Simulation of nonlinear dynamics of radiation formed by high-current beams of charged particles in multidimensional space-periodic structures

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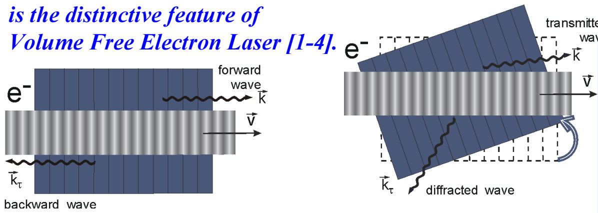
ABSTRACT

The principle of volume free electron lasers (VFEL) is based on the interaction of relativistic electron beam with two or more strong coupled electromagnetic waves generating in essentially non-one-dimensional geometry as a result of dynamical Bragg diffraction inside the resonator. Such resonator is a multidimensional space-periodic structure (natural or artificial electromagnetic (photonic) crystal). Previously VFEL were considered in various two- and three-wave diffraction geometries theoretically and experimentally. Here the general system of equations describing the various variants of multiple beam multi-wave VFEL is proposed. It takes into account multisection resonator, the dispersion of electromagnetic waves in the system, external reflectors etc. The mathematical modeling of two-beam two-wave VFEL was carried out using the proposed system of equations. It is shown that the change of current density of electron beams leads to change of VFEL chaotic dynamics and is one of the ways of chaos control in the system.

Volume (non-one-dimensional) multi-wave distributed feedback where electromagnetic waves and electron beam spread angularly one to other is the distinctive feature of Volume Free Electron Laser [1-4].



Diffraction condition: $2\mathbf{k}\boldsymbol{\tau}+\boldsymbol{\tau}^2\approx 0$

Synchronism condition: $|\omega-\mathbf{k}\mathbf{v}|\approx 0$

One-dimensional distributed feedback   Volume distributed feedback [3]

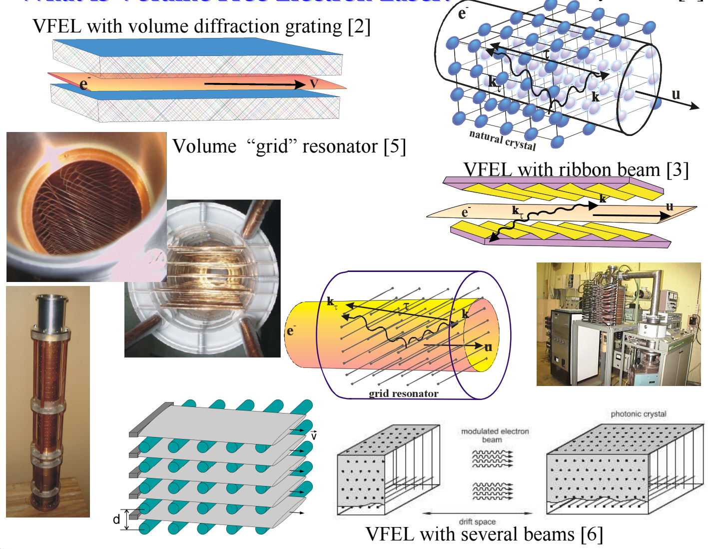
Use of volume distributed feedback makes available:

- The new law of instability for an electron beam passing through a spatially-periodic medium [1] provides the following estimation for threshold current in degeneration points: $j_{threshold}\sim 1\Big/\left[(kL)^3(k\chi_\tau L)^{2s}\right]$,

where **s** is a number of surplus waves appearing due to dynamical diffraction;

- significant reduction of threshold current of electron beam and, as a result, miniaturization of generator ;
- reduction of limits for available output power by the use of wide electron beams and diffraction gratings of large volumes.

What is Volume Free Electron Laser?



Systems of equations for all cases of VFEL are obtained from Maxwell equations in the slowly-varying envelope approximation and contain only first derivatives with respect to electromagnetic field amplitudes [7].

System for two-wave two-beam VFEL*:

$$\frac{\partial E}{\partial t}+\gamma_0 c\frac{\partial E}{\partial z}+0.5il E-0.5i\omega\chi_\tau E_\tau=2\pi j_1\Phi_1\int_0^{2\pi}\frac{2\pi-p}{8\pi^2}\left(e^{-i\theta_1(t,z,p)}+e^{-i\theta_1(t,z,-p)}\right)dp,$$

$$\frac{\partial E_\tau}{\partial t}+\gamma_\tau c\frac{\partial E_\tau}{\partial z}-0.5i\omega\chi_{-\tau}E+0.5i\omega l_1 E_\tau=2\pi j_2\Phi_2\int_0^{2\pi}\frac{2\pi-p}{8\pi^2}\left(e^{-i\theta_2(t,z,p)}+e^{-i\theta_2(t,z,-p)}\right)dp,$$

$$\frac{\partial^2\theta_1(t,z,p)}{\partial z^2}=\frac{e\Phi_1}{m\gamma_1^3\omega^2}\left(k-\frac{\partial\theta_1(t,z,p)}{\partial z}\right)^3\mathrm{Re}\left(E(t-z/u_1,z)\right)\times\exp(i\theta_1(t,z,p)),$$

$$\frac{\partial^2\theta_2(t,z,p)}{\partial z^2}=\frac{e\Phi_2}{m\gamma_2^3\omega^2}\left(k_\tau-\frac{\partial\theta_2(t,z,p)}{\partial z}\right)^3\mathrm{Re}\left(E_\tau(t-z/u_2,z)\right)\times\exp(i\theta_2(t,z,p)),$$

$$E(t,0)=E_0,\quad E_\tau(t,\bullet)=E_{\tau 0},$$

$$\frac{\partial\theta_1(t,0,p)}{\partial z}=k-\omega/u_1,\quad \theta_1(t,0,p)=p,$$

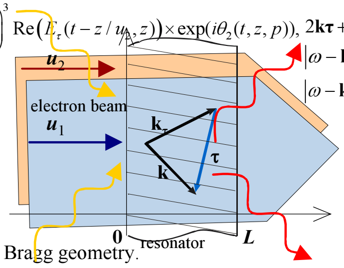
$$\frac{\partial\theta_2(t,\bullet,p)}{\partial z}=k_\tau-\omega/u_2,\quad \theta_2(t,\bullet,p)=p,$$

$$t>0,\quad z\in[0,L],\quad p\in[-2\pi,2\pi],$$

$2\mathbf{k}\boldsymbol{\tau}+\boldsymbol{\tau}^2\approx 0$

$|\omega-\mathbf{k}\mathbf{u}_1|\approx 0$

$|\omega-\mathbf{k}_\tau\mathbf{u}_2|\approx 0$



- • means 0 or L depending on Laue or Bragg geometry.

$E(t,z)$ and $E_\tau(t,z)$ are amplitudes of transmitted and diffracted waves.

$$l_0=\frac{k^2-\omega^2\varepsilon_0}{\omega^2},\quad l_\tau=\frac{k_\tau^2-\omega^2\varepsilon_0}{\omega^2}$$

$\delta_{0,\tau}$ are detuning from synchronism condition.

$\gamma_{0,1}$ are direction cosines, $\beta_0=\gamma_0/\gamma_1$ is an asymmetry factor.

χ_0, $\chi_{\pm\tau}$ are Fourier components of resonator dielectric susceptibility.

$\Phi_{1,2}=\sqrt{l_{0,\tau}+\chi_0-1/(u_{1,2}/c\gamma_{1,2})^2}\,\gamma_{0,\tau}$ are the Lorenz factor.

It easily goes into the system for two-wave one-beam VFEL[7].

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Some numerical results on VFEL chaotic nature [6-10]

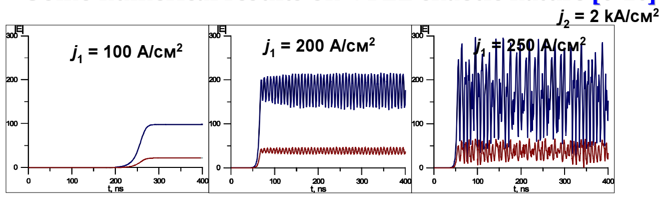
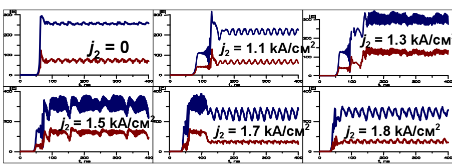


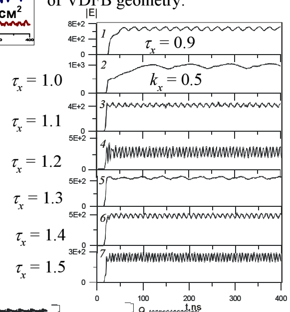
Fig 1. Chaos control of two-wave two-beam VFEL in Bragg geometry



j_1 = 400 A/cм²

Fig 2. Chaos control of two-wave two-beam VFEL

Fig 3. The way of chaos control in VFEL can be realized via changing of VDFB geometry.



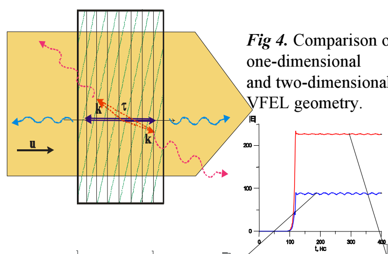


Fig 4. Comparison of one-dimensional and two-dimensional VFEL geometry.

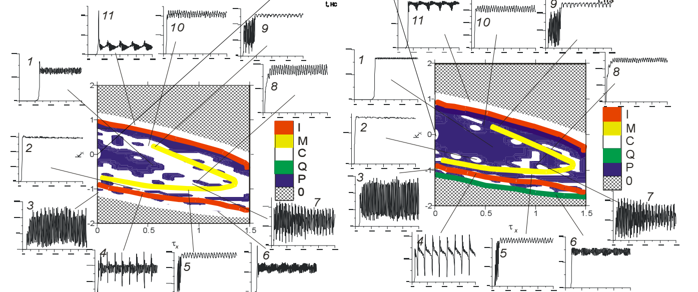


Fig 5. Root to chaotic lasing in two-wave one-beam VFEL. 0 depicts a domain under beam current threshold. P, Q, C, I correspond to periodic regimes, quasiperiodicity, chaos and intermittency, respectively. M describes domains with transitions between large-scale and small-scale amplitudes.