Investigation of chaos in radiation of charged particles moving in non-one-dimensional periodical structures

BSU INP

Svetlana Sytova
Research Institute for Nuclear Problems, Belarusian State University
sytova@inp.bsu.by

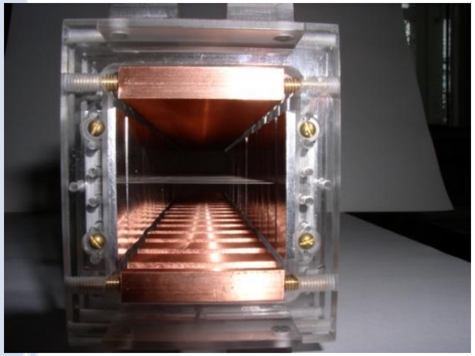
The history of Volume Free Electron Lasers (VFEL) experimental investigation

1996 **Experimental modeling of electrodynamic processes in volume diffraction grating made from dielectric threads**

V.G.Baryshevsky et al., NIM 393A (1997) 71

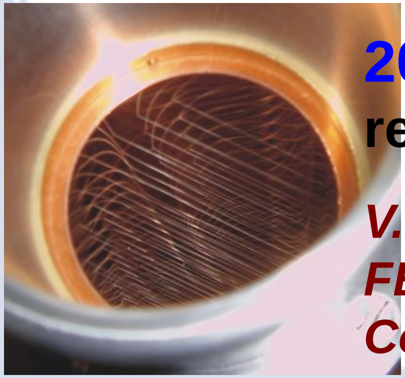
2001 **The first VFEL generation in the millimeter range. Experimental verification of VFEL principles. Demonstration of frequency tuning for a fixed electron energy**

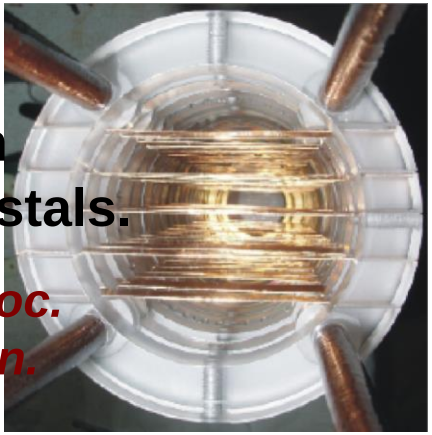
V.G.Baryshevsky et al., NIM 483 A (2002) 21



2004 **VFEL with grid resonator**

V.G. Baryshevsky et al., NIM. B 252 (2006) 86





2009 **VFEL with grid and foil resonators. Such resonators have all properties of photonic crystals.**

V. G. Baryshevsky et al. Proc. IRMMW-THz 2010; Proc. FEL2010. Nuovo Cimento 34 (2011), 199, Nonl. Phen. Complex Syst., vol. 16, no. 3 (2013), 209 - 216

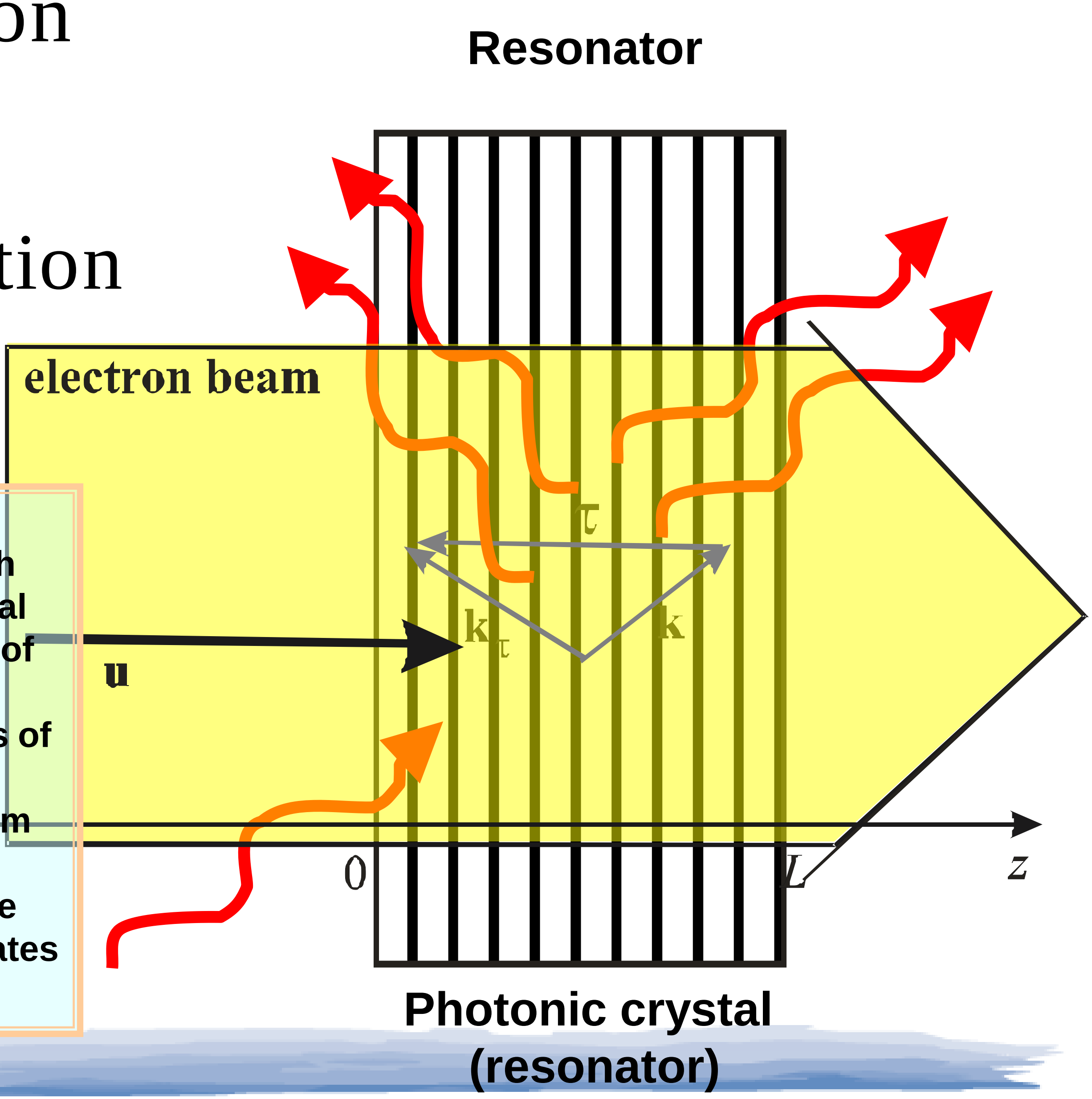
Main physical VFEL principles

Diffraction condition

$$2\mathbf{k}\boldsymbol{\tau} + \boldsymbol{\tau}^2 \approx 0$$

Synchronism condition

$$|\omega - \mathbf{k}\mathbf{u}| = \delta\omega \approx 0$$



Interacting of the electron beam with electromagnetic field in VFEL is much more efficient than in one-dimensional situation because the group velocity of electromagnetic waves decreases sharply due to continuous reflections of them at periodic planes of resonator. Moreover VFEL is an oversized system where relativistic electron beams of broad cross-section can be used. Due to this and VDFB electron beam radiates more effectively.

Maxwell equations:

$$\Delta\mathbf{E} - \nabla(\nabla\mathbf{E}) - \frac{1}{c^2}\frac{\partial^2\mathbf{D}}{\partial t^2} = \frac{\partial\mathbf{j}_\mathrm{b}}{\partial t},$$

n-wave approximation:

$$\mathbf{E} = \sum_{j=1}^{n}\mathbf{e}E_j e^{i(\mathbf{k}_j\mathbf{r}-\omega t)},$$

$$\mathbf{j}_\mathrm{b} = \sum_{l=1}^{m}\mathbf{e}j_l e^{i(\mathbf{k}_j\mathbf{r}-\omega t)},$$

Electrical induction:

$$\mathbf{D}(\mathbf{r},t) \approx \varepsilon(\mathbf{r},\omega)\mathbf{E}(\mathbf{r},t), \qquad \varepsilon(\mathbf{r},\omega) = \sum\varepsilon(\tau,\omega)\exp(-i\tau\mathbf{r}),$$

$$\varepsilon(0,\omega) = 1 + \chi_0,\ \varepsilon(\tau,\omega) = \chi_\tau,\ \varepsilon(-\tau,\omega) = \chi_{-\tau}$$

Motion equation:

$$\frac{d\mathbf{p}}{dt} = e\left\{\mathbf{E} + \frac{1}{c}[\mathbf{v}\times\mathbf{H}]\right\}, \qquad \mathbf{p} = m\gamma\mathbf{v},$$

$$\frac{d^2z}{dt^2} = \frac{e}{m\gamma^3}(\mathbf{en})\mathrm{Re}\left[Ee^{i\theta(t,t_0,\mathbf{r}_\perp)}\right]$$

We use the method of averaging over initial phases of electron entrance in the resonator

$$\theta(t,t_0,\mathbf{r}_\perp) = k_z z + \mathbf{k}_\perp\mathbf{r}_\perp - \omega t(z,t_0)$$ **electron phase in a wave**

Equations for electron beam of broad cross-section

$$\frac{d^2\theta(t,z,p)}{dz^2} = \frac{e\Phi}{m\gamma^3\omega^2}\left(k - \frac{d\theta(t,z,p)}{dz}\right)^3 \mathrm{Re}\big(E(t - z/u, z)\times$$
$$\times\exp(i\theta(t,z,p))\big),$$

$$\frac{d\theta(t,0,p)}{dz} = k - \omega/u, \quad \theta(t,0,p) = p,$$

$$t > 0, \quad z \in [0, L], \quad p \in [-2\pi, 2\pi]$$

$$\Phi = \sqrt{l_0 + \chi_0 - 1/(u/c\gamma)^2}$$

$\theta(t,z,p)$ **is an electron phase in a wave**

Two-wave VFEL

$$\frac{\partial E}{\partial t} + \gamma_0 c \frac{\partial E}{\partial z} + 0.5 i\, l E - 0.5 i \omega \chi_\tau E_\tau = I,$$

$$\frac{\partial E_\tau}{\partial t} + \gamma_1 c \frac{\partial E_\tau}{\partial z} - 0.5 i \omega \chi_{-\tau} E + 0.5 i \omega\, l_1 E_\tau = 0,$$

$$I = 2\pi j \Phi \int_0^{2\pi} \frac{2\pi - p}{8\pi^2} \left(e^{-i\theta(t,z,p)} + e^{-i\theta(t,z,-p)} \right) dp,$$

$$E(t,0) = E_0, \quad E_\tau(t,L) = E_{\tau 0}$$

$$\Phi = \sqrt{l_0 + \chi_0 - 1/(u/c\gamma)^2},$$

$$l_0 = \frac{\mathbf{k}^2 c^2 - \omega^2 \varepsilon_0}{\omega^2},$$

$$l_1 = \frac{\mathbf{k}_\tau^2 c^2 - \omega^2 \varepsilon_0}{\omega^2},$$

$$l = l_0 + \delta,$$

$$\mathbf{k}_\tau = \mathbf{k} + \boldsymbol{\tau}$$

$\gamma_{0,1}$ **are direction cosines,**

δ **is departure from synchronism conditions.**

χ_0, $\chi_{\pm\tau}$ **are Fourier components of the dielectric susceptibility of the target.**

Numerical algorithm*

$$\hat{\theta}^{j}_{\bar{z}z} = \Psi\left[\left(k - \hat{\theta}^{j}_{\overset{o}{z}}\right)^{3}\right] \mathrm{Re}\left(\tilde{E}_0 \exp\left(i\theta^{j}\right)\right),$$

$$E_{t} + a_1 \hat{E}_{\bar{z}} + b_{11}\hat{E} + b_{12}\hat{E}_{\tau} =$$

$$= \Phi \sum_{j=0}^{N} c_j \left(\exp(-i\hat{\theta}^{j}) + \exp(-i\hat{\theta}^{-j})\right),$$

$$E_{\tau t} + a_2 \hat{E}_{\tau\tilde{z}} + b_{21}\hat{E} + b_{22}\hat{E}_{\tau} = 0$$

*** Batrakov K., Sytova S. Computational Mathematics and Mathematical Physics 45: 4 (2005) 666–676**

Energy conservation law

Using the Chu's kinetic power theorem for coupled electromagnetic waves we obtain

$$\frac{\partial W}{\partial t} + P + P_{\tau} = C\eta,$$

$$\eta = \int_0^{2\pi} \frac{2\pi - p}{8\pi^2} \frac{2u^2 - v^2(z = L, p) - v^2(z = L, -p)}{u^2} dp$$ is the electron efficiency

$$W = EE^* + E_{\tau}E_{\tau}^* = |E|^2 + |E_{\tau}|^2$$ is the electromagnetic energy stored in the resonator

$$P = C_1|E(L)|^2 \text{ and } P_{\tau} = C_2|E_{\tau}(0)|^2$$

represent radiation losses associated with the transmitted wave and diffracted wave respectively

Two-beam two-wave VFEL

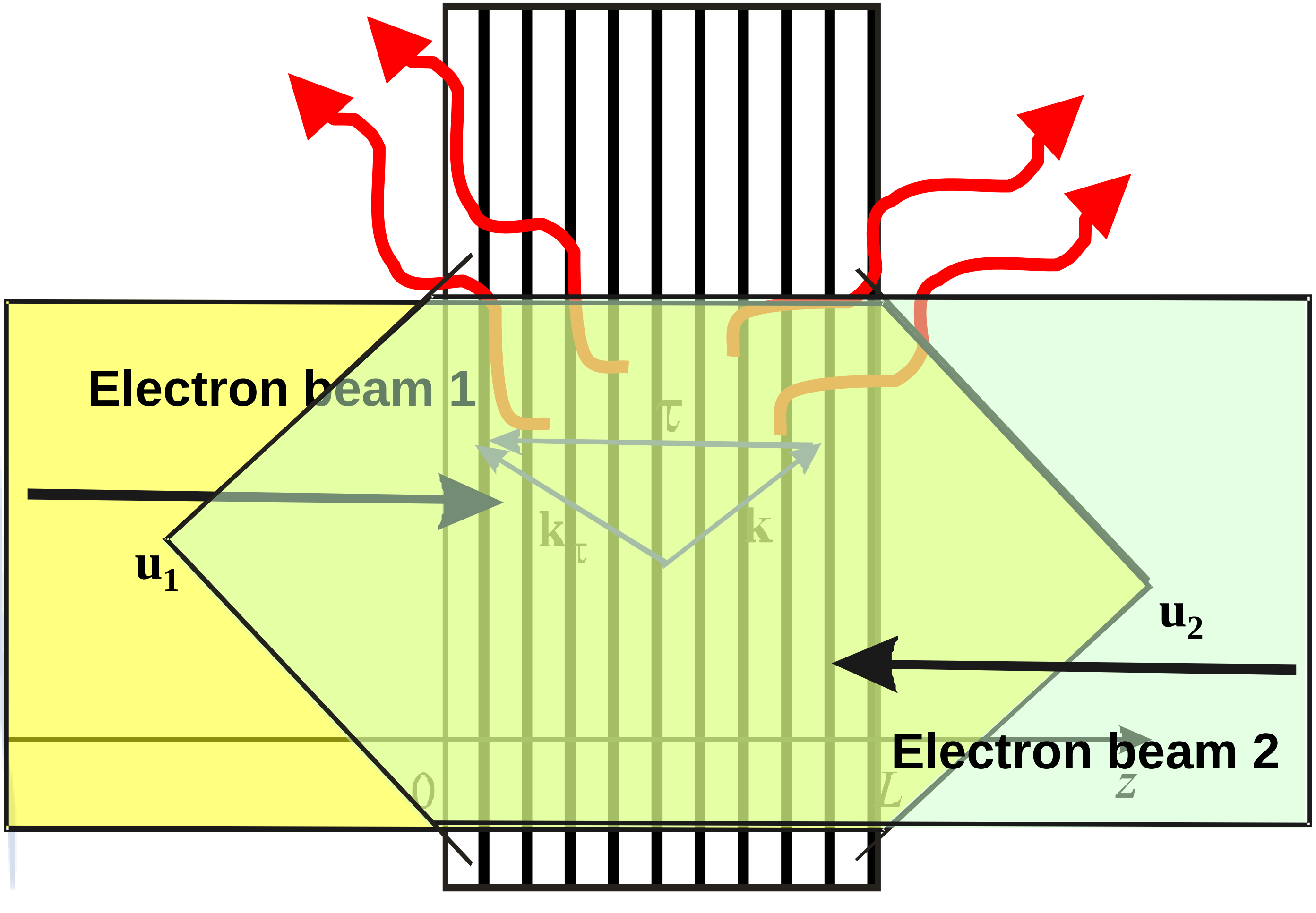
Diffraction condition

$$2\mathbf{k\tau} + \boldsymbol{\tau}^2 \approx 0$$

Synchronism conditions:

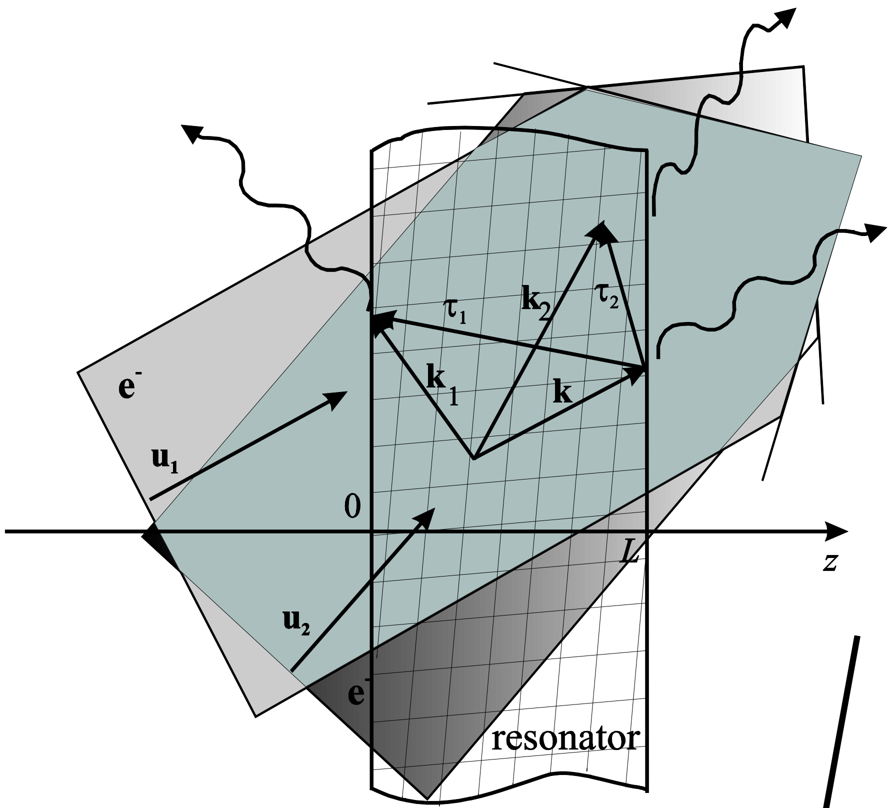
$$|\omega - \mathbf{k u}_1| \approx 0,$$

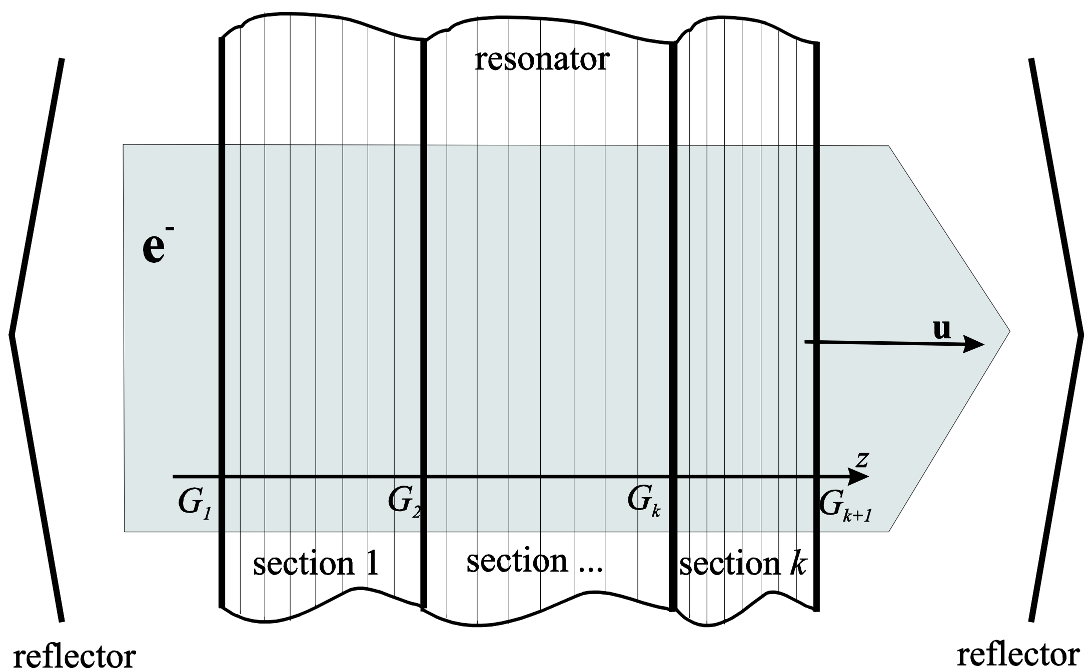
$$|\omega - \mathbf{k}_\tau \mathbf{u}_2| \approx 0$$



Photonic crystal

Other VFEL schemes





Common system for m-beam n-wave VFEL

$$\mathbf{A}\frac{\partial \mathbf{E}}{\partial t}+\mathbf{B}\frac{\partial \mathbf{E}}{\partial z}+\mathbf{C}\mathbf{E}=\mathbf{I},$$

$$\mathbf{E}(t,\Gamma_1)=\mathbf{E}^0+\mathbf{D}\mathbf{E}(t,\Gamma_2),$$

$$I_l=\Phi_l\int_0^{2\pi}\frac{2\pi-p}{8\pi^2}\left(e^{-i\theta_l(t,z,p)}+e^{-i\theta_l(t,z,-p)}\right)dp,$$

$$\mathbf{E}(t,z)=(E_1,\ldots,E_n)^T,\quad \mathbf{I}=(I_1,\ldots,I_m,0,\ldots0)^T,$$

$$j=1,\ldots,n,\quad l=1,\ldots,m,\quad m\le n;$$

$$\frac{\partial^2\theta_l(t,z,p)}{\partial z^2}=\Psi_l\left(k_l-\frac{\partial\theta_l(t,z,p)}{\partial z}\right)^3\mathrm{Re}\left(E_l(t-z/u,z)e^{i\theta_l(t,z,p)}\right),$$

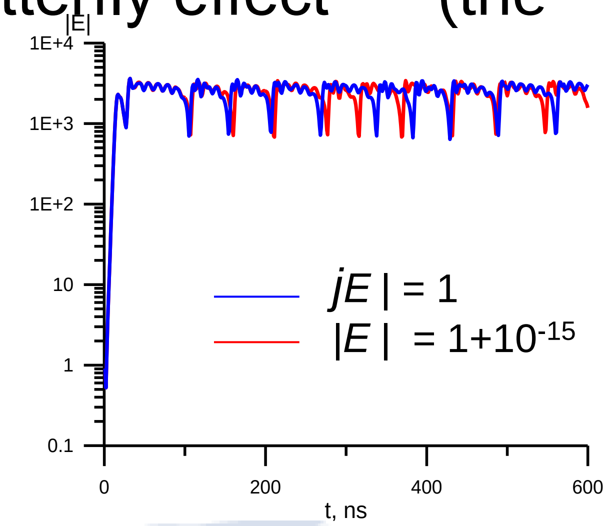
$$\frac{\partial\theta_l(t,0,p)}{\partial z}=k_l-\omega/u_l,\quad \theta_l(t,0,p)=p,\quad l=1,\ldots,m$$

$$t>0,\quad z\in[0,L],\quad p\in[-2\pi,2\pi]$$

Dynamical systems*

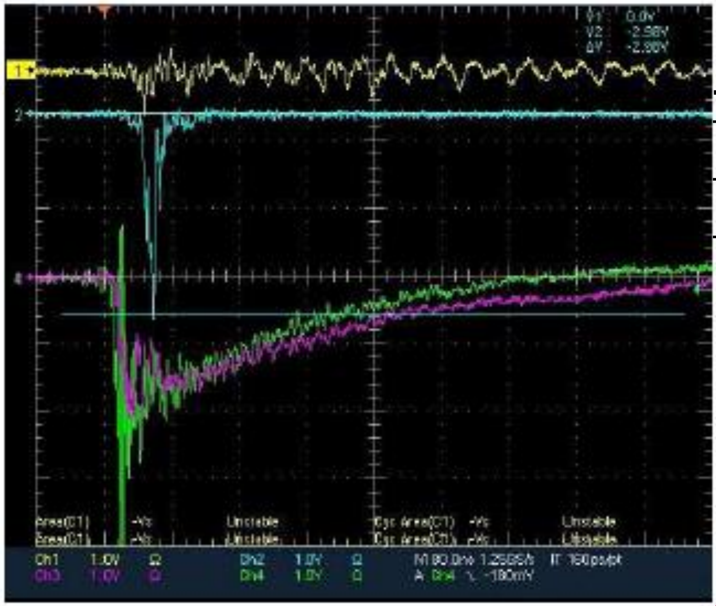
Chaotic dynamics means the tendency of wide range of systems to transition into states with deterministic behavior and unpredictable behavior. Bifurcation is any qualitative changes of the system when control parameter μ passes through the bifurcational value μ_0.

Nonlinearity is necessary but non-sufficient condition for chaos in the system. The main origin of chaos is the exponential divergence of initially close trajectories in the nonlinear systems. This is so-called the “Butterfly effect”** (the sensibility to initial conditions).



H.-G. Schuster, "Deterministic Chaos" An Introduction, *Physik Verlag, (1984)
**** E.N. Lorenz, *J. Atmos. Sci. 20 (1963), 130***

What happens in electronic vacuum devices

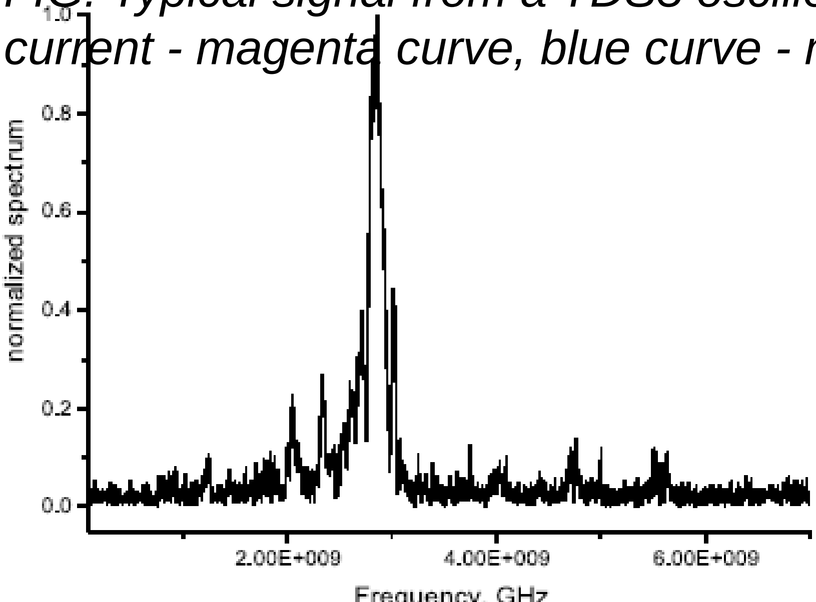


➢<u>Investigation of transfer from a chaotic to a single-frequency regime of generation by the relativistic beams moving in one-dimensional periodic structures in regime of BWO:</u>

- **shifting the operating point of generation close to the upper end of a pass band wave and using the adjustable feedback reflector for a backward wave;**
- **suppress the parasitic generation waves and obtain a single-mode oscillation.***

**** V. G. Baryshevsky, P. V. Molchanov, NPCS, 16, (2013), 209***

FIG. Typical signal from a TDS5 oscilloscope: diode voltage - green curve, diode current - magenta curve, blue curve - microwave signal.



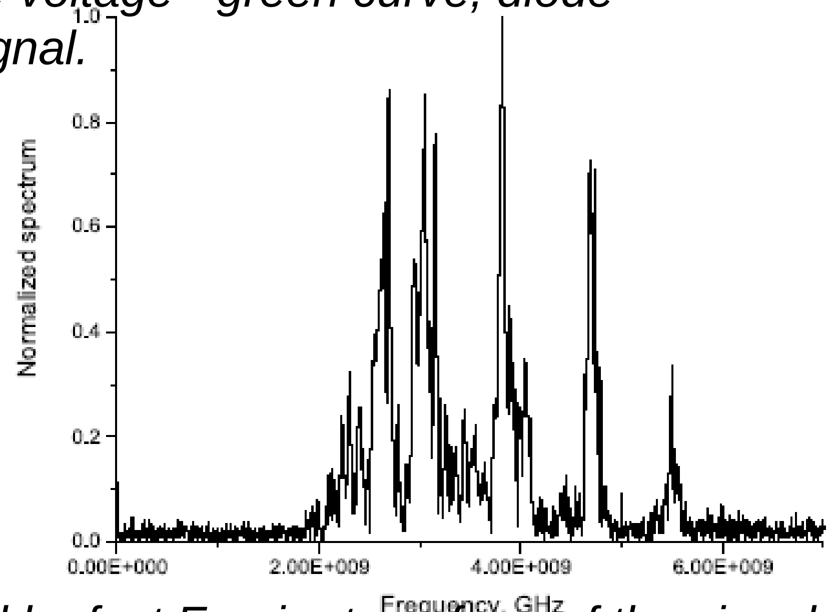
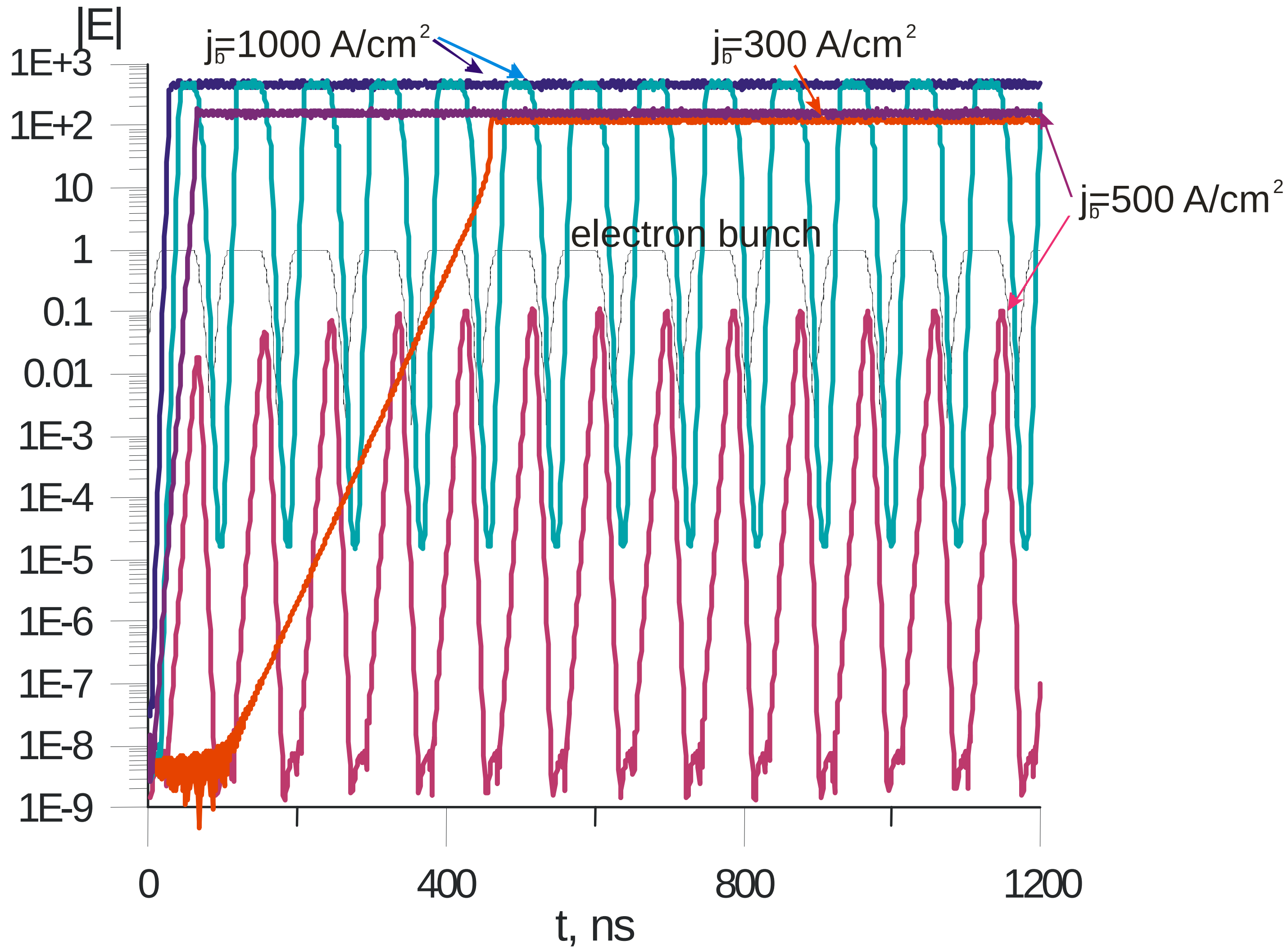


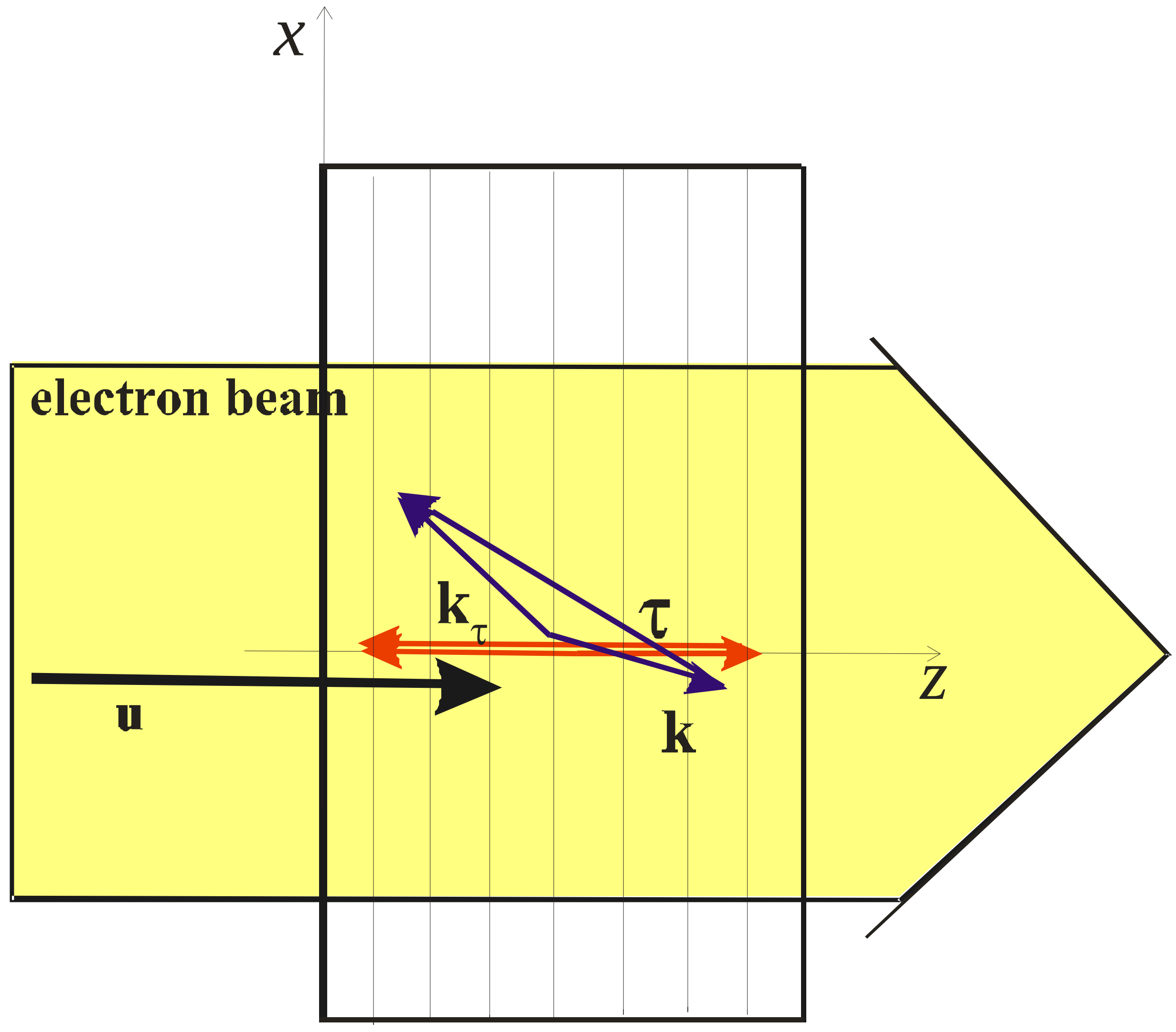
FIG. Typical single mode radiation spectrum obtained by fast Fourier transform of the signal.

Comparison of continuous electron beam and electron bunches



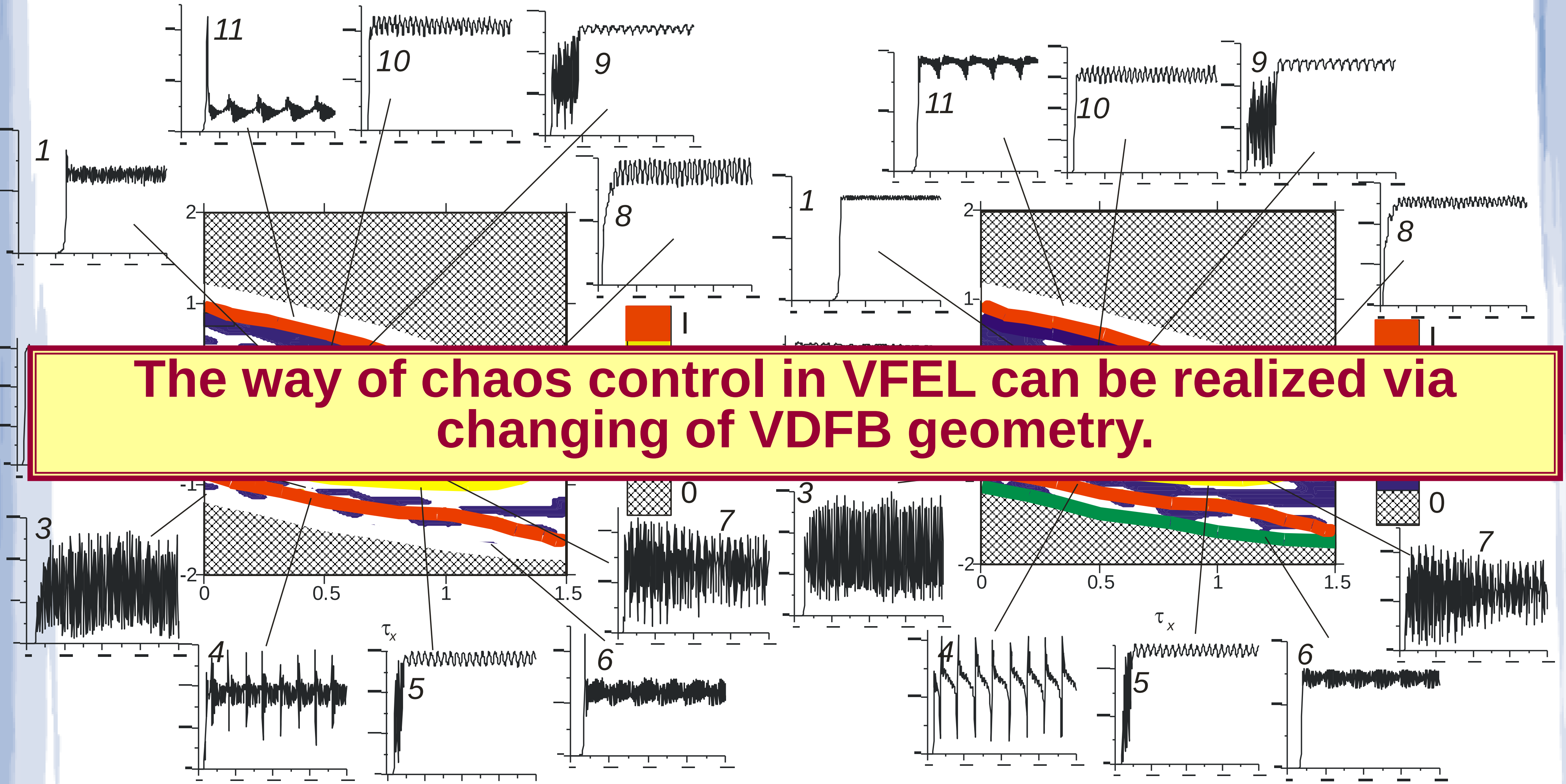
It is shown that the beam parameters affect the generation regime.

Comparison of one-dimensional and two-dimensional distributed feedback



We will change one-dimensional geometry by varying transverse components of vectors $\mathbf{k}$ and $\boldsymbol{\tau}$ as well as detuning from synchronism condition δ.

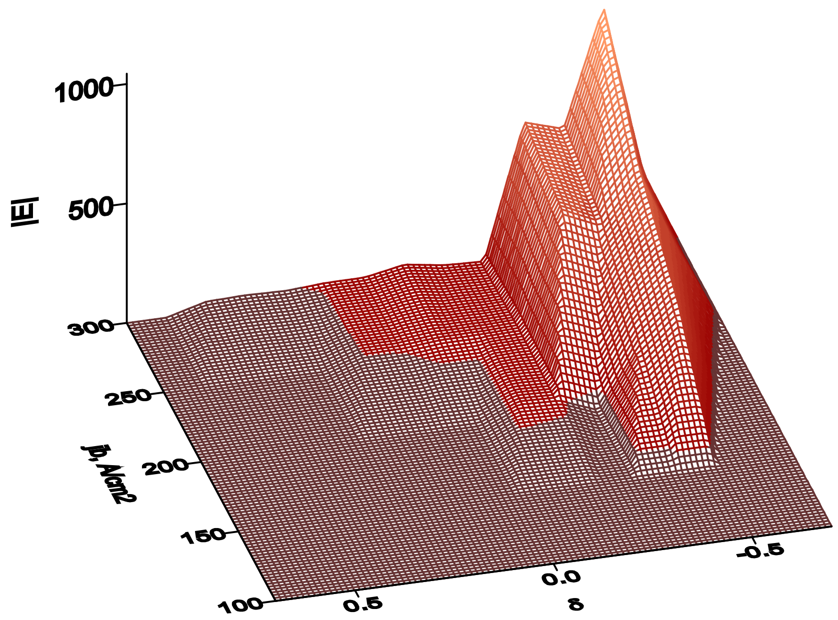
Parametric maps of the transition to chaos in dependence of the diffraction geometry



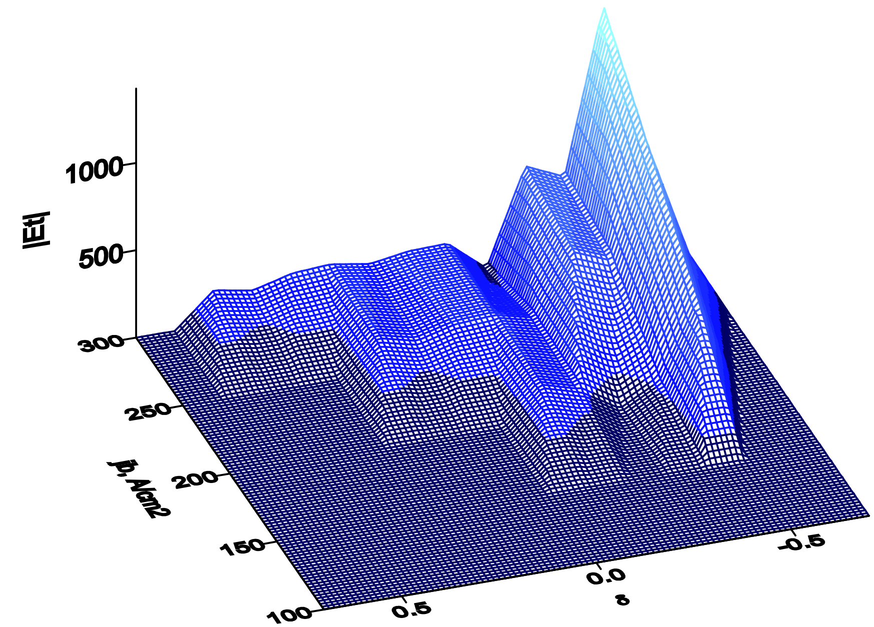
The way of chaos control in VFEL can be realized via changing of VDFB geometry.

0 depicts a domain under generation threshold. P, Q, C correspond to periodic regimes, quasiperiodicity and chaos, respectively. M describes domains with transitions between large-scale and small-scale amplitudes. I stands for intermittency. On edges the most typical dependencies of amplitudes on going crystal on time are presented.

Dependence of amplitudes at VFEL boundaries from beam current j and departure from synchronism condition δ



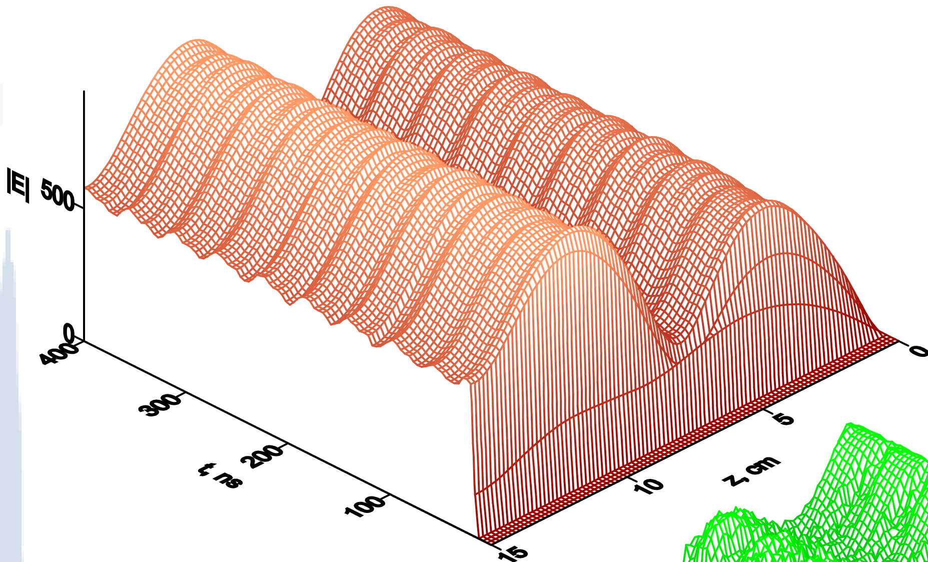
for transmitted wave

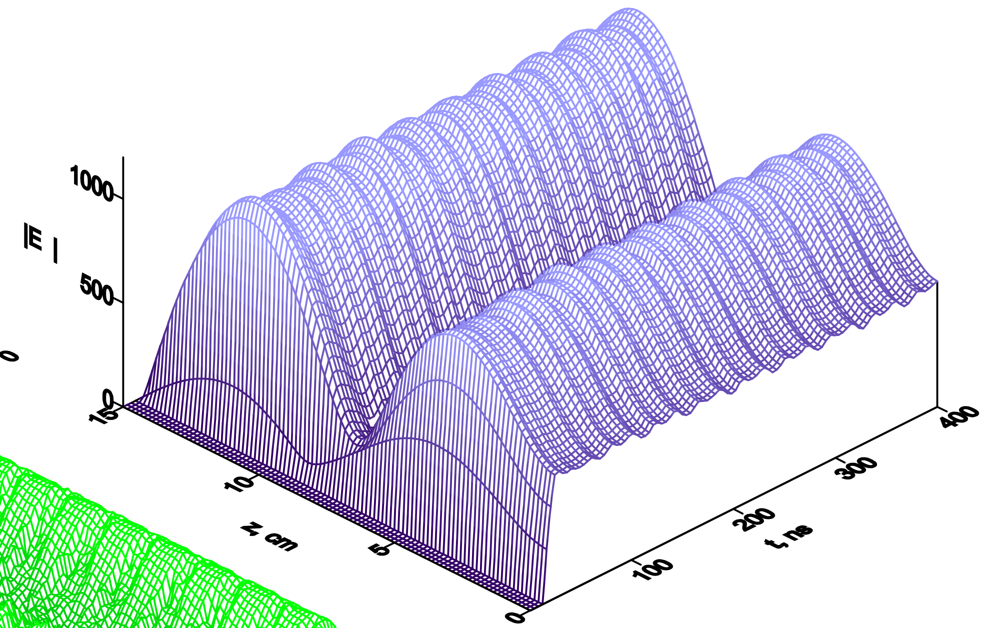


for diffracted wave

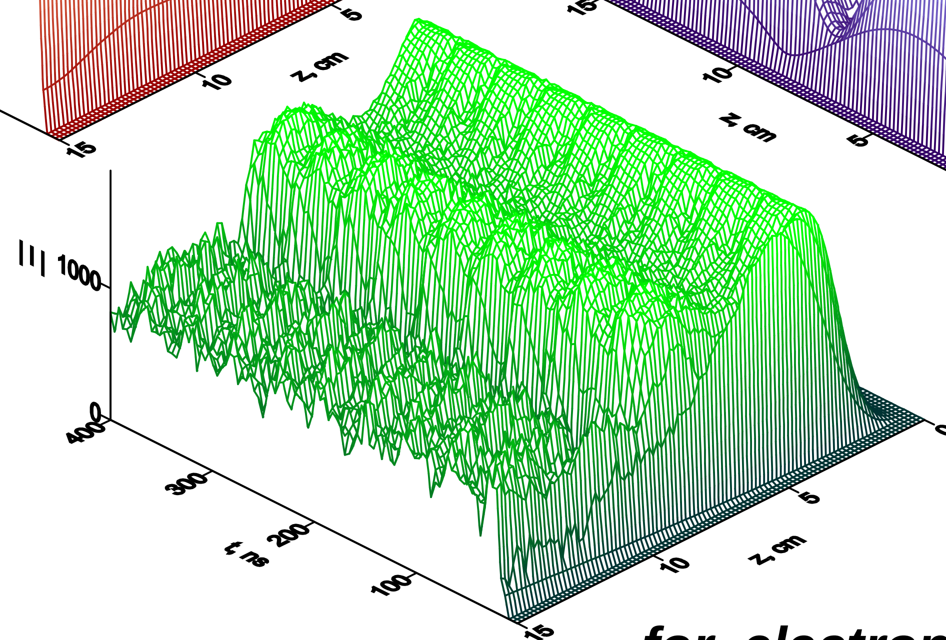
Suppression of parasitic modes inside VFEL

for transmitted wave



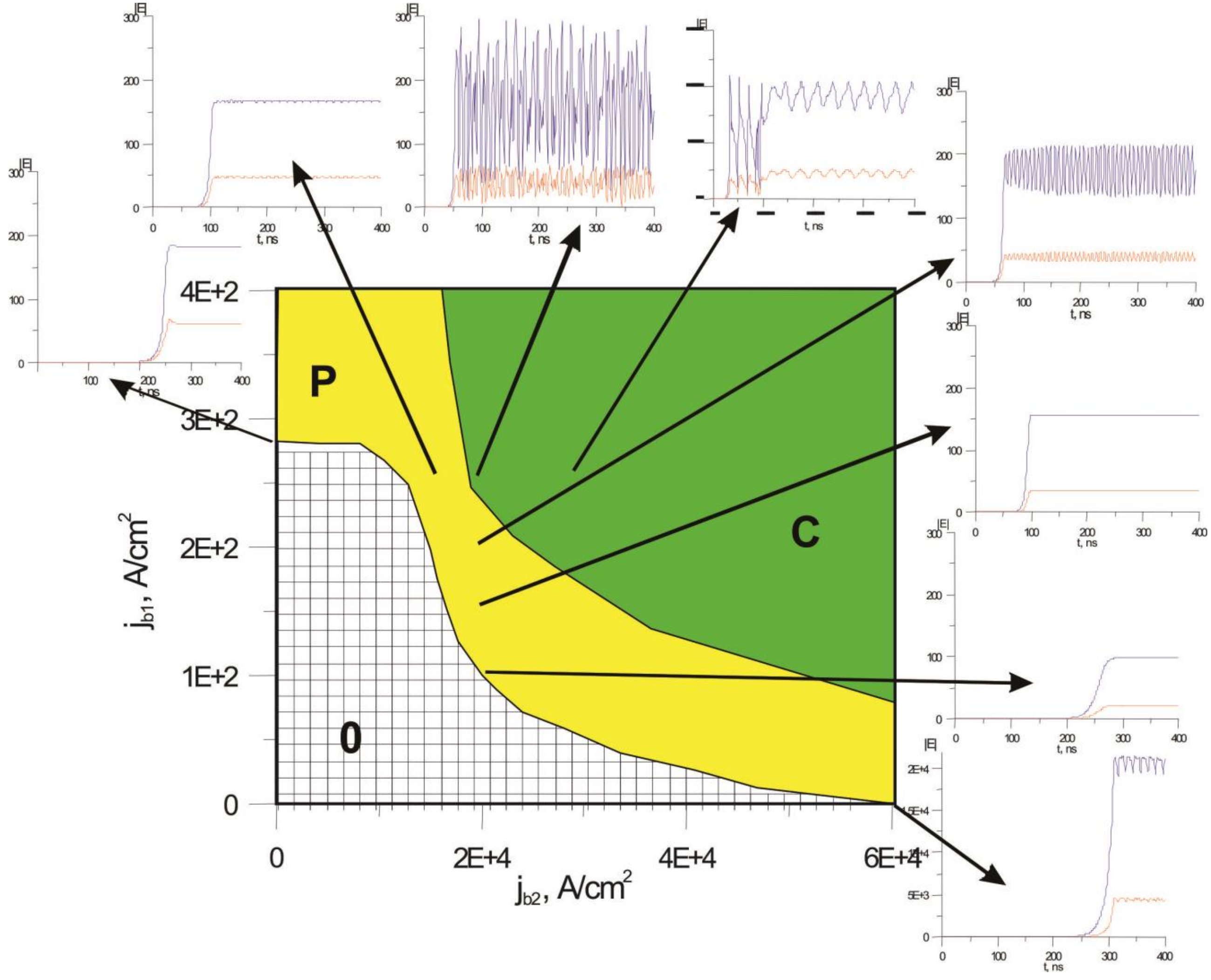


for diffracted wave



for electron beam

Two-beam two-wave VFEL



transmitted wave
diffracted wave

Conclusions

- As VFEL physical principles differ from ones of other vacuum electronic devices VFEL is a new object of investigation, that is the source of powerful electromagnetic radiation in different wavelength ranges.
- Different types of VFEL chaotic dynamics and their explanation are given.
- It is shown that the beam parameters affect the generation regime.
- The way of chaos control in VFEL for self-modulation elimination is changing of VDFB geometry.
- The effect of suppressing parasitic modes of electromagnetic waves in VFEL is demonstrated.
- Two-beam two-wave VFEL was investigated numerically.
- So, each step in investigation of VFEL nonlinear dynamics will profit some new results.

Thank you for attention!

sytova@inp.bsu.by