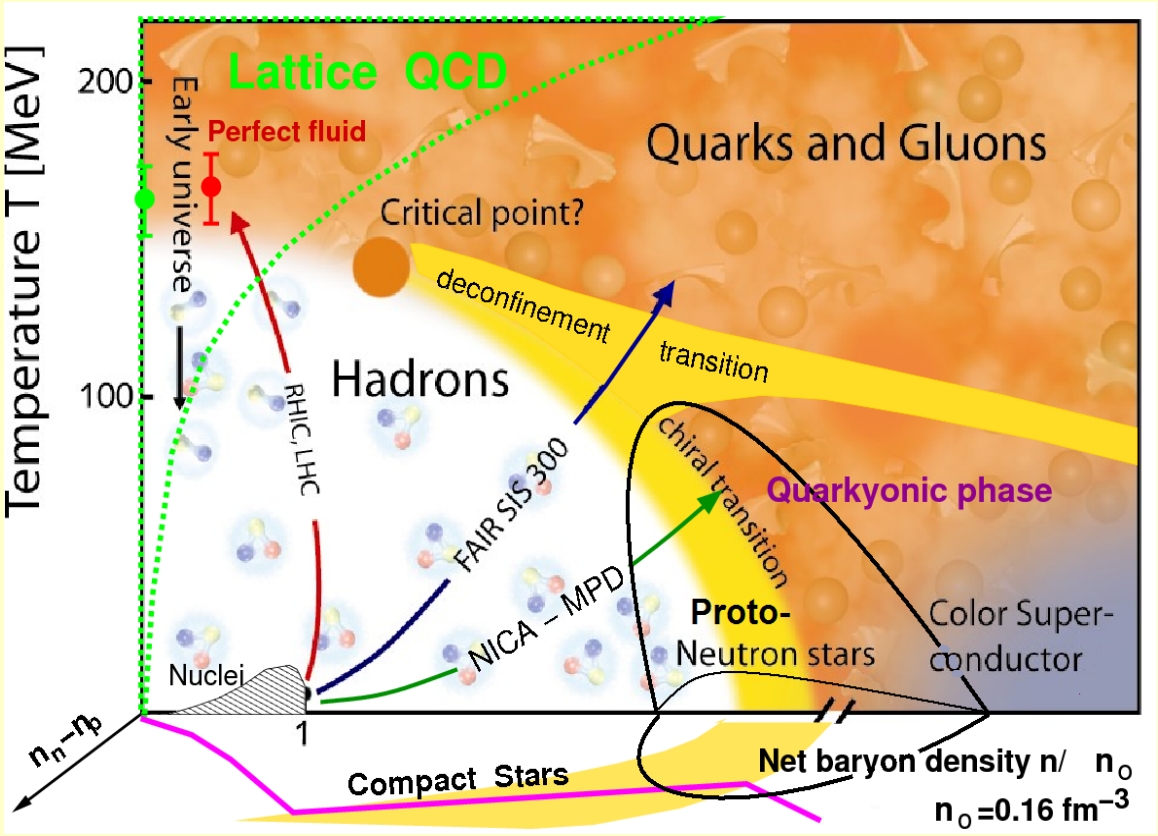
Implementation of Higuchi method of fractal dimension measurement for the multiparticle event

XXIV International Seminar Nonlinear Phenomena in Complex Systems

Oleg Rogachevsky
Bychkov A.
JINR

May 16, 2017

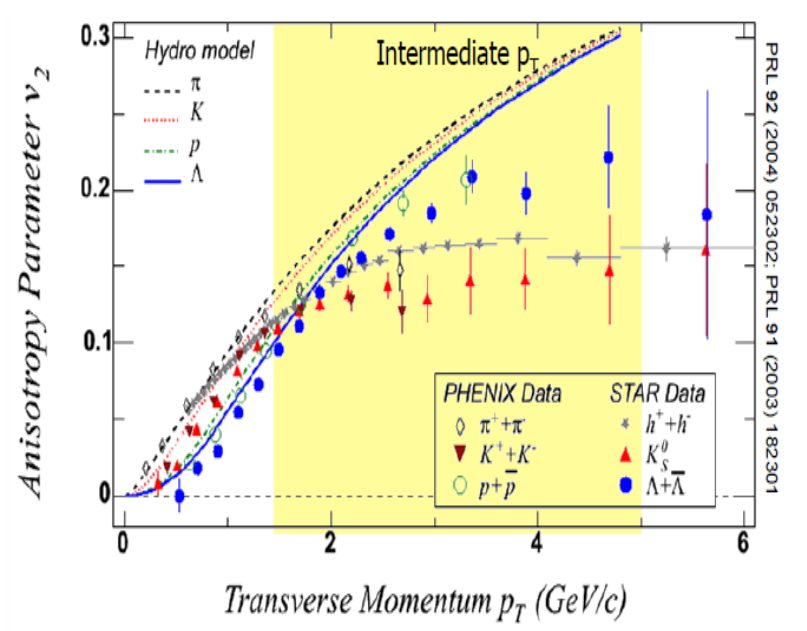
QGP phase diagram

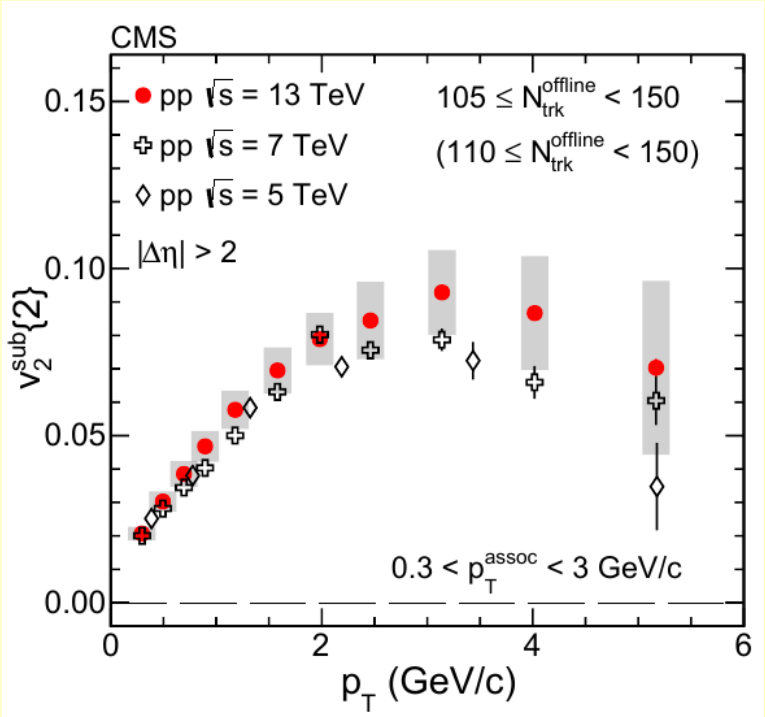


Collective behaviour

STAR 200 GeV

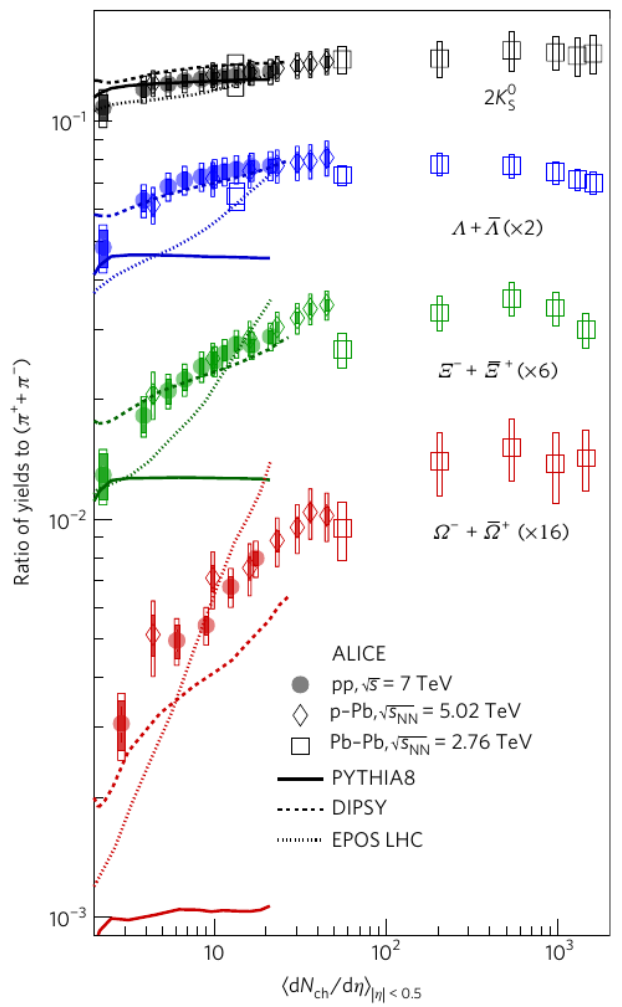
arXiv:1606.06198v1
Evidence for collectivity in pp collisions at the LHC





$v_2 \quad vs \quad p_t$

Strangeness enhancement



nature physics
LETTERS
PUBLISHED ONLINE: 24 APRIL 2017 | DOI: 10.1038/NPHYS4111

p_T -integrated yield ratios to pions ($\pi^+ + \pi^-$) as a function of (dNch /dη) measured in |y| < 0.5.

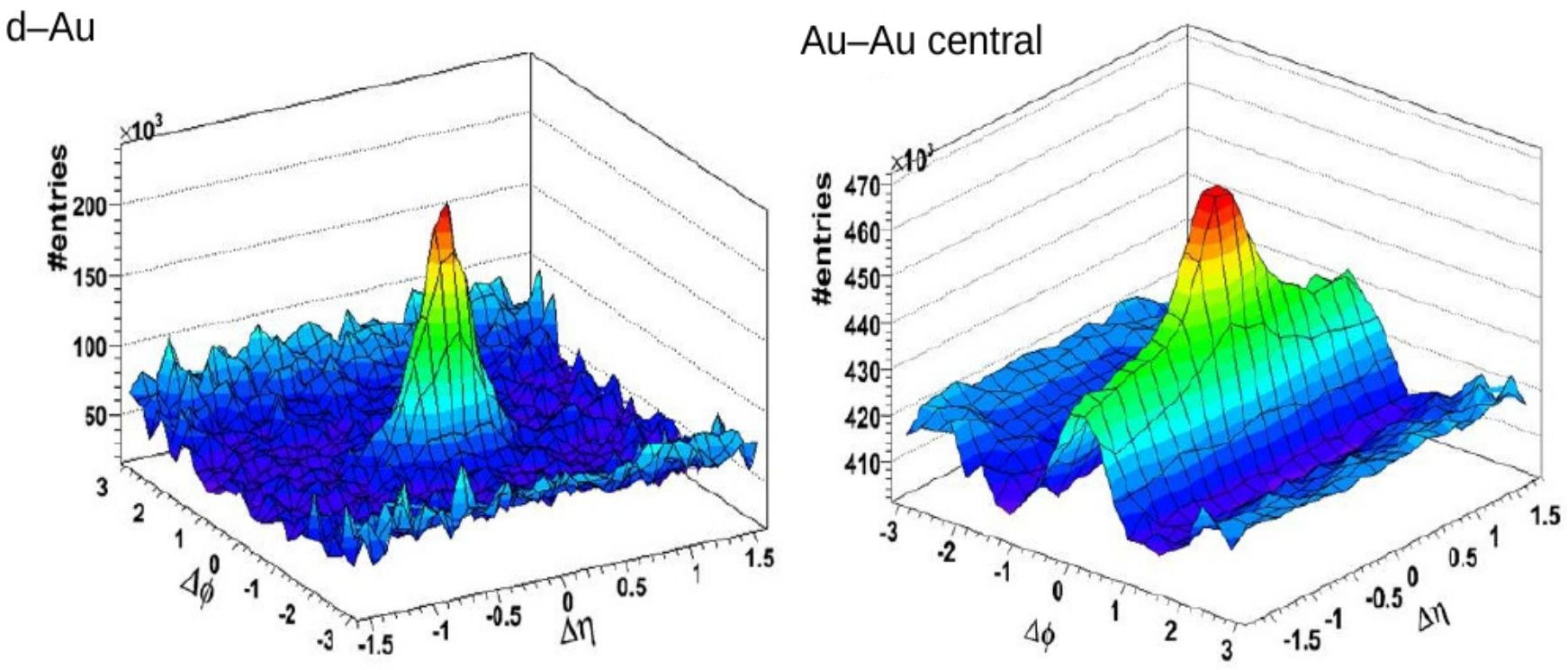
The error bars show the statistical uncertainty, whereas the empty and dark-shaded boxes show the total systematic uncertainty and the contribution uncorrelated across multiplicity bins, respectively.

The values are compared to calculations from MC models and to results obtained in p–Pb and Pb–Pb collisions at the LHC.

For Pb–Pb results the ratio $2\Lambda/(\pi^+ + \pi^-)$ is shown. The indicated uncertainties all represent standard deviations

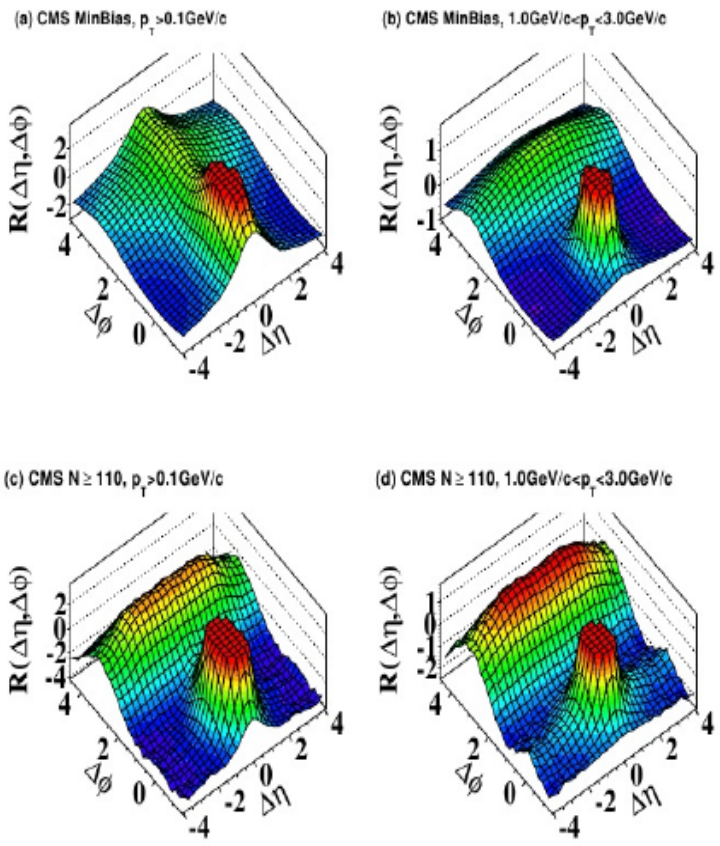
Ridge @ RHIC

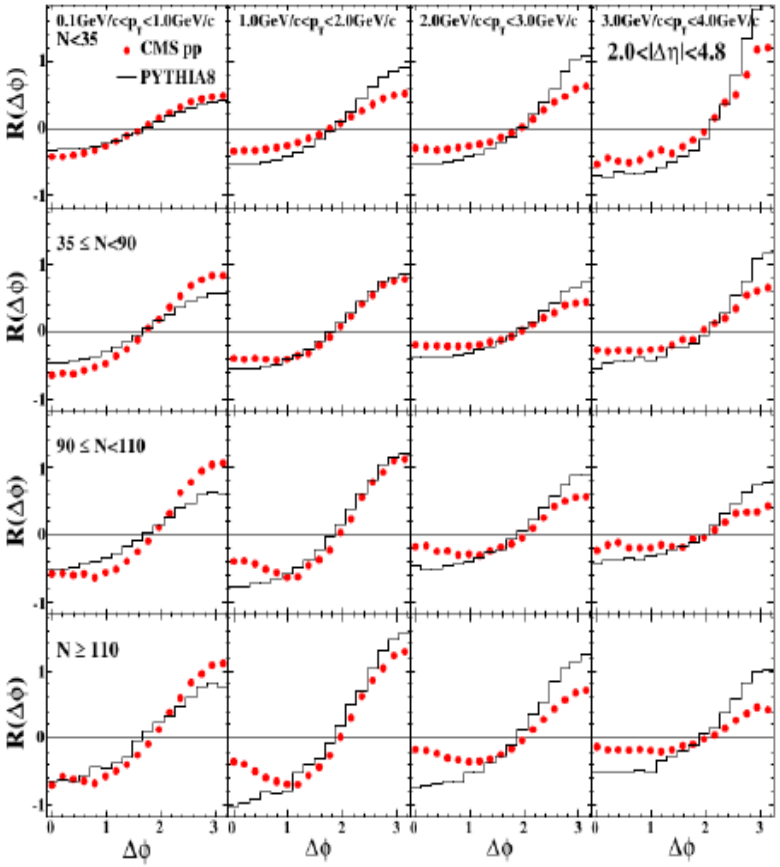
B. Abelev et al., Phys. Rev. C80, 064912 (2009).



Two particle correlation function at 200 GeV/c/N from STAR experiment

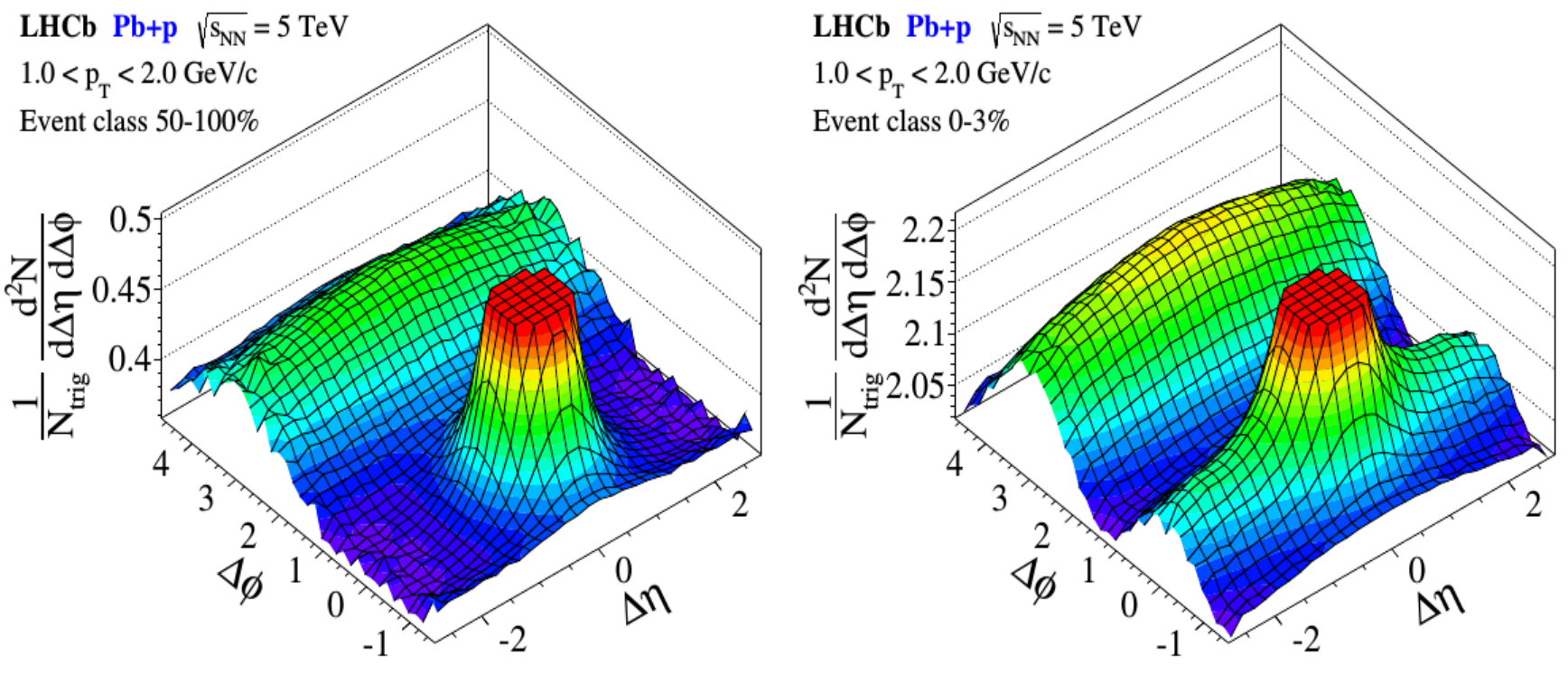
Ridge @ LHC pp & TeV





Ridge @ LHCb pAu & √s=5TeV

arXiv:1512.00439v1



Ridge

What are the meaning of the cuts ?

For RHIC AuAu

$$3\ \text{GeV/c} < p_T^{trig} < 4\ \text{GeV/c}$$

$$2\ \text{GeV/c} < p_T^{assoc} < p_T^{trig}$$

For LHC pp

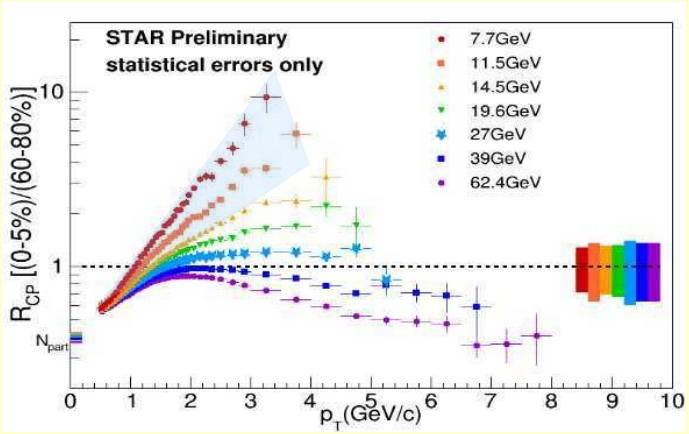
$$0.1\ \text{GeV/c} < \ldots < p_T < \ldots < 5.\ \text{GeV/c}$$

Multiplicity N > 110

STAR BES I results

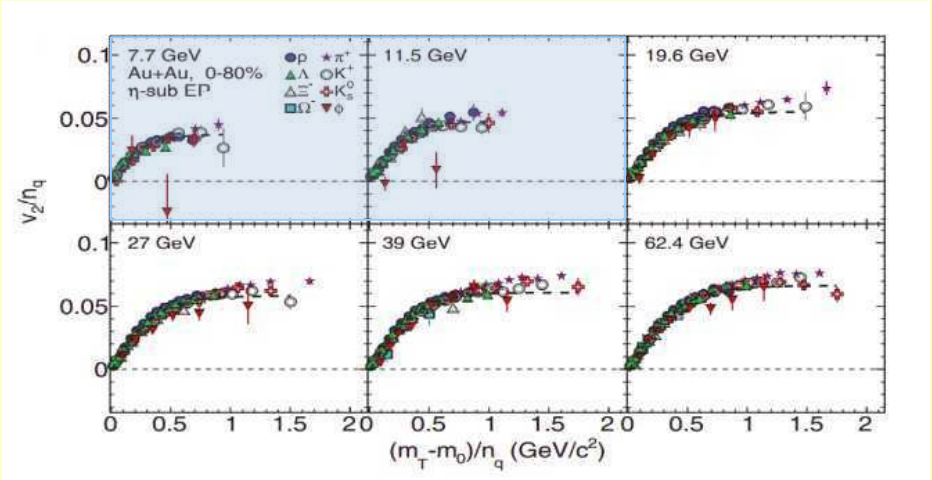
High P_T suppression

Stephen Horvat Quark Matter 2015



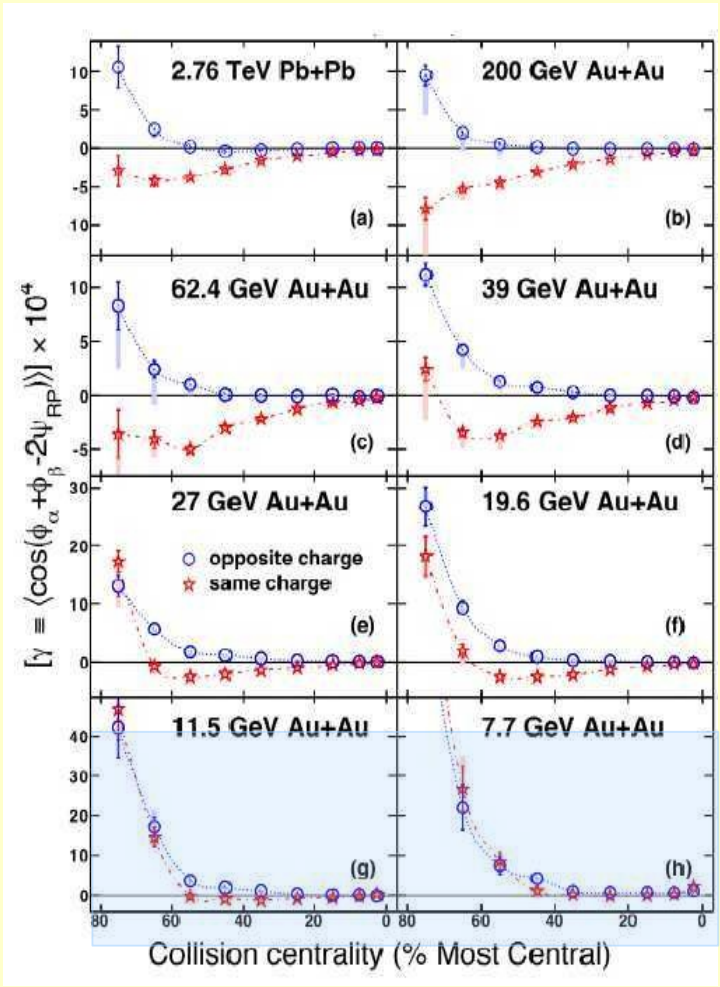
Number of constituent quarks scaling

Phys. Rev. C88, (2013), 014902



Chiral Magnetic Effect

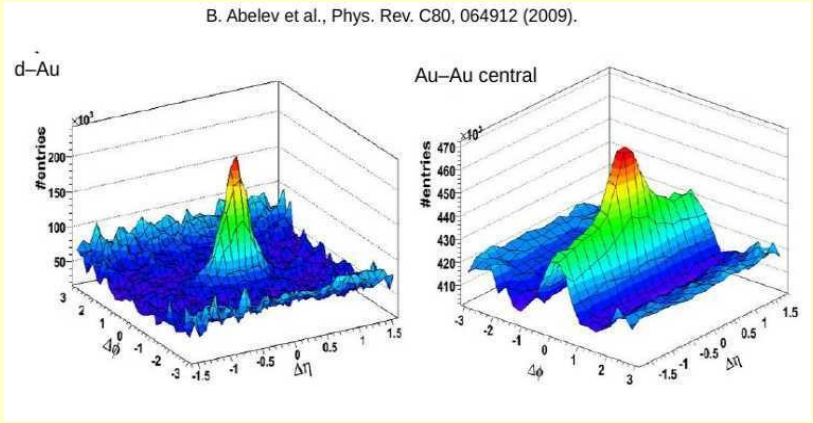
STAR, Phys. Rev. Lett. 113, 052302 (2014)

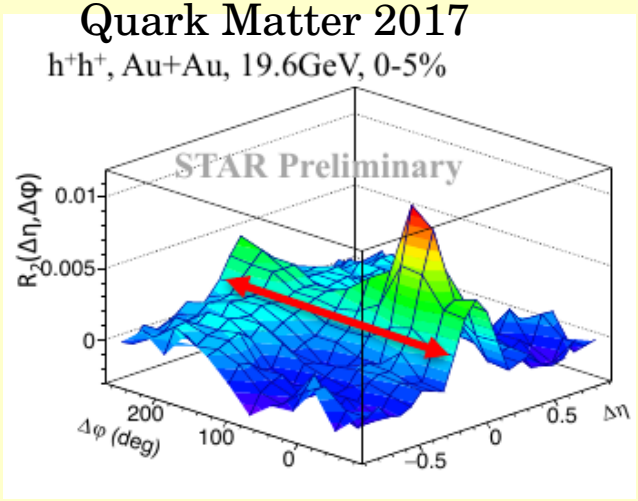


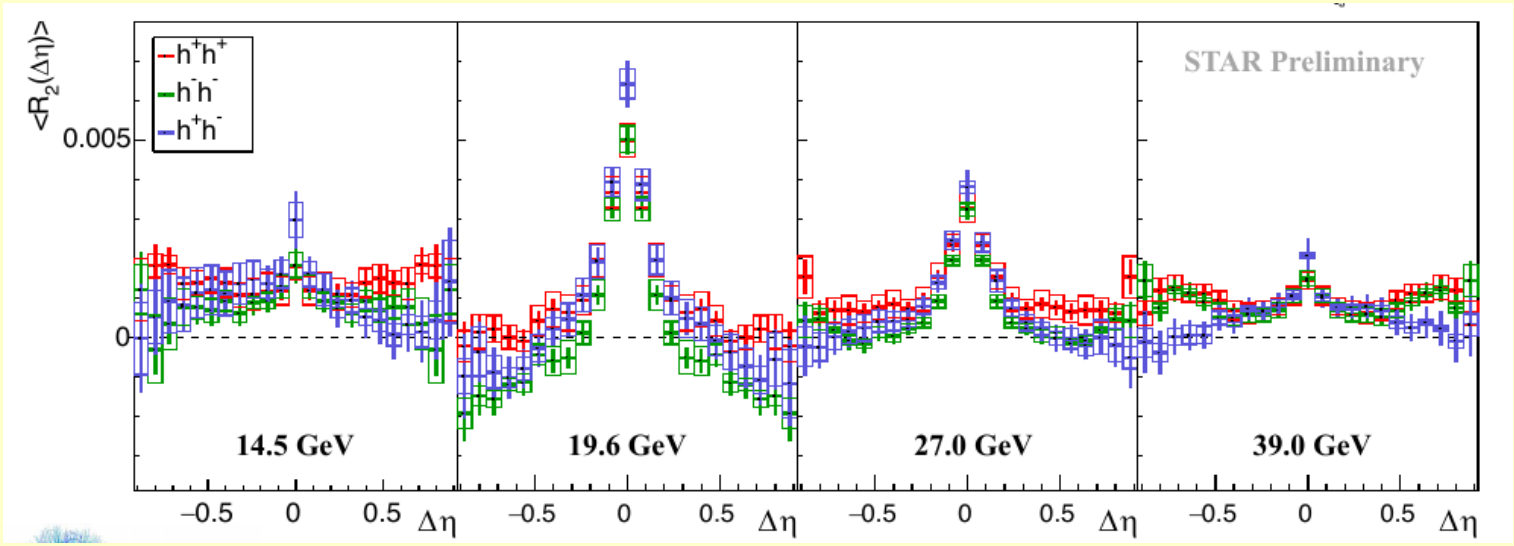
STAR BES I results

S. Jowzaee,
Quark Matter 2017

Ridge effect

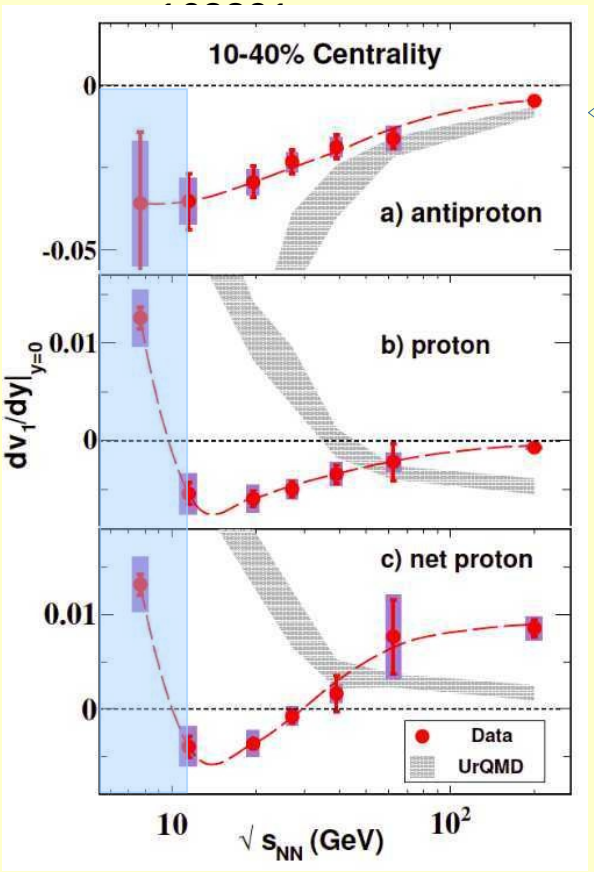






STAR BES I results

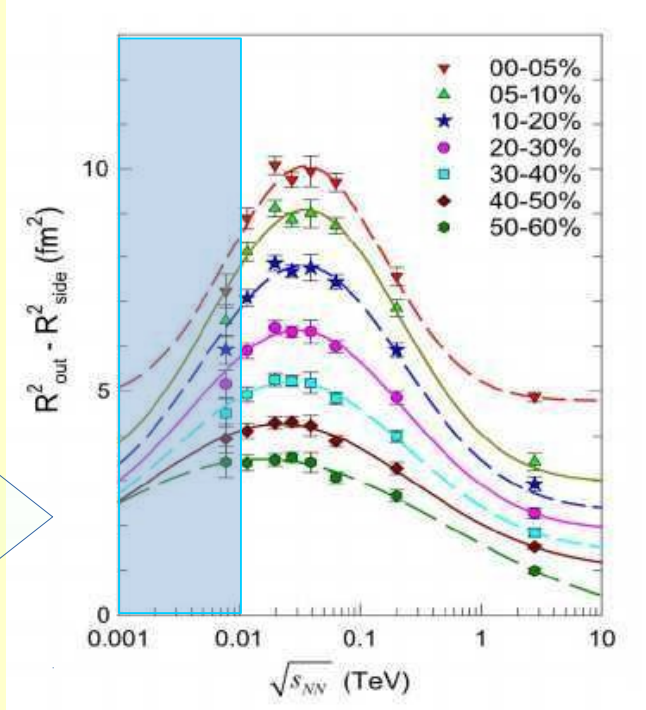
PRL 112 (2014)



The rapidity-slope of the net proton directed flow v_1 , dv_1 /dy. This quantity is sensitive to early pressure gradients in the medium.

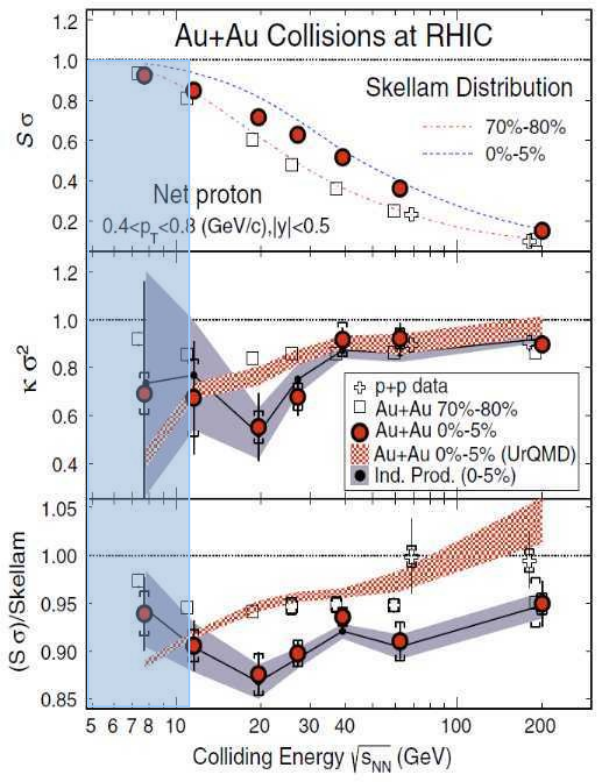
$R^2_{out} - R^2_{side}$ – reflects the lifetime of the collision fireball and was predicted to reach a maximum for collisions in which a hydrodynamic fluid forms at temperatures where the equation of state is softest.

R. Lacey, PRL 114, 142301 (2015)



STAR BES I results

STAR, PRL 112, 032302 (2014)



The kurtosis of the event-by-event distribution of the net proton (i.e. proton minus antiproton) number per unit of rapidity, normalized such that Poisson fluctuations give a value of 1.

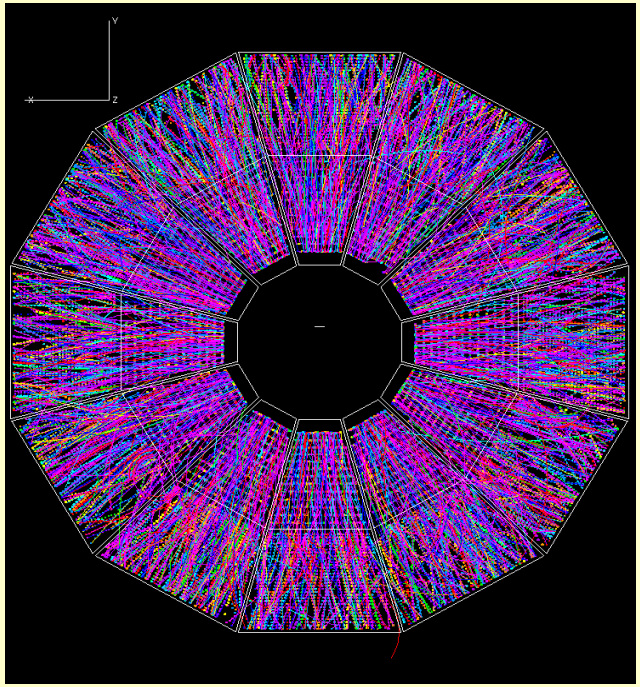
In central collisions, published results in a limited kinematic range show a drop below the Poisson baseline around $\sqrt{s_{NN}}$ =27 and 19.6 GeV.

New preliminary data over a larger p_T range, although at present still with substantial error bars, hint that the normalized kurtosis may, in fact, rise above 1 at lower $\sqrt{s_{NN}}$, as expected from critical fluctuations..

The grey band shows the much reduced uncertainties anticipated from BES-II in 2018-2019, for the 0-5% most central collisions.

Deterministic chaos

Collections of small entities (particles or whatever) behave haphazardly, even though physical laws govern the particles individually



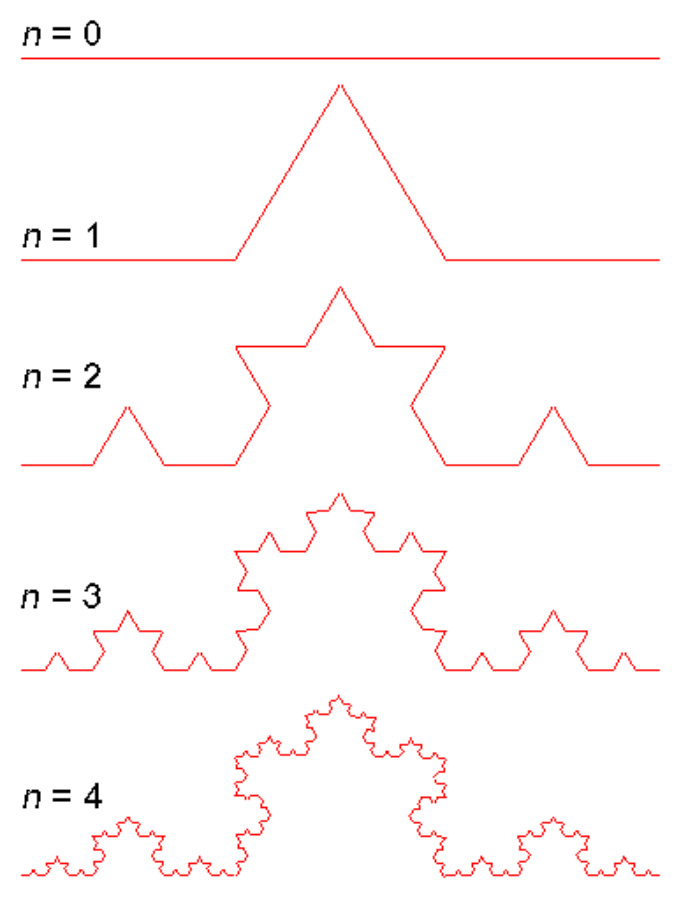
Deterministic chaos – chaos results from deterministic process

R. Hagedorn, “Remarks on the Thermodynamical Model of Strong Interactions”,
Nucl. Phys. B (1970).

A firebal is
a statistical equilibrium of undetermined numbers of fireballs, each of which, in turn, is considered to be ...

Fractal dimension

Koch's arc



The **fractal dimension** of sets observed in the dynamics can be used as quantitative measure of the chaoticity

$$D = \lim_{r \to 0} \frac{\log N(r)}{\log (1/r)}$$

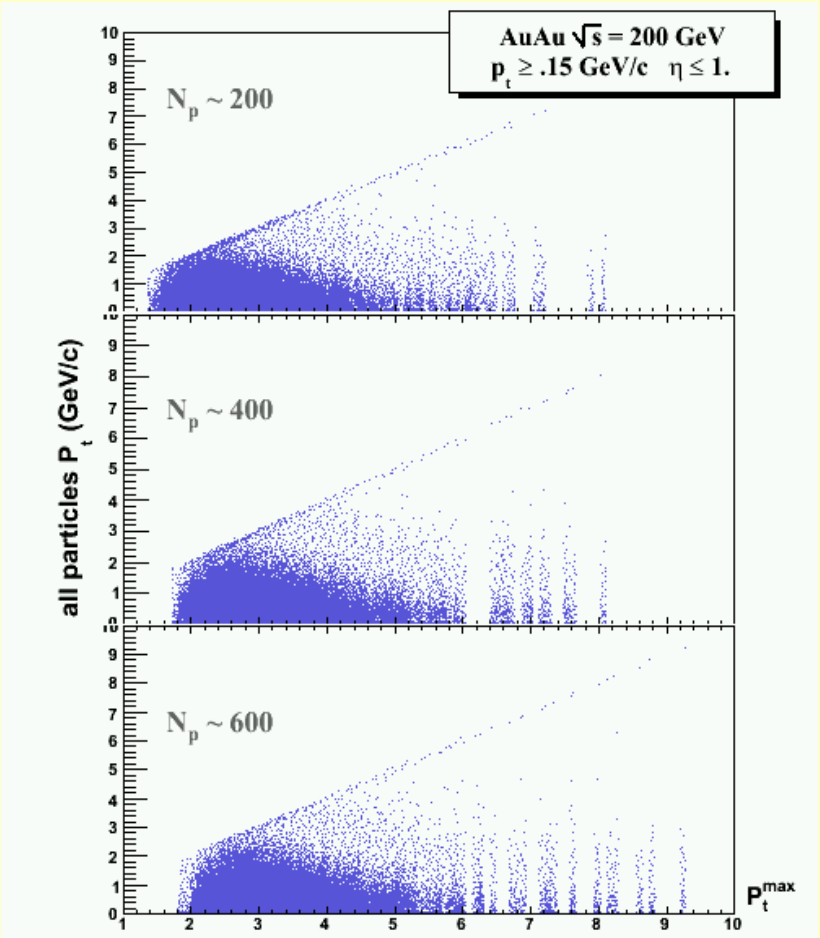
r - scale

N(r) - number of elements in fractal at given scale

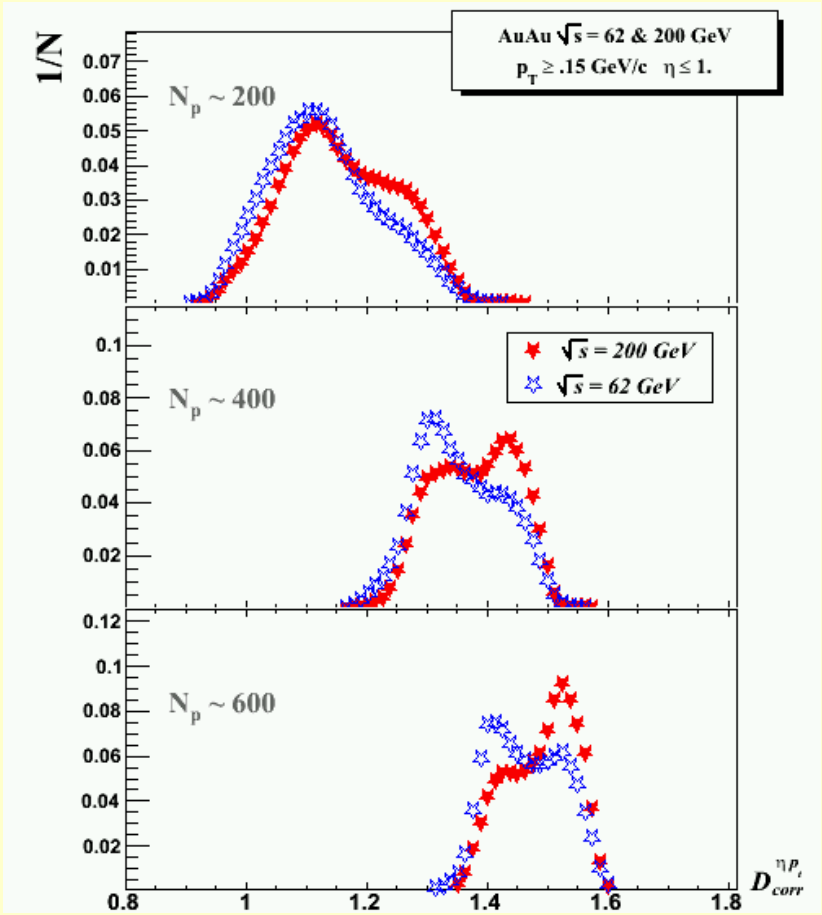
$$D^{Koch's\ arc} = \lim_{n \to \infty} \frac{\log 4^n}{\log 3^n} = \frac{\log 4}{\log 3} = 1.261\ldots$$

STAR data: 62 vs 200 GeV

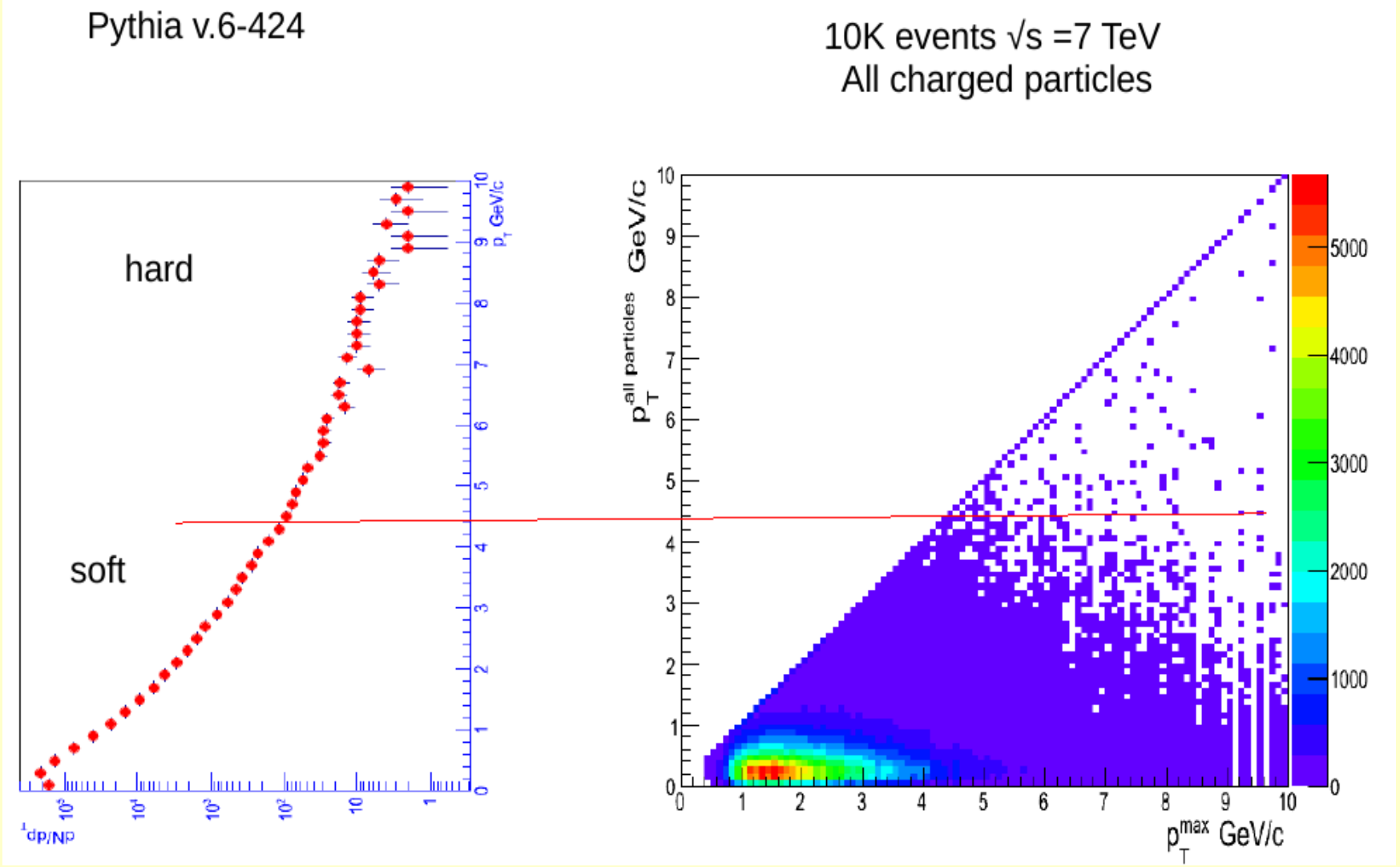
O.R. ICHEP 2006



$FD \ in \ \eta p_t - space$



High & low p_T particles



Higuchi's method I

T.Higuchi Physica D 31 (1988) 277-283
Approach to an irregular time series on the basis of the fractal theory

We now consider a finite set of time series observations taken at a regular interval:

$$X(1),\ X(2),\ X(3),\ldots,\ X(N).$$

From given time series, we first construct a new time series, X_k^m, defined as follows:

$$X_k^m;\ X(m),\ X(m+k),\ X(m+2k),\ldots,\ X\left(m+\left[\frac{N-m}{k}\right]\cdot k\right)\quad (m=1,2,\ldots,k),$$

where [ ] denotes the Gauss' notation and both k and m are integers. m and k indicate the initial time and the interval time, respectively. For a time interval equal to k, we get k sets of new time series. In the case of $k=3$ and $N=100$, three time series obtained by the above process are described as follows:

$$X_3^1;\ X(1),\ X(4),\ X(7),\ldots,\ X(97),\ X(100),$$

$$X_3^2;\ X(2),\ X(5),\ X(8),\ldots,\ X(98),$$

$$X_3^3;\ X(3),\ X(6),\ X(9),\ldots,\ X(99).$$

Higuchi's method II

T.Higuchi Physica D 31 (1988) 277-283

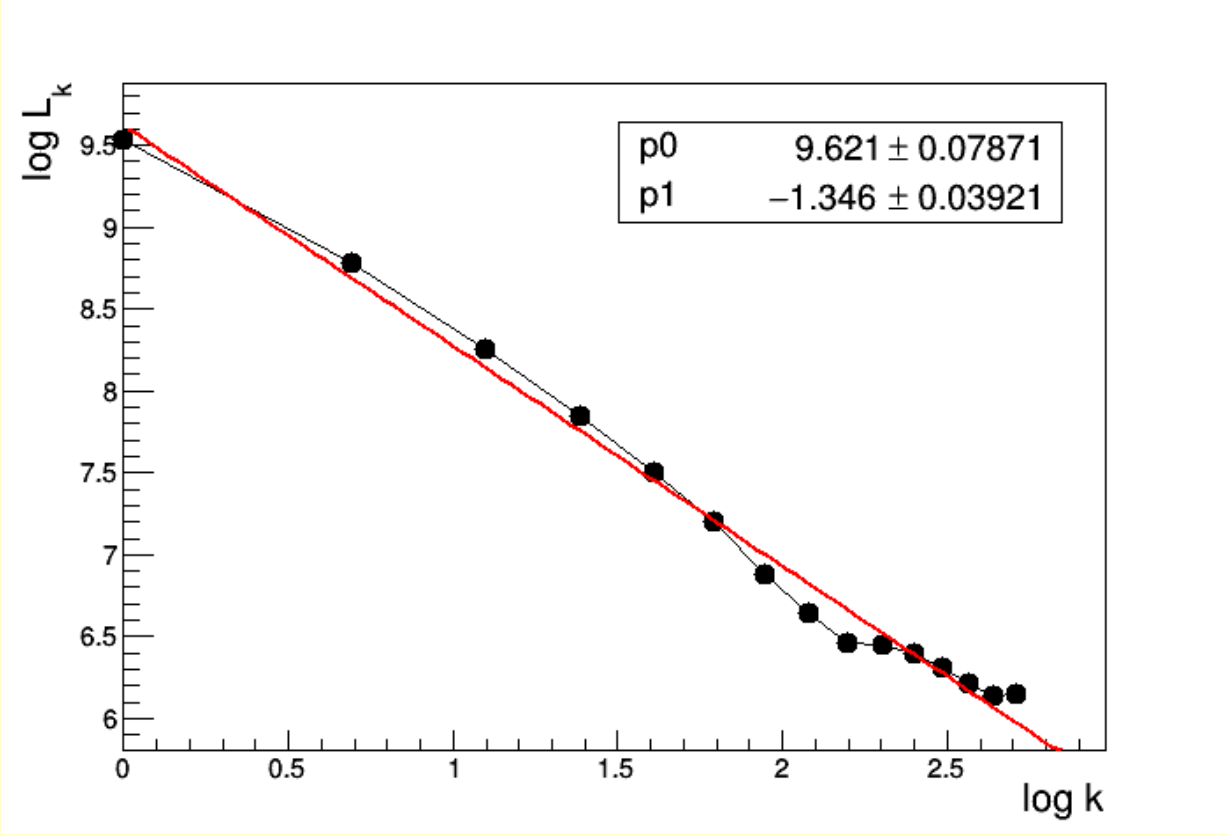
Approach to an irregular time series on the basis of the fractal theory

We define the length of the curve, X_k^m, as follows:

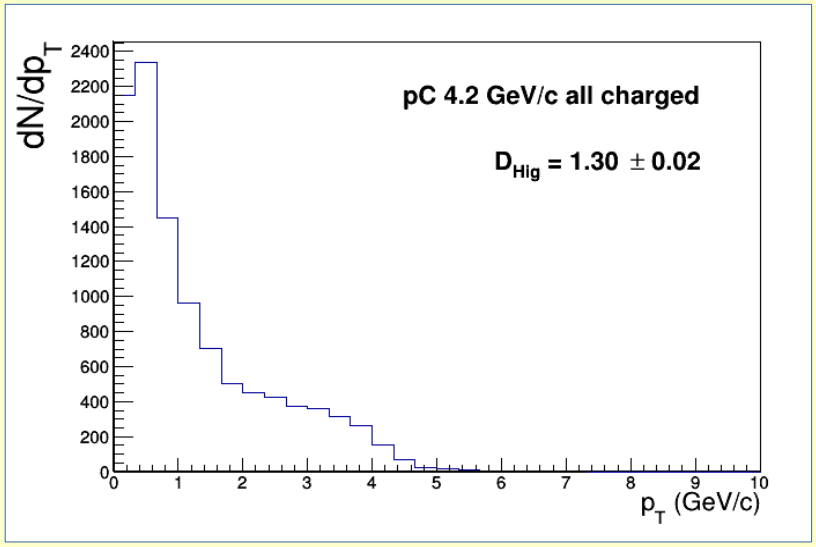
$$L_m(k) = \left\{ \left(\sum_{i=1}^{\left[\frac{N-m}{k}\right]} |X(m+ik) - X(m+(i-1)\cdot k)| \right) \frac{N-1}{\left[\frac{N-m}{k}\right]\cdot k} \right\} \Big/ k.$$

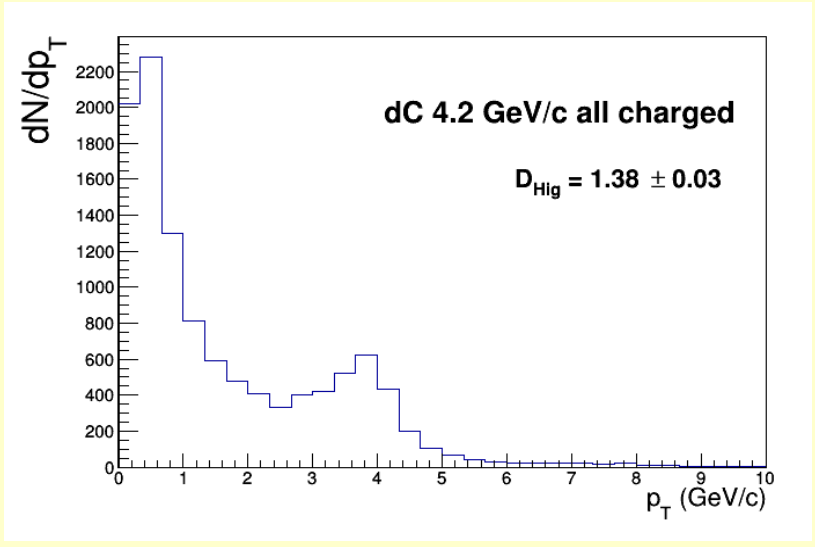
The term, $N-1/[(N-m)/k]\cdot k$ represents the normalization factor for the curve length of subset time series. We define the length of the curve for the time interval k, $\langle L(k)\rangle$, as the average value over k sets of $L_m(k)$. If $\langle L(k)\rangle \propto k^{-D}$, then the curve is fractal with the dimension D.

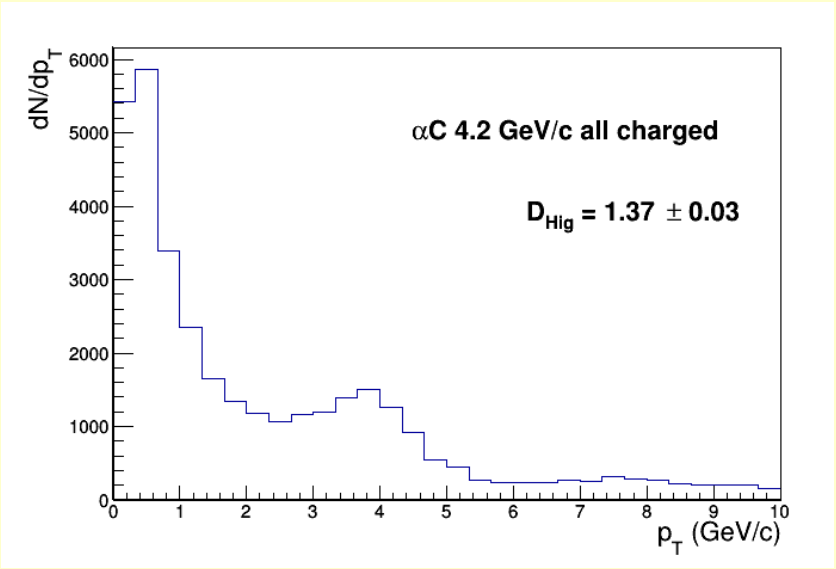
Higuchi's method III

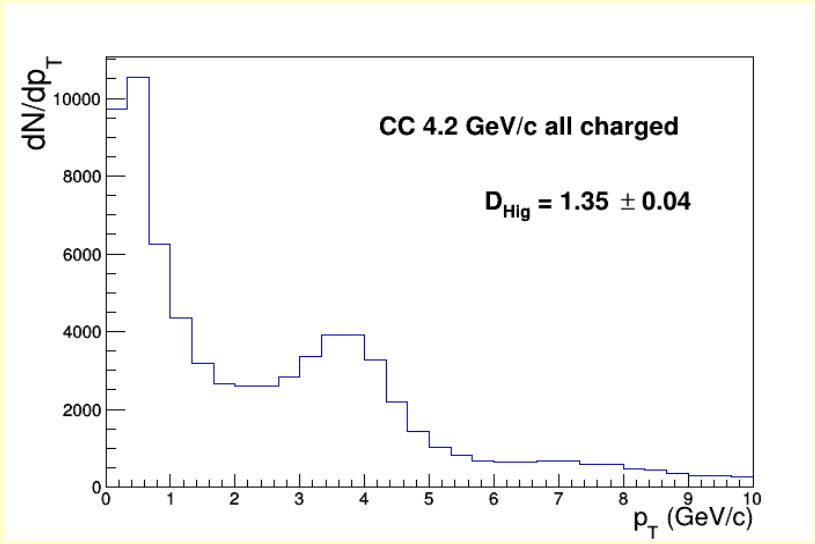


Higuchi's method p_T all charged

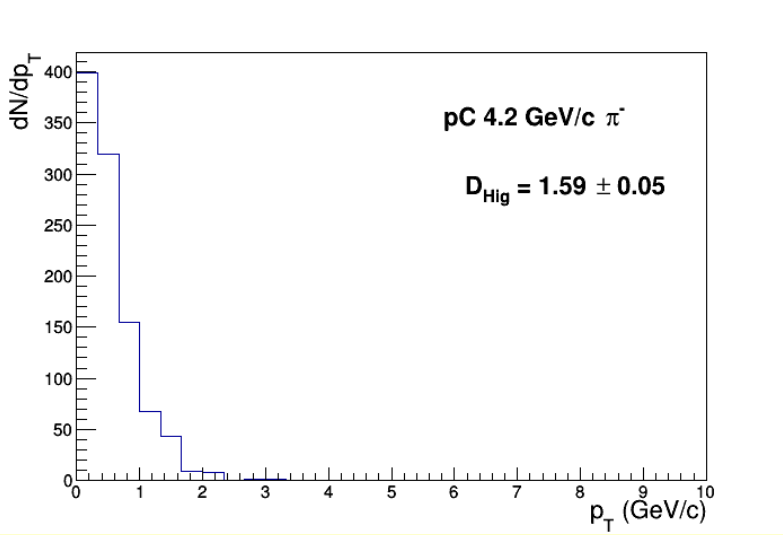


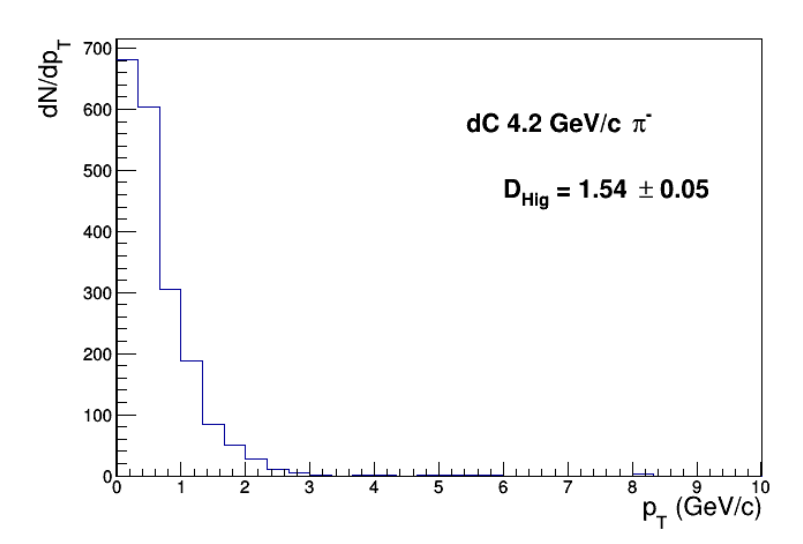


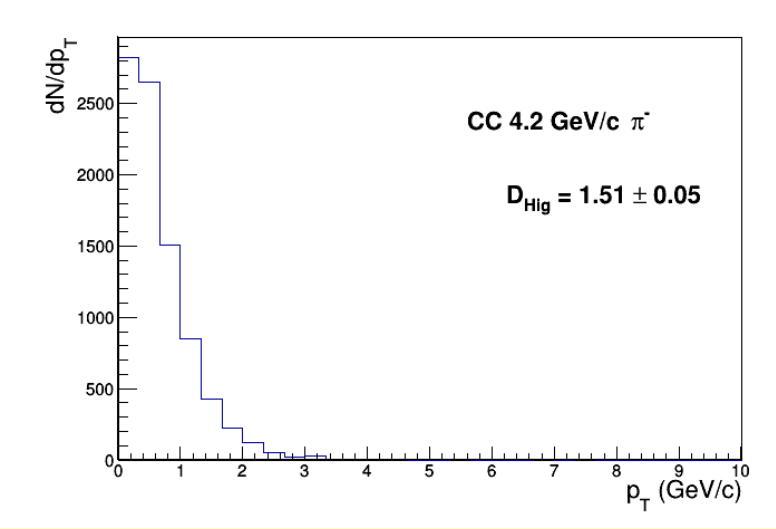


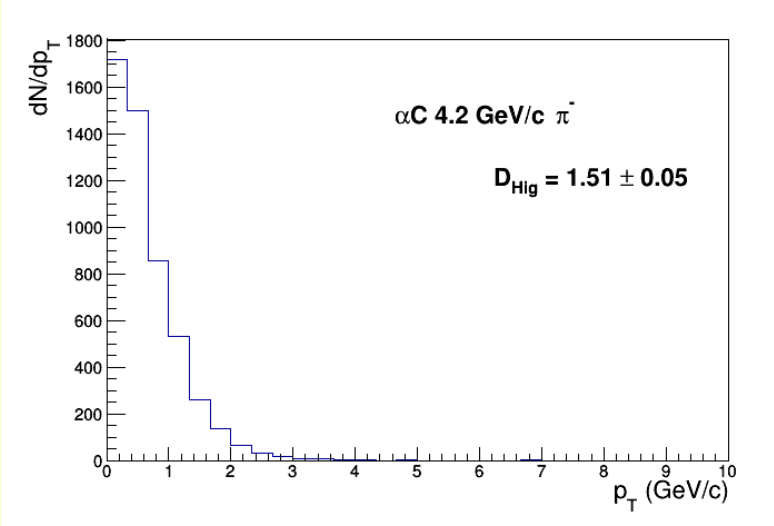


Higuchi's method p_T π^-







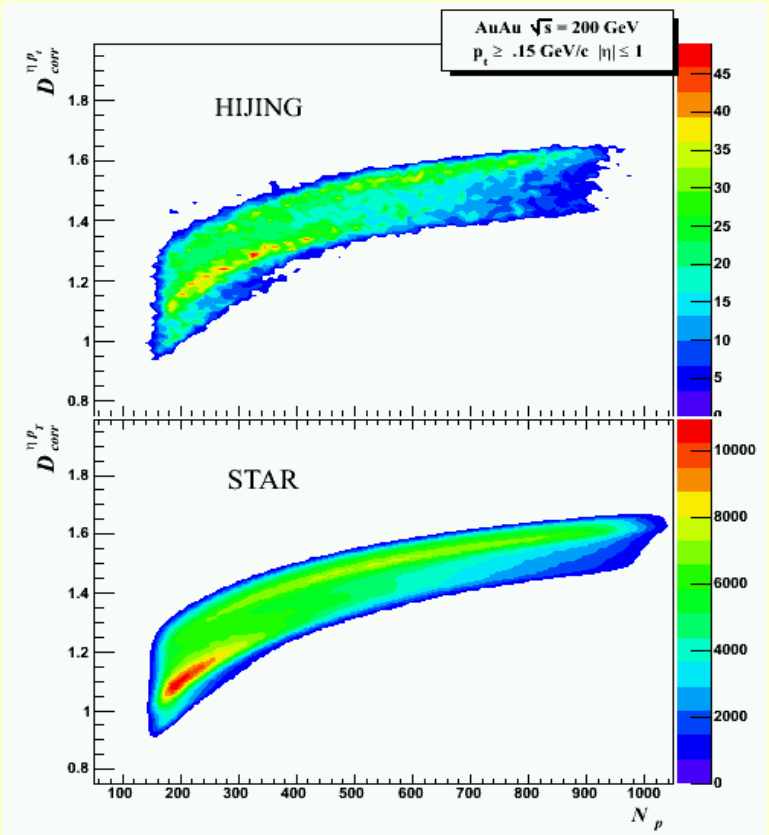


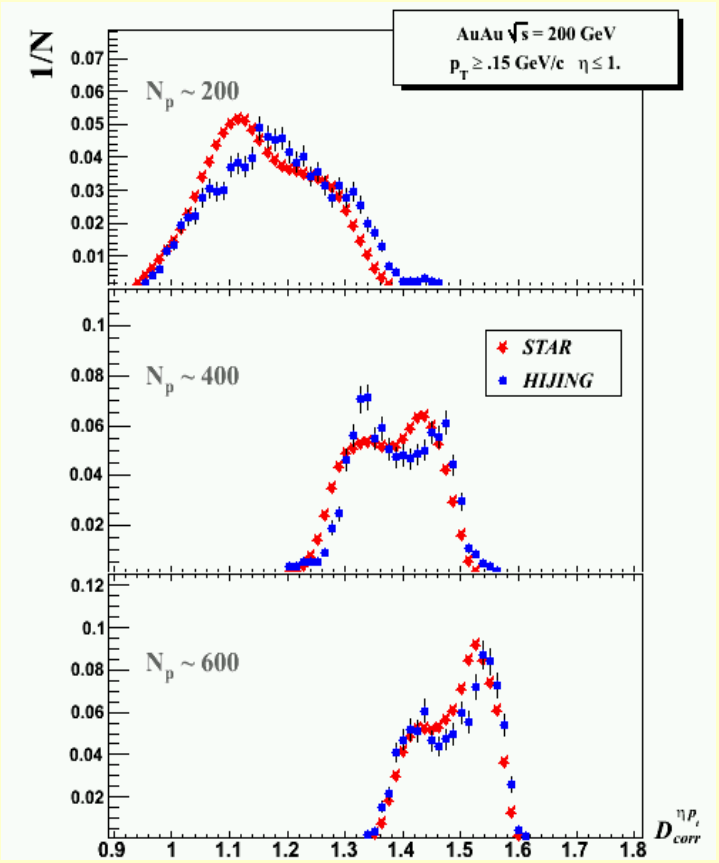
Summary

- ***Fractal dimension is sensitive to the global a structure***
- ***Fractal dimension analysis is useful to understand features in events***
- ***More specific methods which could be implemented to small number of the particles are needed***

STAR = 200 GeV

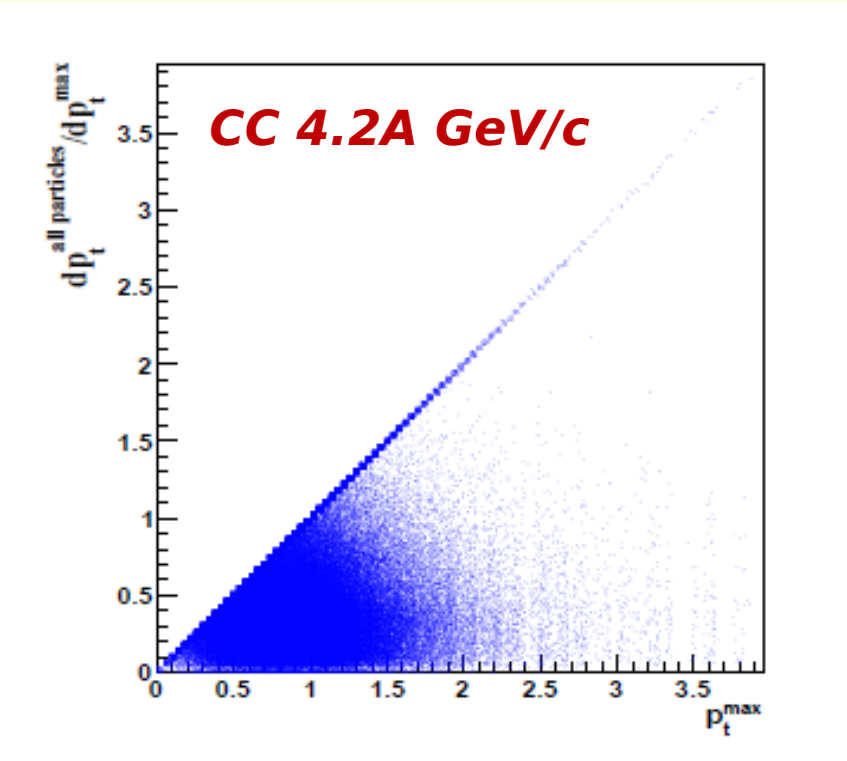
O.R. Proceedings of ICHEP 2006

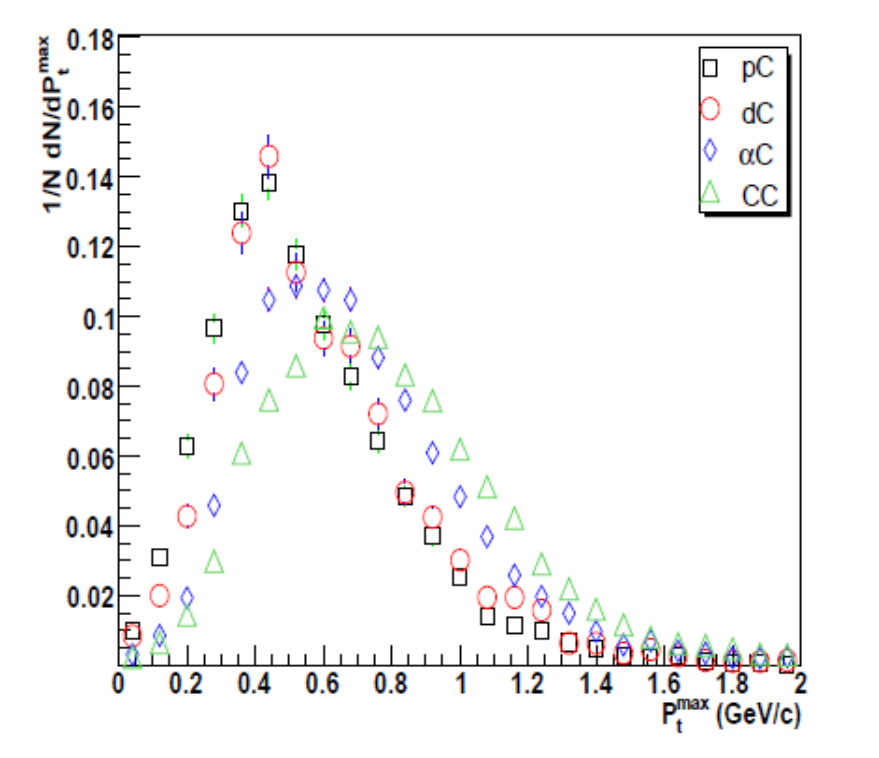




$FD \ in \ \eta p_t - space$

AA events @ 4.2A GeV/c

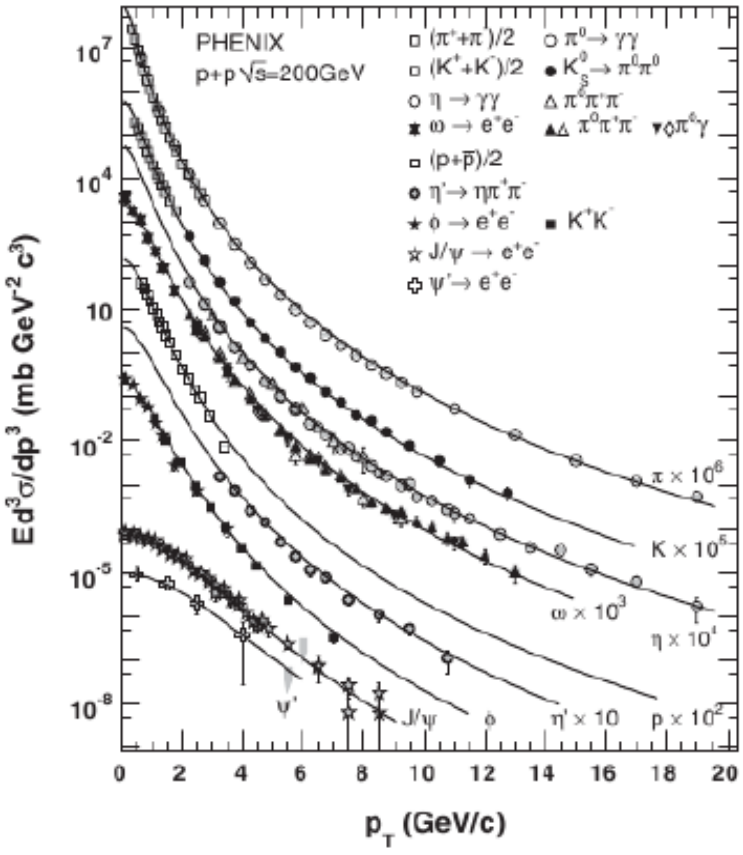




Shifts with A
Broadening with A

RHIC PP collisions

PHENIX collaboration
PRD 83, 052004 (2011)



Tsallis distribution

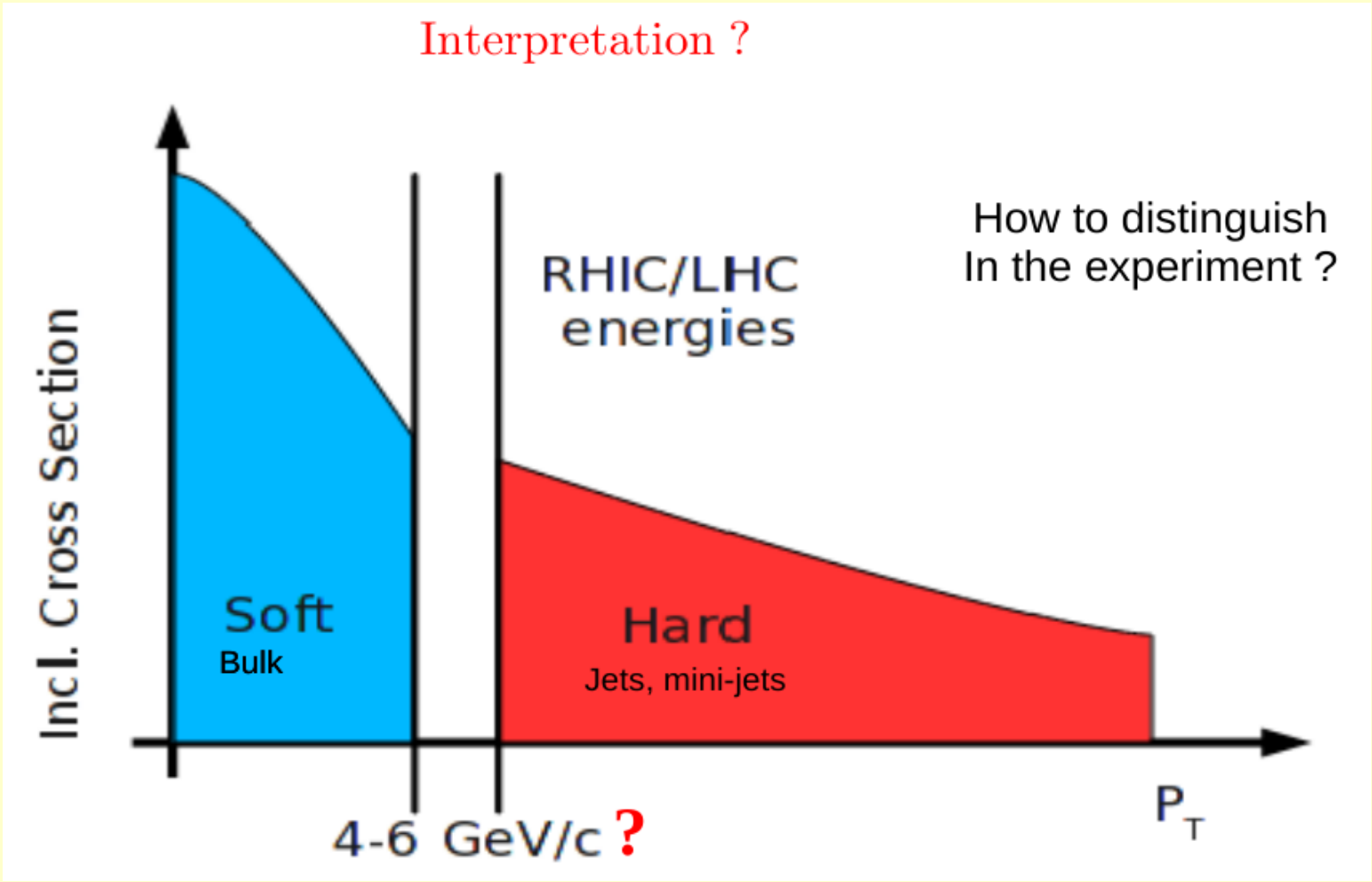
$$G_q(E) = C_q\left(1-(1-q)\frac{E}{T}\right)^{1/(1-q)},$$

$$n = -\frac{1}{1-q}.$$

$$E\frac{d^3\sigma}{dp^3} = \frac{1}{2\pi}\frac{d\sigma}{dy}\frac{(n-1)(n-2)}{(nT+m_0(n-1))(nT+m_0)}\left(\frac{nT+m_T}{nT+m_0}\right)^{-n}$$

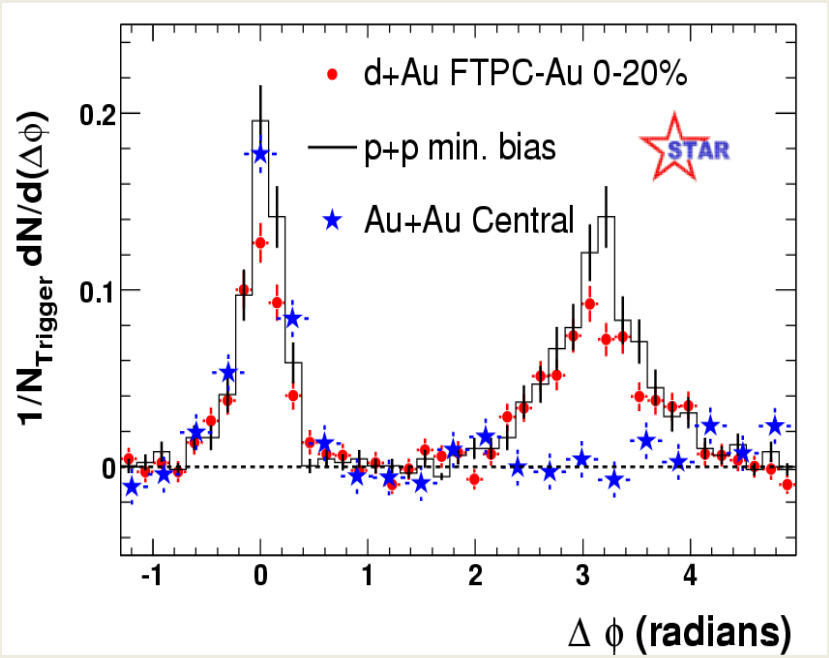
Invariant differential cross sections of different particles measured in p + p collisions at √s = 200 GeV in various decay modes.

High & low p_T hadron spectra



Disappearance of away side jet

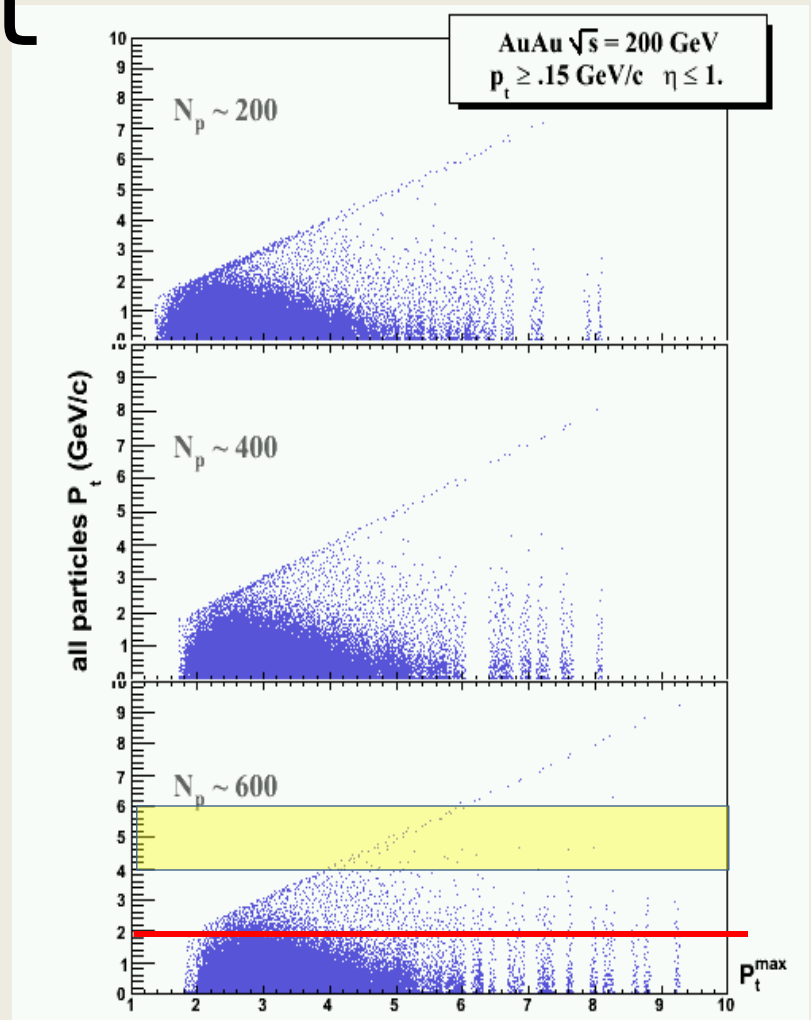
Phys. Rev. Lett. 91, 072304 (2003).



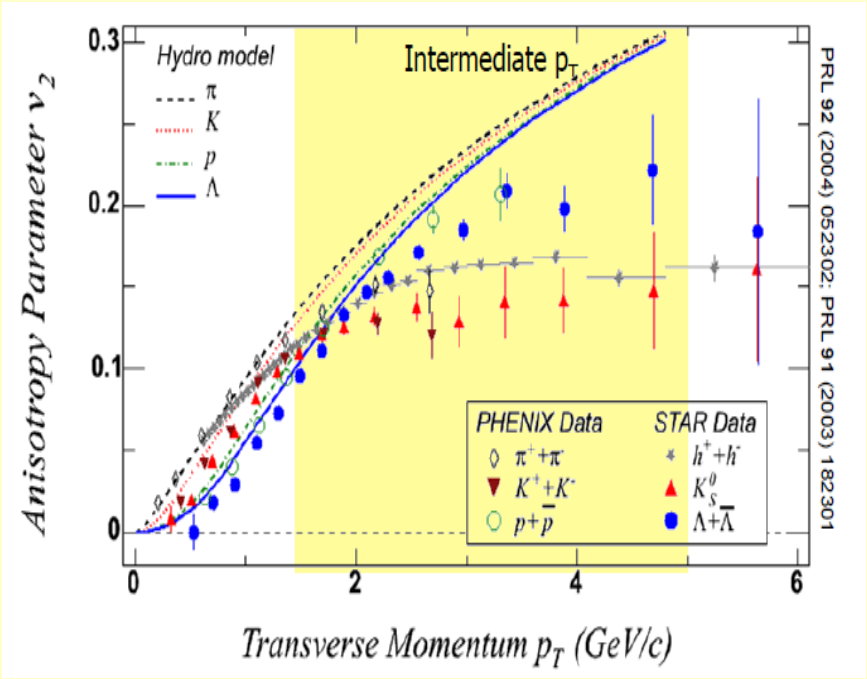
$|\eta| < 0.7$

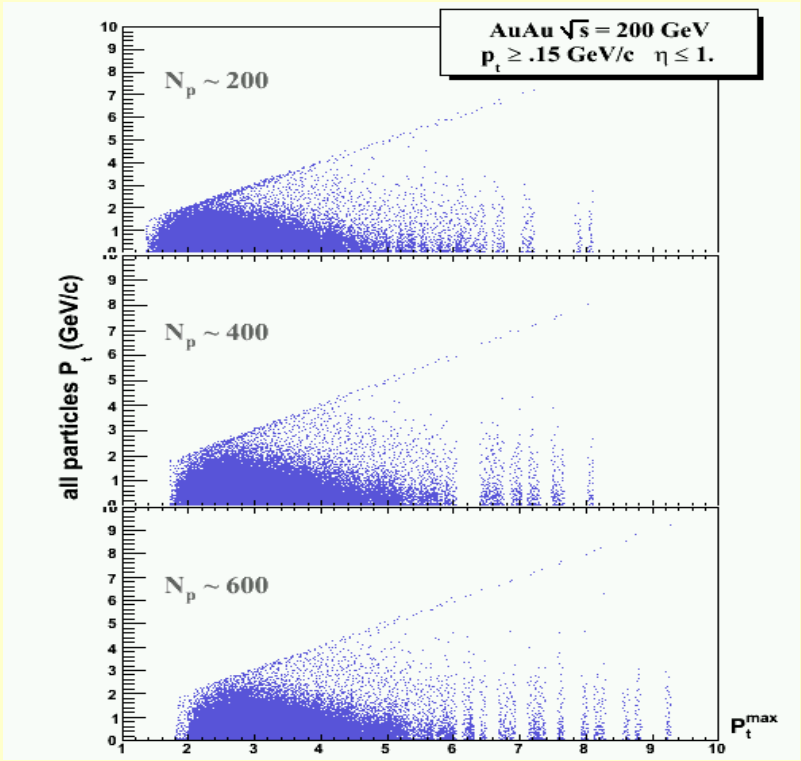
$4 < p_t^{trig} < 6$ GeV/c,

$2 < p_t^{associated} < p_t^{trig}$



Azimuthal anisotropy

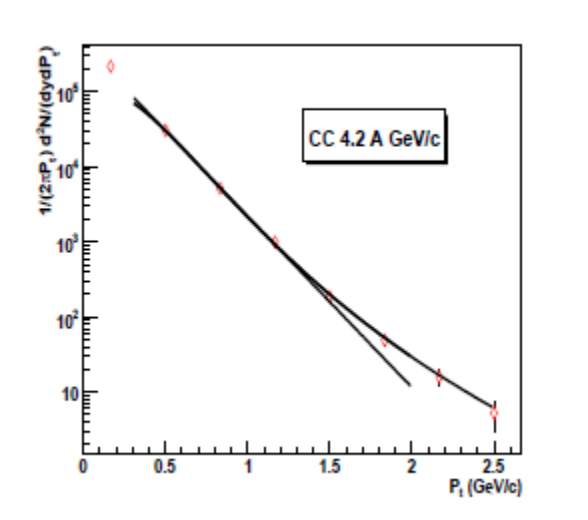




sensitivity to the system geometry for production at all measured

pC, dC, αC, CC @ 4.2A GeV/c

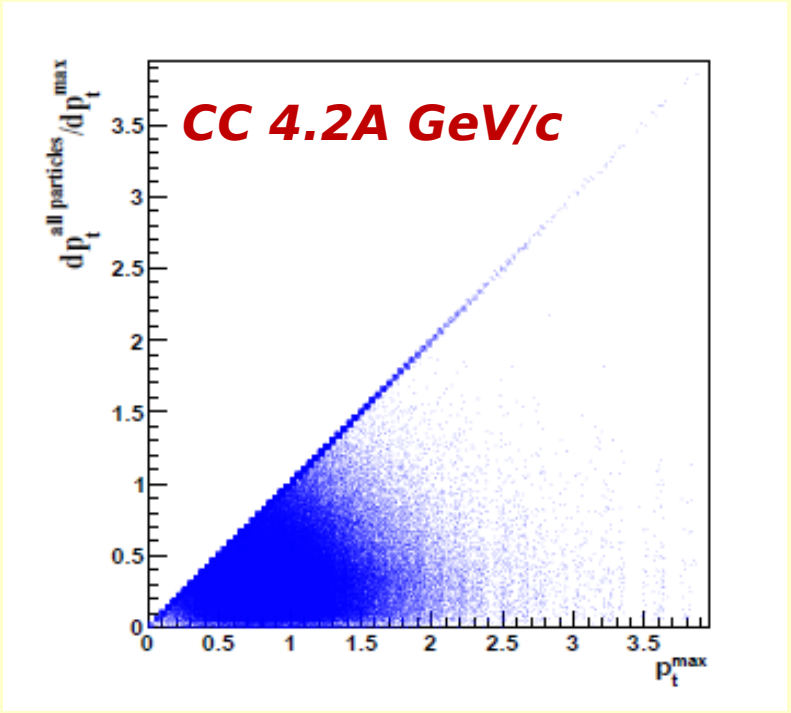
Interaction	Number of events
pC	5722
dC	3826
αC	9643
CC	15842



Levy distribution

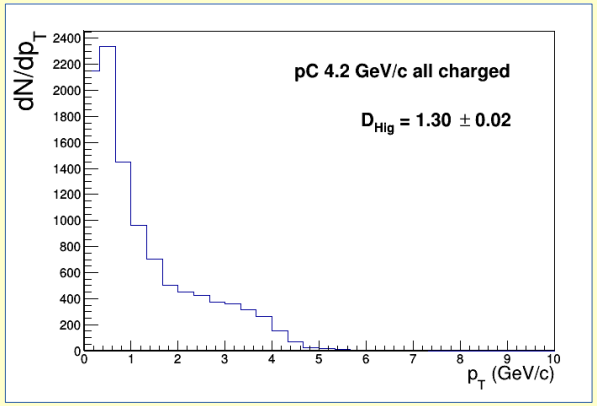
$$\frac{d^2N}{2\pi p_t dp_t dy} = \frac{B}{(1+(m_t - m_0)/nT)^n}$$

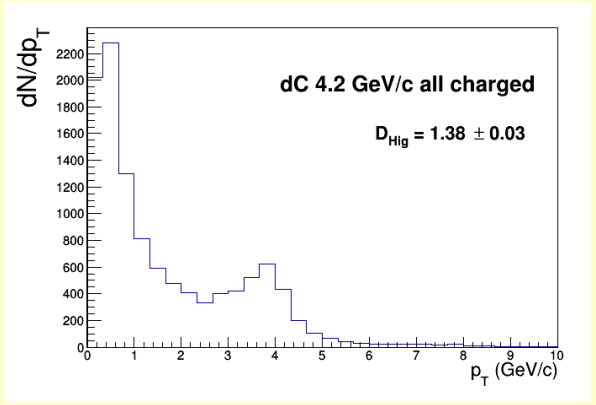
	parameter value			
	n	T	B	χ^2/ndf
pC	7.45 ± 1.22	6.86e-02 ± 6.04e-03	2.29e+04 ± 2.40e+03	0.19
dC	6.68 ± 1.10	6.63e-02 ± 6.78e-03	1.85e+04 ± 2.31e+03	0.19
αC	6.59 ± 0.57	7.08e-02 ± 3.66e-03	6.10e+04 ± 3.70e+03	1.20
CC	6.84 ± 0.39	7.69e-02 ± 2.39e-03	1.30e+05 ± 4.55e+03	3.20

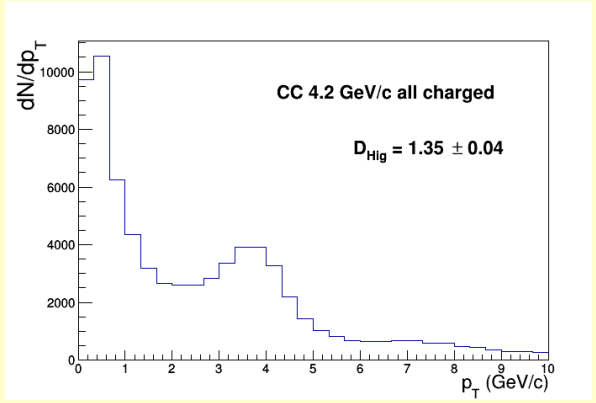


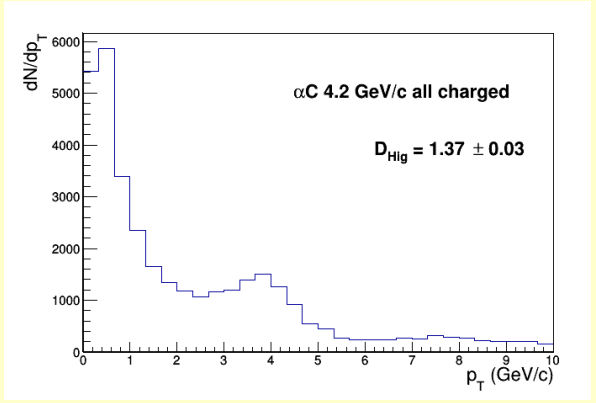
Higuchi’s method

Higuchi's method



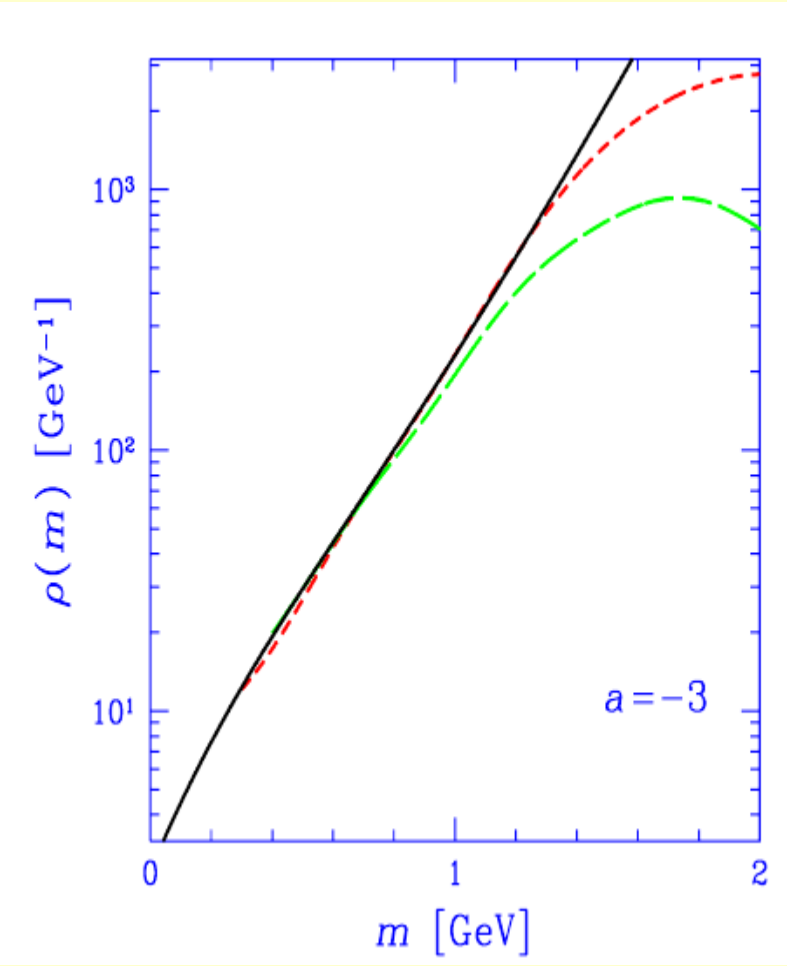






Thanks for attention
&
To organizers
for invitation

Hagedorn temperuture



The solid line is the fit:

$$\rho(m) \approx c(m_a^2 + m^2)^{a/2} \exp(m/T_{\rm H})$$

with $a = -3$, $m_a = 0.66$ **GeV**, $T_{\rm H} = 0.158$ **GeV**.
Long-dashed line: 1411 states of 1967.
Short-dashed line: 4627 states of 1996.

Experimental lines include Gaussian smoothing:

$$\rho(m) = \sum_{m^*=m_\pi, m_\rho, \ldots} \frac{g_{m^*}}{\sqrt{2\pi}\sigma_{m^*}} \exp\left(-\frac{(m-m^*)^2}{2\sigma_{m^*}^2}\right).$$

$\sigma = \Gamma/2$, $\Gamma = \mathcal{O}(200)$ MeV is the assumed width of the resonance, excluding the 'stable' pion, a special case.