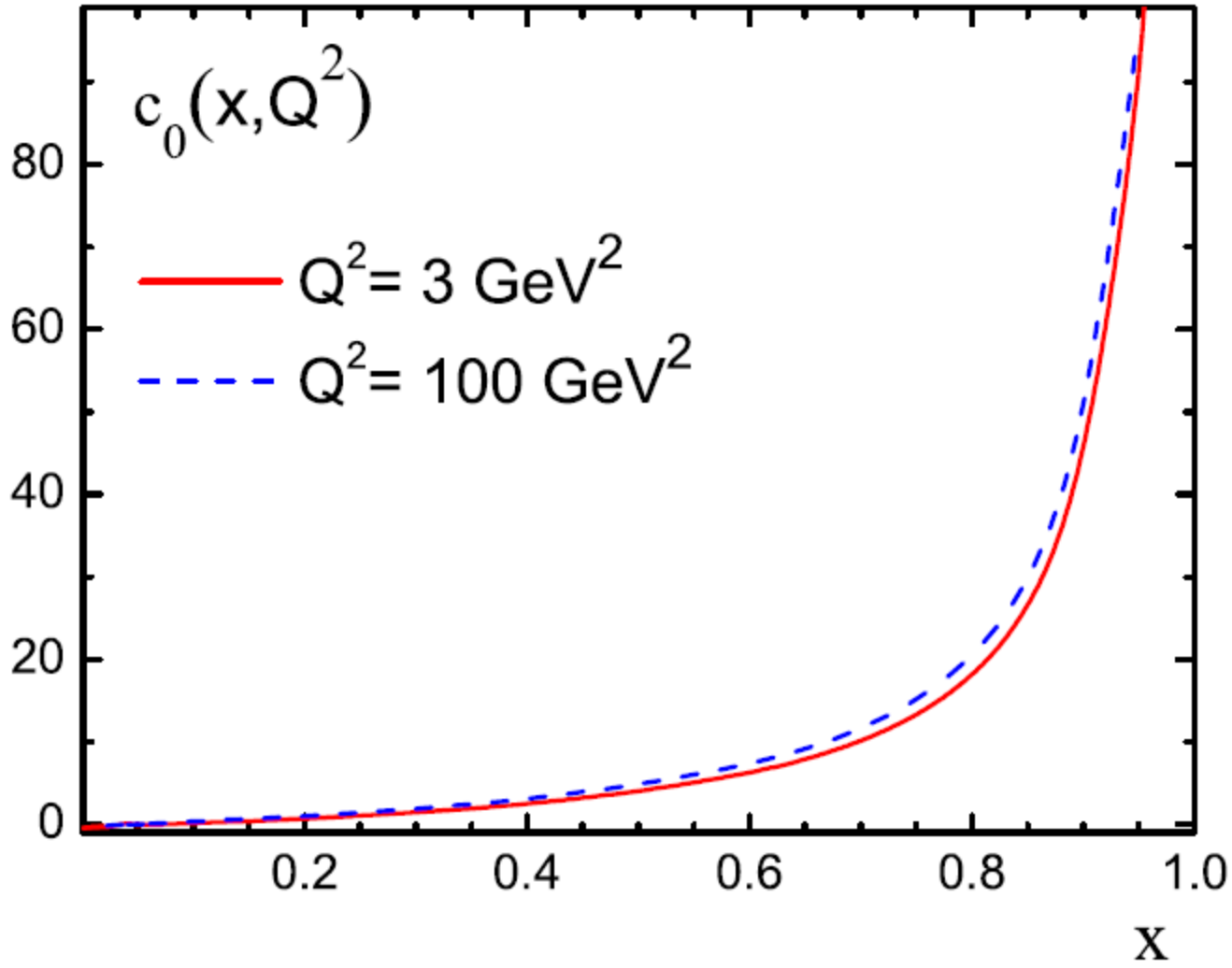
$$xF_3(x,Q^2) = \frac{1}{2\pi i}\int_C dz\; x^{-z} M_3(z,Q^2)\,.$$

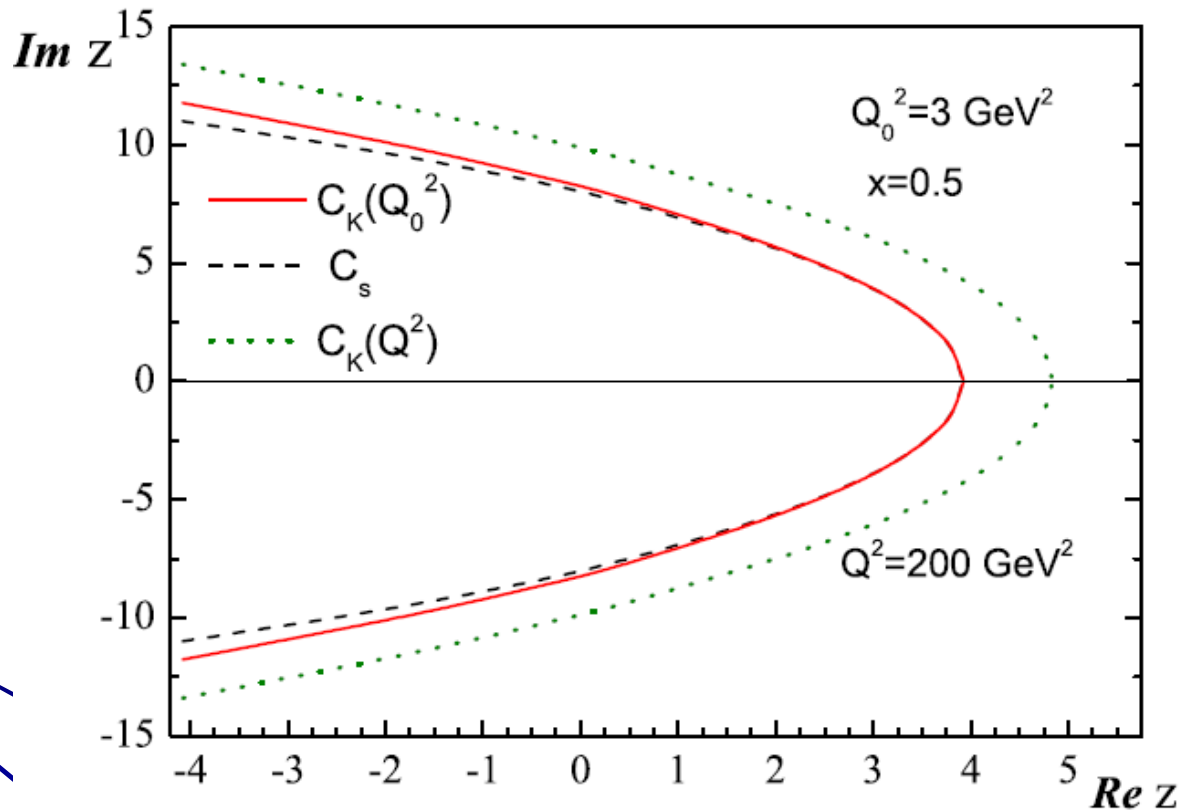
$$M_3^{NS}(z,Q^2) = M_3(z,Q_0^2)\; e^{ln[\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)}]\frac{16}{33-2f}\left[\frac{1}{2\,(z+1)\,(z+2)}+\frac{3}{4}-\gamma-\psi(z+2)\right]}$$

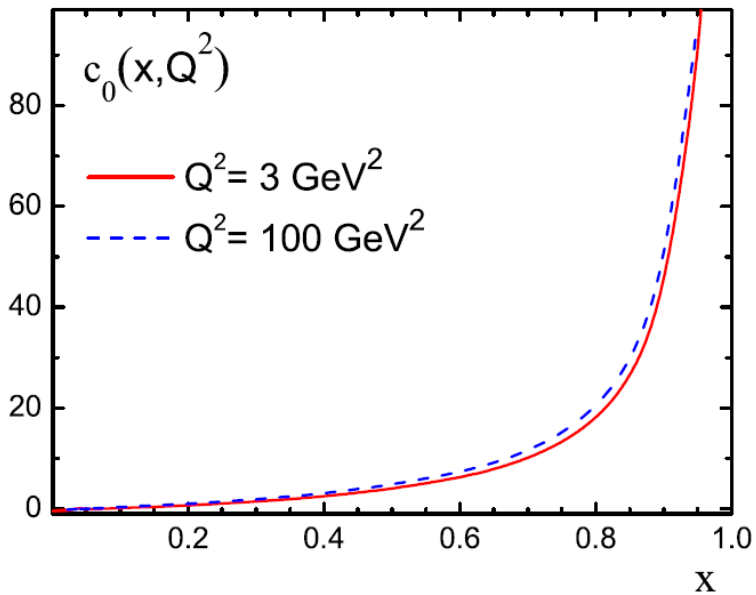

Im z
5
4
3
2
1
0
-1
-2
-3
-4
-5
x=0.1
C_S
$C_K^{(6)}$
$C_K^{3)}$
Pade
-5
-4
-3
-2
-1
0
Re z

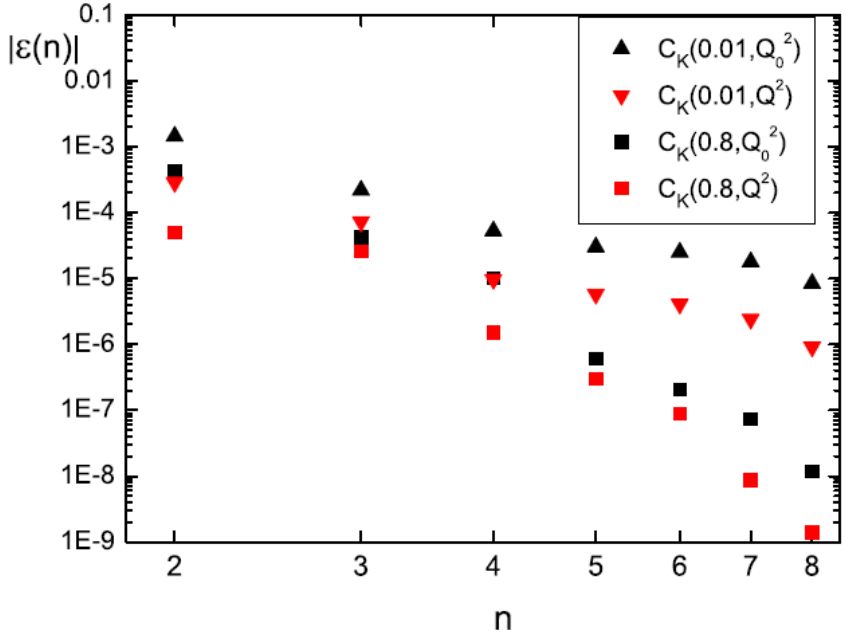

Im z
x=0.5
C_S
C_K^(6)
C_K^(3)
10
5
0
-5
-10
-5
-4
-3
-2
-1
0
1
2
3
4
Re z


Im z
Re z
15
10
5
0
-5
-10
-15
-5
-4
-3
-2
-1
0
1
2
3
4
5
6
7
$C_K(Q^2)$
$C_S(Q^2)$
Q^2=3 GeV^2
Q^2=100 GeV^2

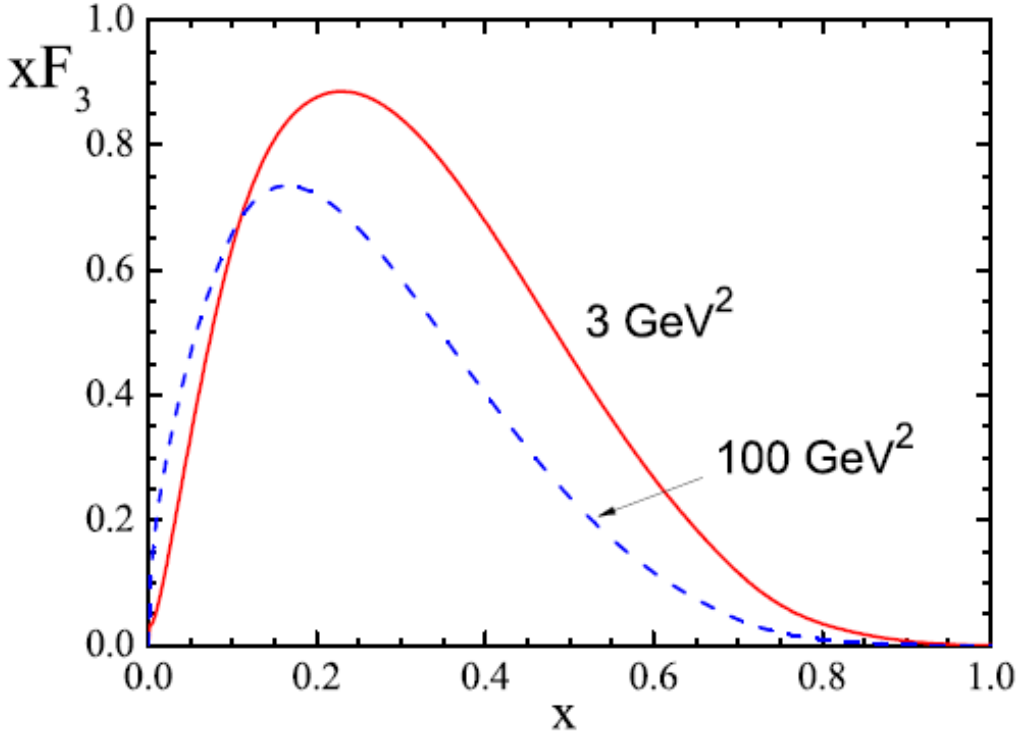

$c_0(x,Q^2)$
Q^2= 3 GeV^2
Q^2= 100 GeV^2
0
20
40
60
80
0.2
0.4
0.6
0.8
1.0
x


Im z
Re z
15
10
5
0
-5
-10
-15
-4
-3
-2
-1
0
1
2
3
4
5
Q_0^2=3 GeV^2
x=0.5
$C_K(Q_0^2)$
C_s
$C_K(Q^2)$
Q^2=200 GeV^2





x	точное	n = 1	n = 2	n = 4	n = 6	n = 8	n = 10
0,001 z_0=0.469	0.04300014819	0.040	$0.26 * 10^{-2}$	$-1.1 * 10^{-4}$	$-0.37 * 10^{-4}$	$0.53 * 10^{-7}$	$5.3 * 10^{-8}$
0,01 $z_0 = -0.352$	0.18906984253	0.025	$0.19 * 10^{-2}$	$-0.66 * 10^{-4}$	$-0.25 * 10^{-5}$	$0.36 * 10^{-5}$	$3.7 * 10^{-8}$
0,1 $z_0 = 0.128$	0.736598029113	$-0.34 * 10^{-2}$	$0.84 * 10^{-3}$	$0.62 * 10^{-5}$	$0.32 * 10^{-5}$	$0.60 * 10^{-5}$	$7.1 * 10^{-8}$
0,5 $z_0 = 3.92$	0.462524624315	$-0.78 * 10^{-2}$	$-0.19 * 10^{-3}$	$0.22 * 10^{-5}$	$0.68 * 10^{-7}$	$0.20 * 10^{-8}$	$0.22 * 10^{-9}$
0,8 $z_0 = 17.66$	0.030607902006	0.012	$-0.46 * 10^{-4}$	$-0.24 * 10^{-5}$	$0.13 * 10^{-6}$	$-0.98 * 10^{-9}$	$-0.33 * 10^{-9}$
0,95 $z_0 = 86.9$	0.000255232463	0.012	$-0.28 * 10^{-4}$	$-0.35 * 10^{-5}$	$0.35 * 10^{-6}$	$-0.30 * 10^{-9}$	$-0.39 * 10^{-9}$


xF3
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0
x
3 GeV2
100 GeV2

	$\varepsilon = 10^{-4}$		$\varepsilon = 10^{-5}$	
x	C_{Q^2}	$C_{Q_0^2}$	C_{Q^2}	$C_{Q_0^2}$
0.001	4	6	7	8
0.01	3	4	5	8
0.1	3	4	5	7
0.5	3	3	4	4
0.8	2	3	4	5

Conclusions

We consider inverse Mellin transformation method for evaluation Q^2-dependence of the xF_3 structure function based on the efficient contour corresponding saddle point proposed by Kosower.

Мы подтверждаем предположение, сделанное в работе Косовера 1997 г. об универсальном характере контура $C_0(Q_0^2)$. Этот контур эффективно работает для Q^2 не равных Q_0^2. В области $x > 0.5$ контура $C_0(Q_0^2)$и $C(Q_0^2)$дают близкие результаты по точности восстановления. Для малых x контур $C(Q_0^2)$ дает более точный результат, однако это преимущество компенсируется при использовании контура $C_0(Q_0^2)$, если увеличить число членов в квадратурной формуле на 2-3 единицы.

Включение в выражение, определяющее форму контура, следующего члена ~ t^4 не приводит к заметному увеличению точности восстановления структурной функции.

Контур $C_0(Q_0^2)$ одинаково успешно описывает структурную функцию $xF_3^{TMC}(x,Q^2)$ -- с учетом и без учета ТМС.

В целом рассмотренный метод обладает быстрой сходимостью к точному значению структурной функции с ростом числа полиномов. Для восстановления $xF_3(x,Q^2)$ с относительной точностью 10^{-4} в области доступных экспериментальных данных достаточно использовать пароболический контур $C_0(Q_0^2)$ и ограничиться в квадратурной формуле 6 полиномами. Это качественно согласуется с выводами работы Косовера для параметризации соответствующей выбору $\gamma = 0$ и позволяет в разы ускорить вычисление интеграла, а контур $C_0(Q_0^2)$ рассматривать, как универсальный, то есть применимый для любого значения Q^2 при фиксированном значении x.

Thanks for your attention !