CHAOTIC DYNAMICS IN VOLUME FREE ELECTRON LASER (VFEL)

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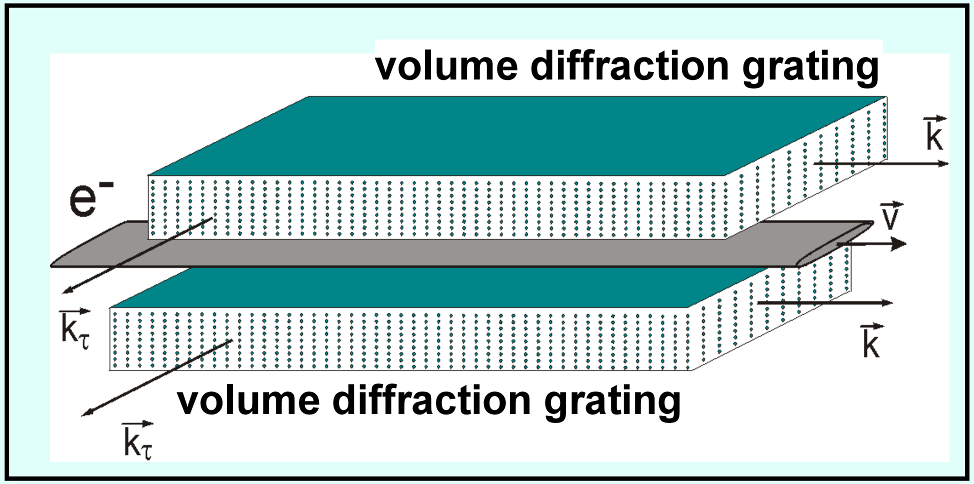
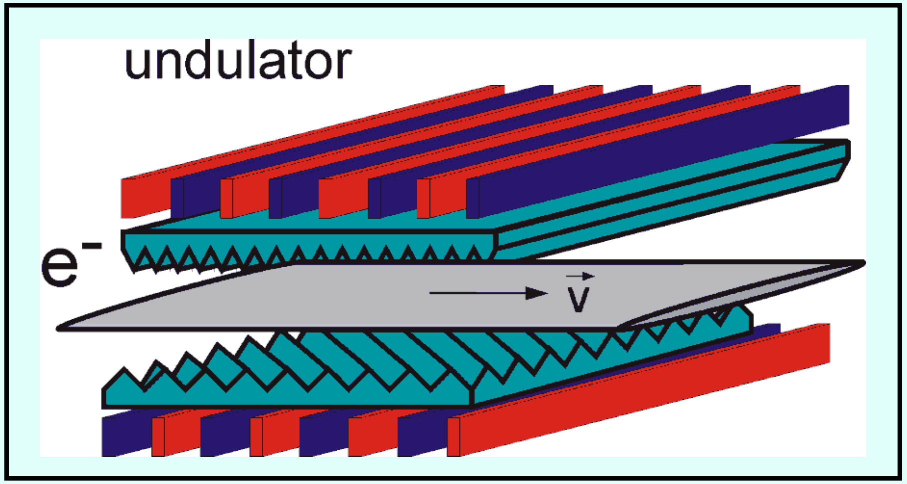
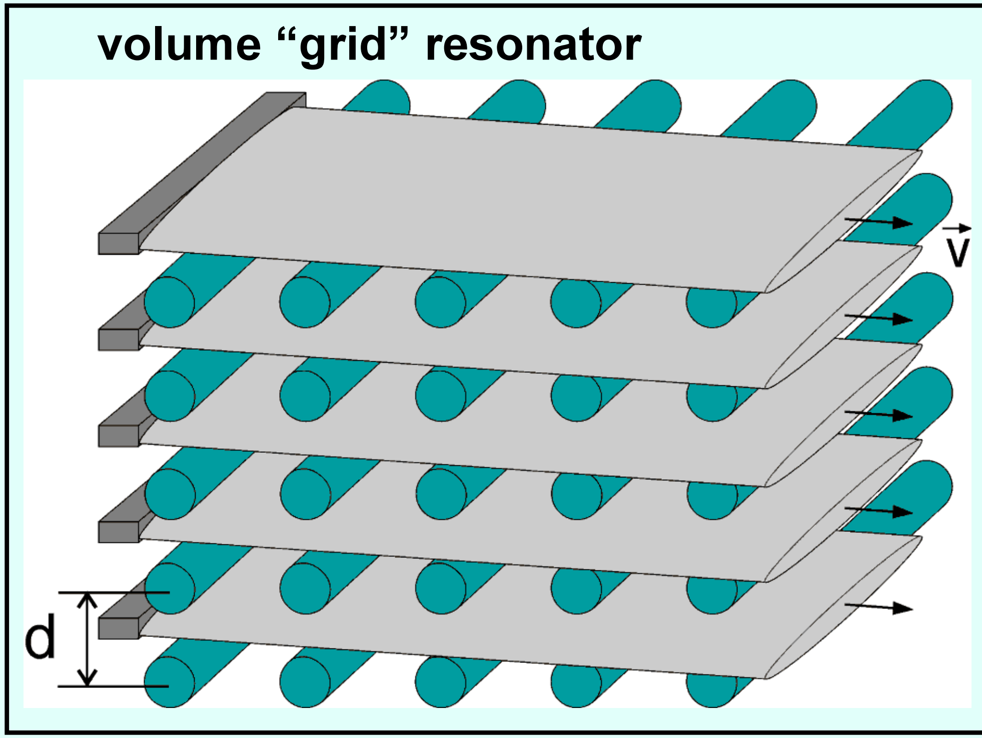
Outline

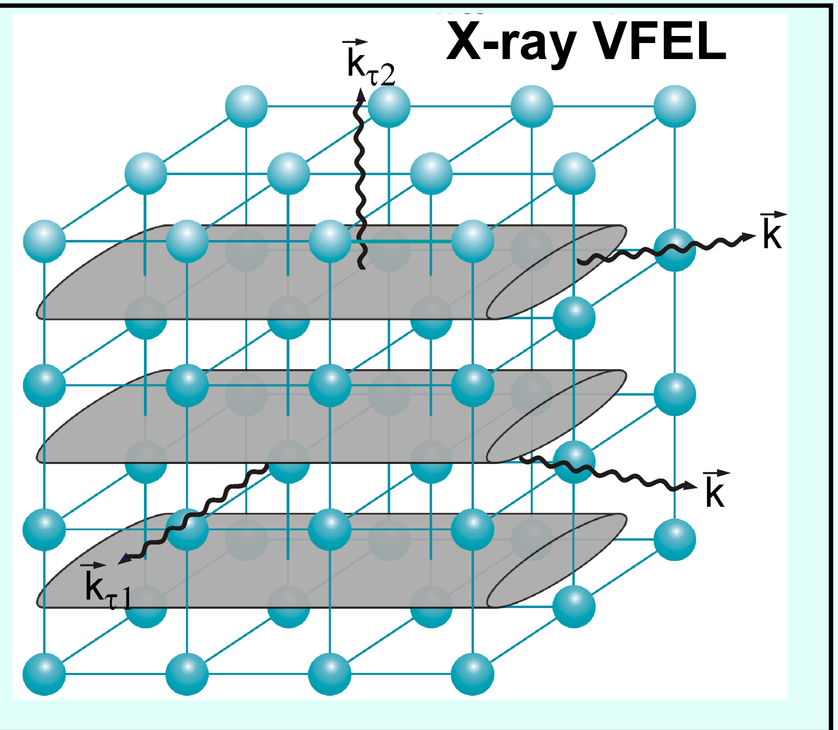
- What is new
- What is Volume Free Electron Laser
- VFEL physical and mathematical models
- Code VOLC for VFEL simulation
- Some numerical results
- Examples of different chaotic regimes of VFEL intensity
- Sensibility to initial conditions – the “Butterfly” effect
- The largest Lyapunov exponent
- Two-parametric analysis of root to chaotic lasing in VFEL

What is Volume Free Electron Laser ? *









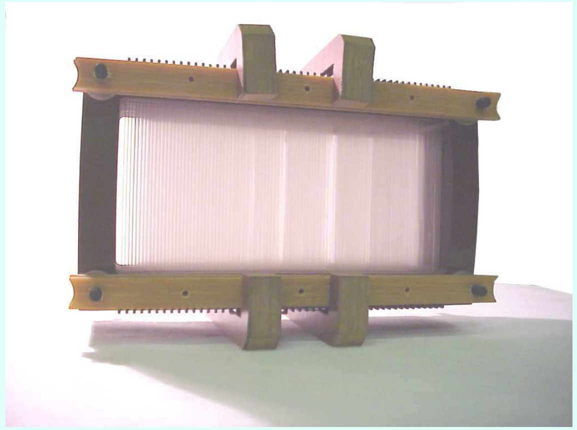
*** Eurasian Patent no. 004665**

Benefits of volume distributed feedback*

- frequency tuning at fixed energy of electron beam in significantly wider range than conventional systems can provide
- more effective interaction of electron beam and electromagnetic wave allows significant reduction of threshold current of electron beam and, as a result, miniaturization of generator
- reduction of limits for available output power by the use of wide electron beams and diffraction gratings of large volumes
- simultaneous generation at several frequencies

VFEL experimental history

1996



Experimental modeling of electrodynamic processes in the volume diffraction grating made from dielectric threads

V.G.Baryshevsky et al., *NIM 393A (1997) 71*

2001

First lasing of volume free electron laser in mm-wavelength range. Demonstration of validity of VFEL principles. Demonstration of possibility for frequency tuning at constan electron energy

V.G.Baryshevsky et al., *NIM 483 A (2002) 21*

2004

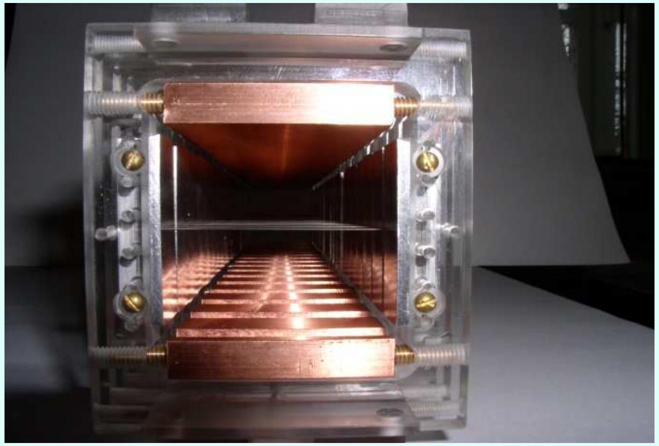
New VFEL generator with a volume “grid” resonator.

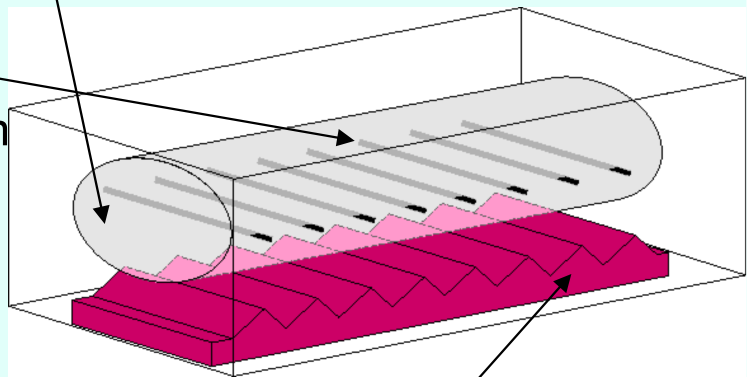
V.G. Baryshevsky et al., *NIM. B 252 (2006) 86*

VFEL generator with a "grid" volume resonator*:

Main features:

- electron beam of large cross-section
- electron beam energy 180-250 keV
- possibility of gratings rotation
- operation frequency 10 GHz
- tungsten threads with diameter 100 μm





Resonant grating provides VDFB of generated radiation with electron beam

* **V.G. Baryshevsky et al., *Nucl. Instr. Meth. B 252 (2006) 86***

V.G.Baryshevsky et al., *Proc. FEL06, p.331*

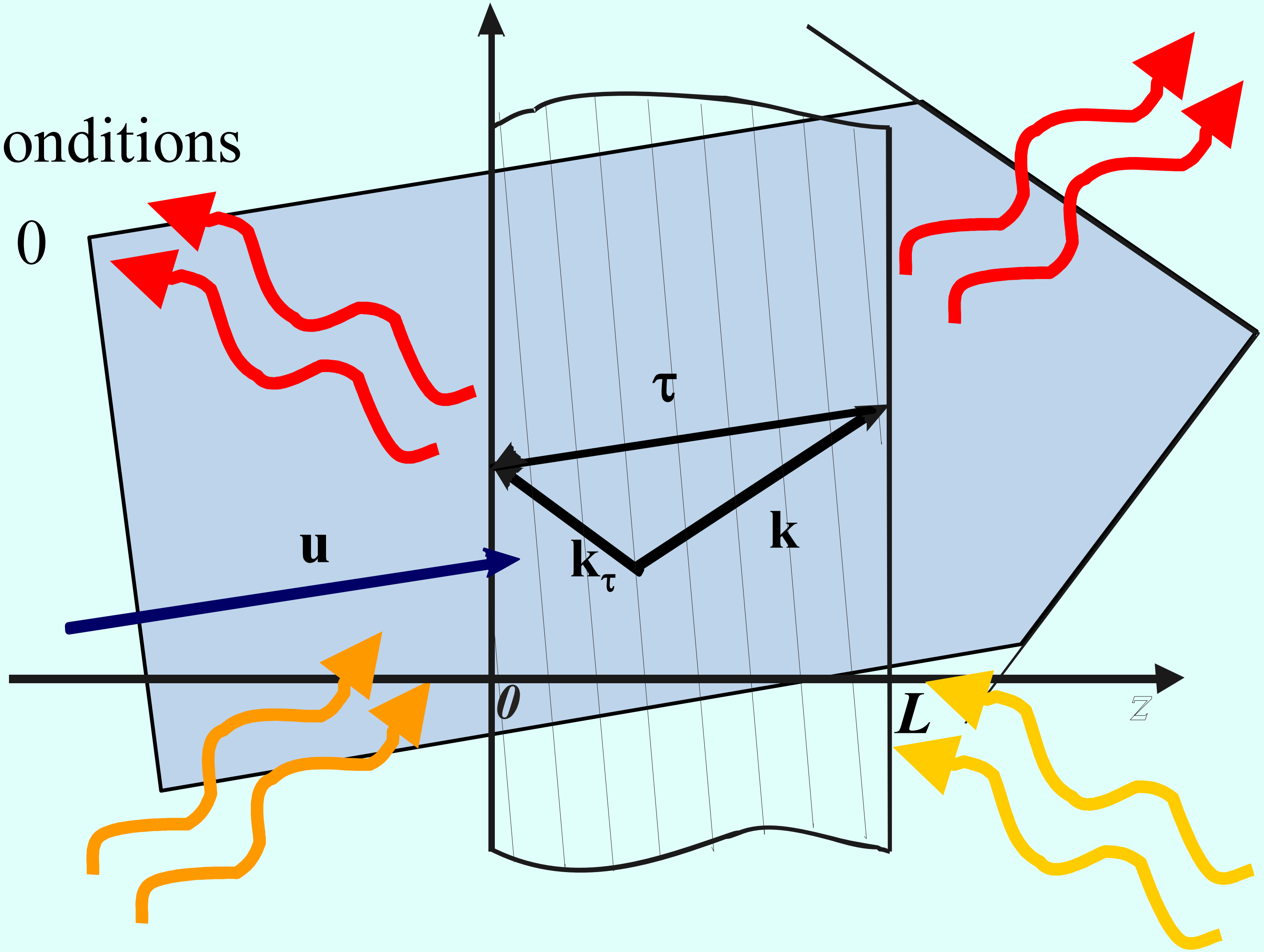
Two-wave VFEL

Diffraction conditions

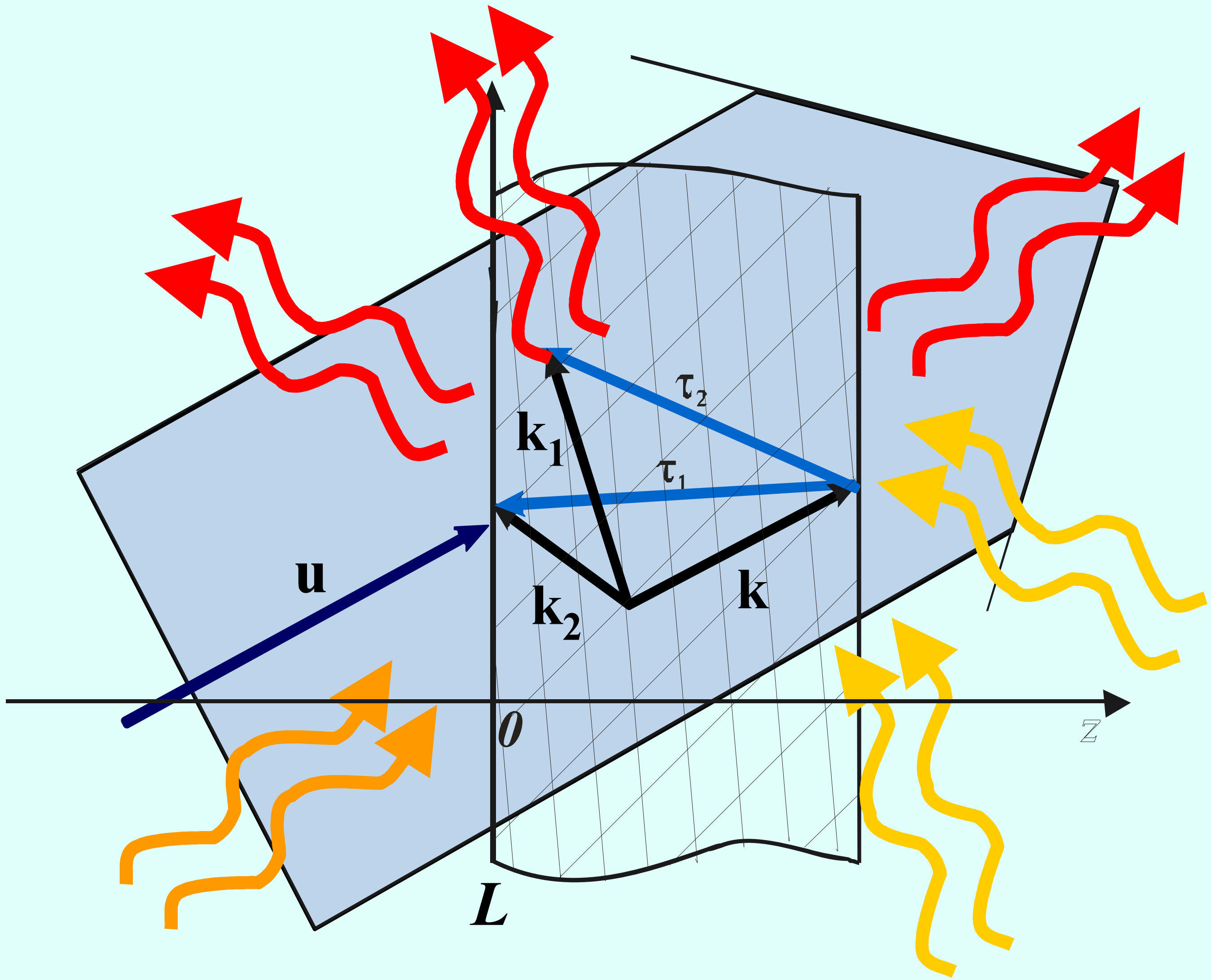
$$2\mathbf{k\tau} + \boldsymbol{\tau}^2 \approx 0,$$

Synchronism conditions

$$|\omega - \mathbf{ku}| = \delta\omega \approx 0$$



Three-wave VFEL



Equations for electron beam

$$\frac{d^2\theta(t,z,p)}{dz^2} = \frac{e\Phi}{m\gamma^3\omega^2}\left(k - \frac{d\theta(t,z,p)}{dz}\right)^3 \mathrm{Re}\Big(E(t - z/u, z)\Big)\times$$

$$\times\exp(i\theta(t,z,p)),$$

$$\frac{d\theta(t,0,p)}{dz} = k - \omega/u,$$

$$\theta(t,0,p) = p,$$

$$t > 0, \quad z \in [0, L], \quad p \in [-2\pi, 2\pi]$$

$\theta(t,z,p)$ is an electron phase in a wave

System for two-wave VFEL:

$$\frac{\partial E}{\partial t} + \gamma_0 c \frac{\partial E}{\partial z} + 0.5 i\omega l E - 0.5 i\omega \chi_\tau E_\tau =$$

$$= 2\pi j \Phi \int_0^{2\pi} \frac{2\pi - p}{8\pi^2} \left(e^{-i\theta(t,z,p)} + e^{-i\theta(t,z,-p)} \right) dp,$$

$$\frac{\partial E_\tau}{\partial t} + \gamma_1 c \frac{\partial E_\tau}{\partial z} - 0.5 i\omega \chi_{-\tau} E + 0.5 i\omega l_1 E_\tau = 0$$

$l_i = \frac{k_i^2 c^2 - \omega^2 \varepsilon_0}{\omega^2}$ **are system parameters,** $\Phi = \sqrt{l_0 + \chi_0 - 1/(u/c\gamma)^2}$

$l = l_0 + \delta,\quad \delta$ **- detuning from synchronism condition**

$\gamma_{0,1}$ **are direction cosines,** $\beta = \gamma_0/\gamma_1$ **is an asymmetry factor**

χ_0, $\chi_{\pm 1}$ **are Fourier components of the dielectric susceptibility of the target**

System of equations for BWT, TWT etc. *

$$\partial^2\theta/\partial\zeta^2 = -\mathrm{Re}\left[F\exp(i\theta)\right], \;\; \partial F/\partial\tau - \partial F/\partial\zeta = \tilde{I}, \;\; \tilde{I} = -\frac{1}{\pi}\int\limits_0^{2\pi} e^{-i\theta} d\theta_0,$$

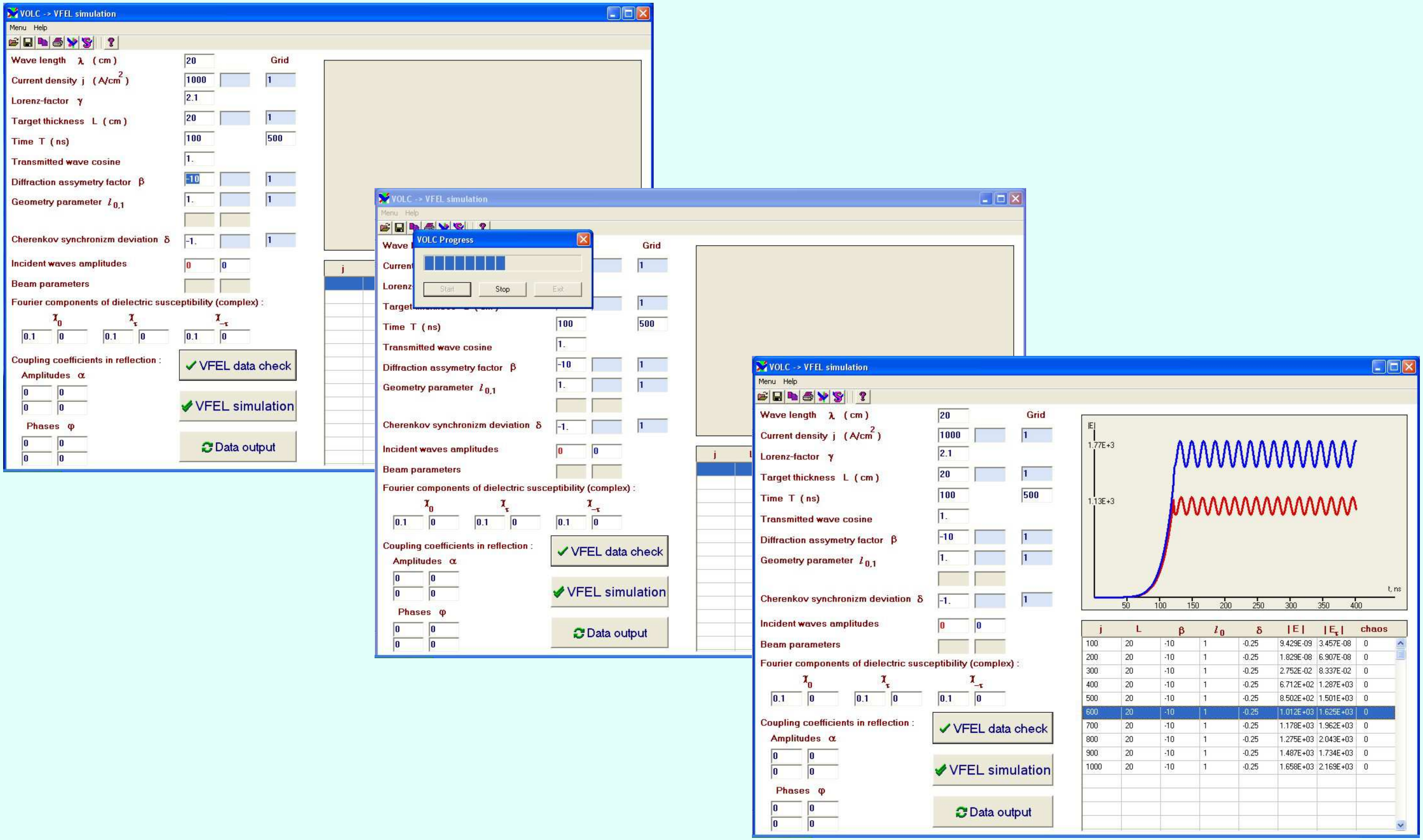
$$\theta|_{\zeta=0} = \theta_0, \quad \partial\theta/\partial\zeta|_{\zeta=0} = 0, \quad F|_{\zeta=L} = 0,$$

System is versatile in the sense that they remain the same within some normalization for a wide range of electronic devices (FEL, BWT, TWB etc).

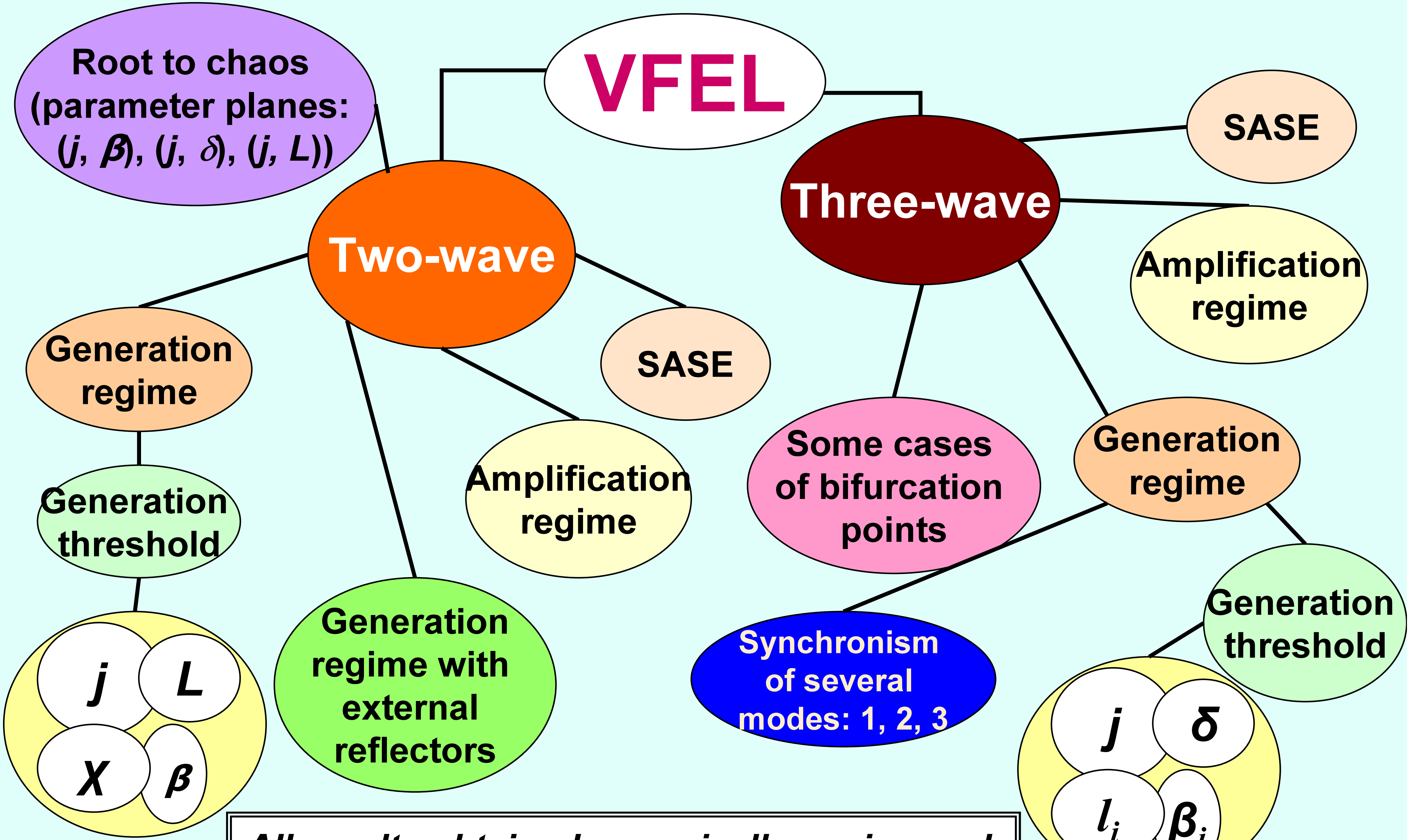
N.S.Ginzburg, S.P.Kuznetsov, T.N.Fedoseeva.** ***Izvestija VUZov - Radiophysics, 21 (1978), 1037 (in Russian).

Right-hand side of our system is more complicated than cited here, because it takes into account two-dimensional distributions with respect to spatial coordinate and electron phase *p*. So, they allow to simulate electron beam dynamics more precisely. This is very important when electron beam moves angularly to electromagnetic waves.

Code **VOLC** ("***VOL***ume ***C***ode"), for VFEL simulation



Results of numerical simulation (2002-2007):



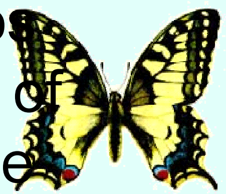
All results obtained numerically are in good agreement with analytical predictions.

Dynamical systems

In electronic devices such as FEL, TWT, BWT etc. self-oscillations are due to interaction of electron beam and electromagnetic field under distributed feedback. Investigation of chaos in nonlinear optical devices, accelerators, FEL etc. is of great interest in modern physics *.

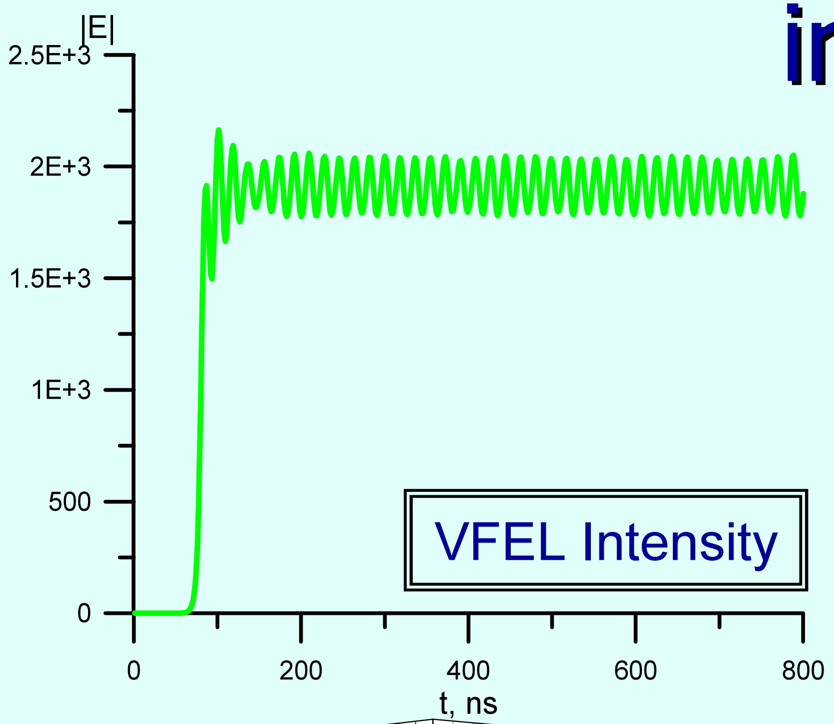
In VFEL simulation we faced with chaotic behaviour of electromagnetic field intensities too. Here chaotic dynamics is induced by complicated interaction of electron beam bunches with electromagnetic field under VDFB. Investigation of chaos in VFEL is important in the light of its experimental development.

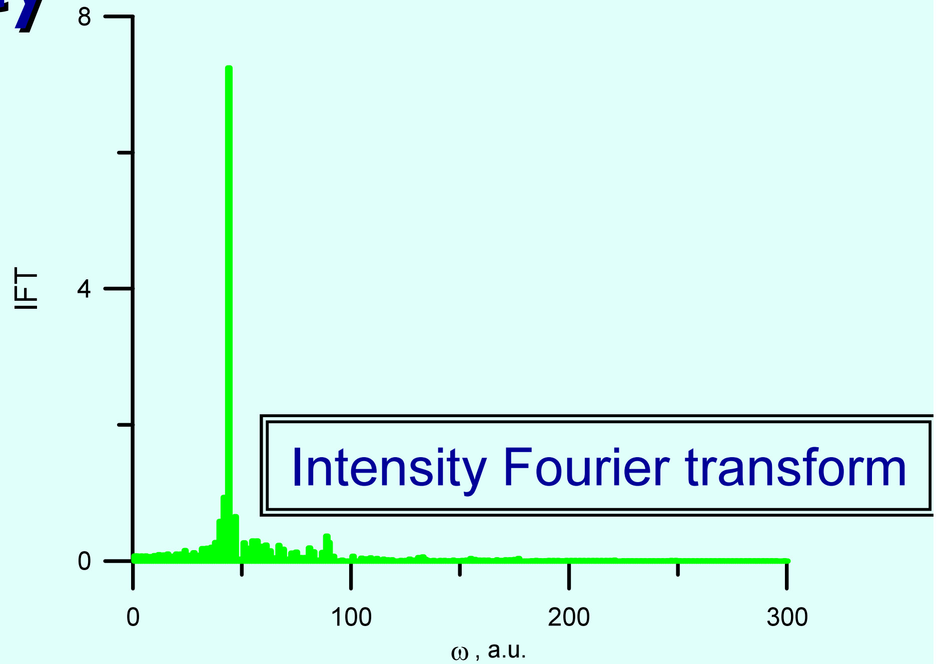
Nonlinearity is necessary but non-sufficient condition for chaos in the system. The main origin of chaos is the exponential divergence of initially close trajectories in the nonlinear systems. This is so-called the "Butterfly effect"** (the sensibility to initial conditions).

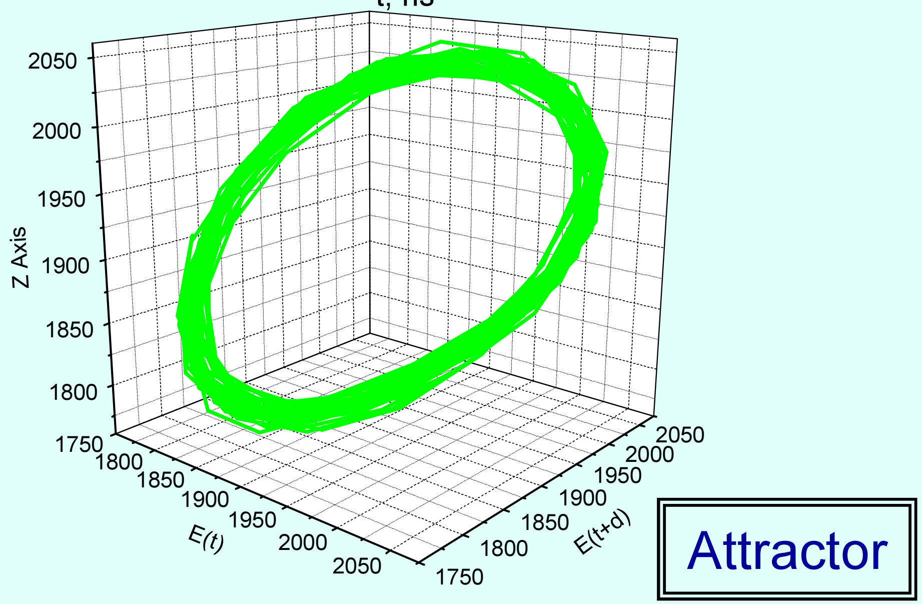


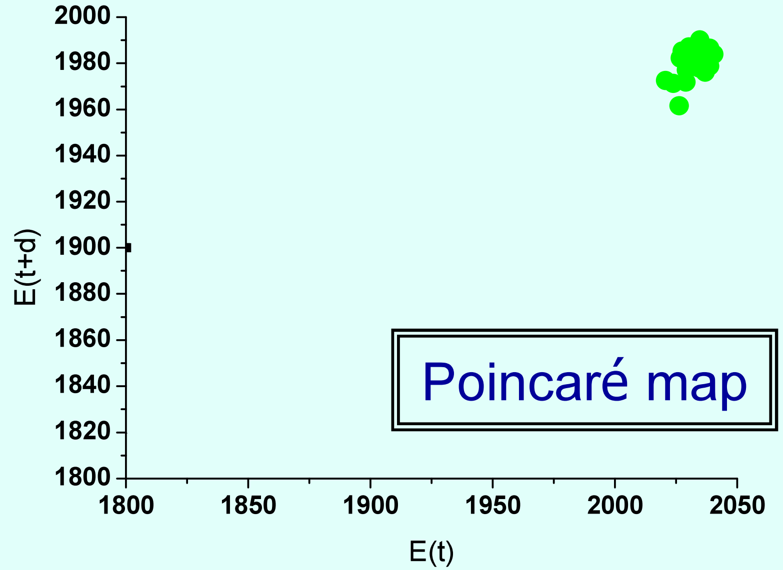
*** M.E.Couprie, *Nucl. Instr. Meth. A507 (2003), 1***
M.S.Hur, H.J. Lee, J.K.Lee., *Phys. Rev. E58 (1998), 936*
N.S.Ginzburg, R.M.Rosental, A.S.Sergeev, *Tech. Phys. Lett.,* 29 (2003) 71
**** E.N. Lorenz, *J. Atmos. Sci. 20 (1963), 130***

Example of periodic regimes of VFEL intensity

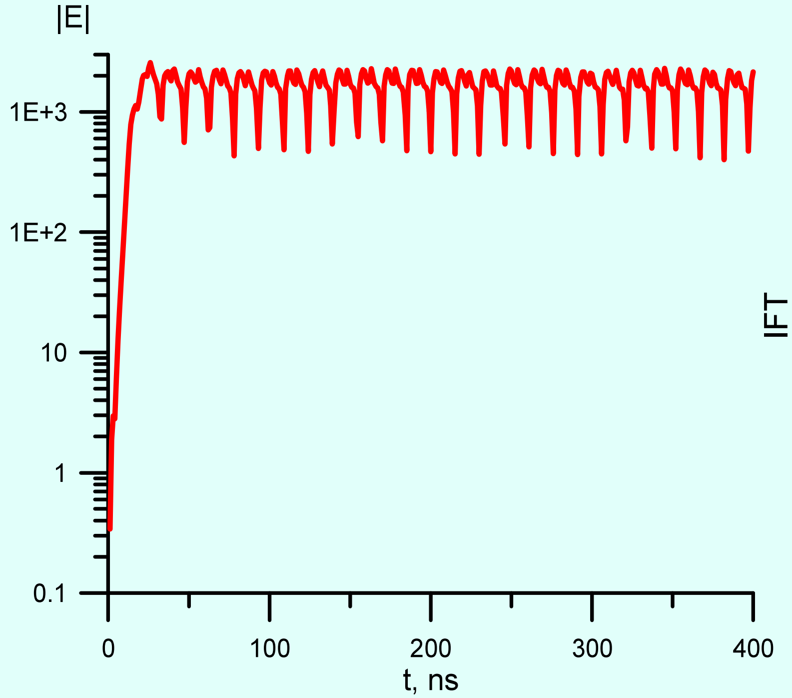


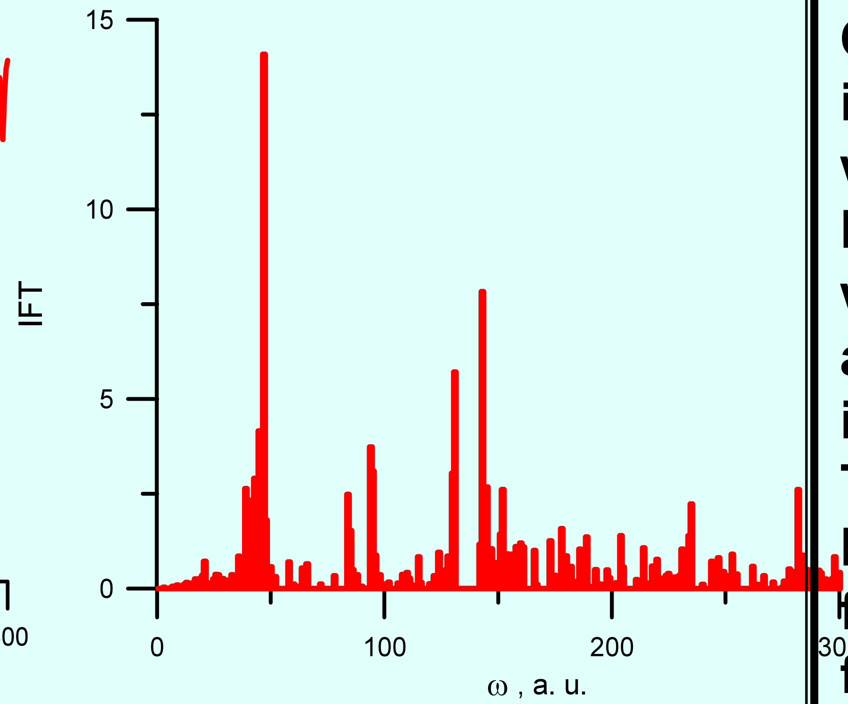




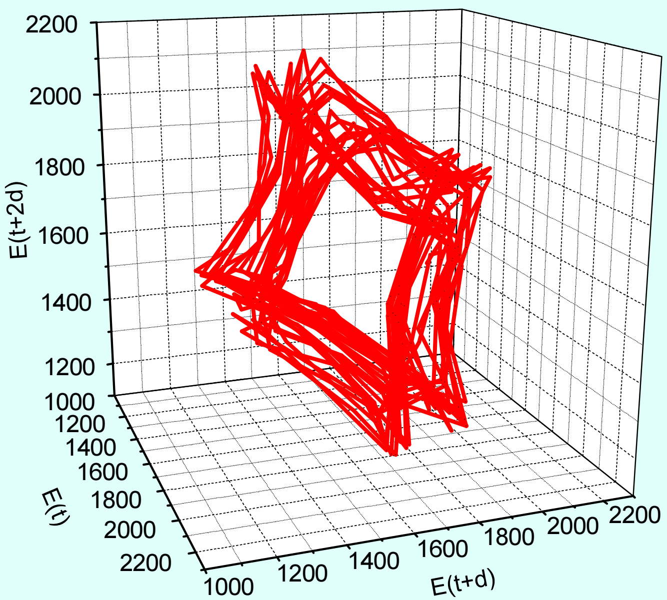


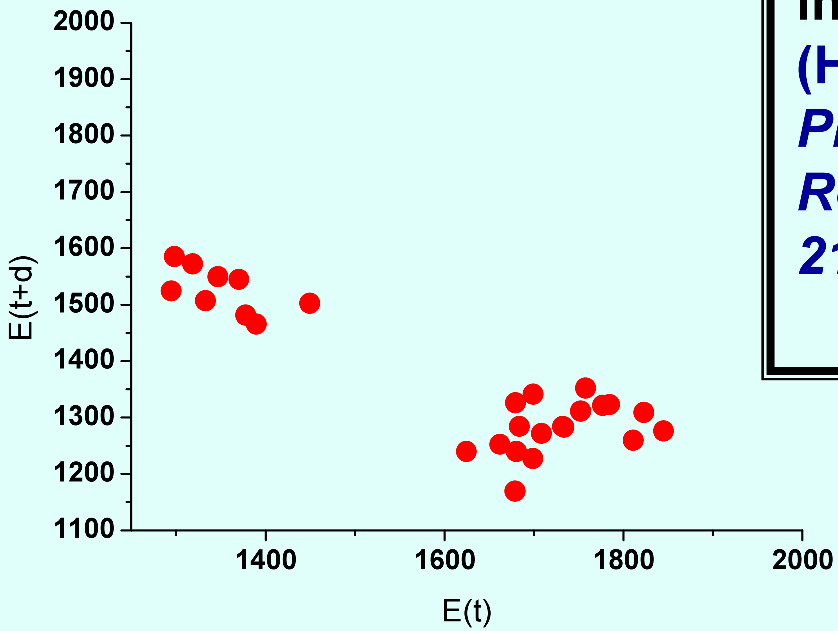
Quasiperiodic oscillations



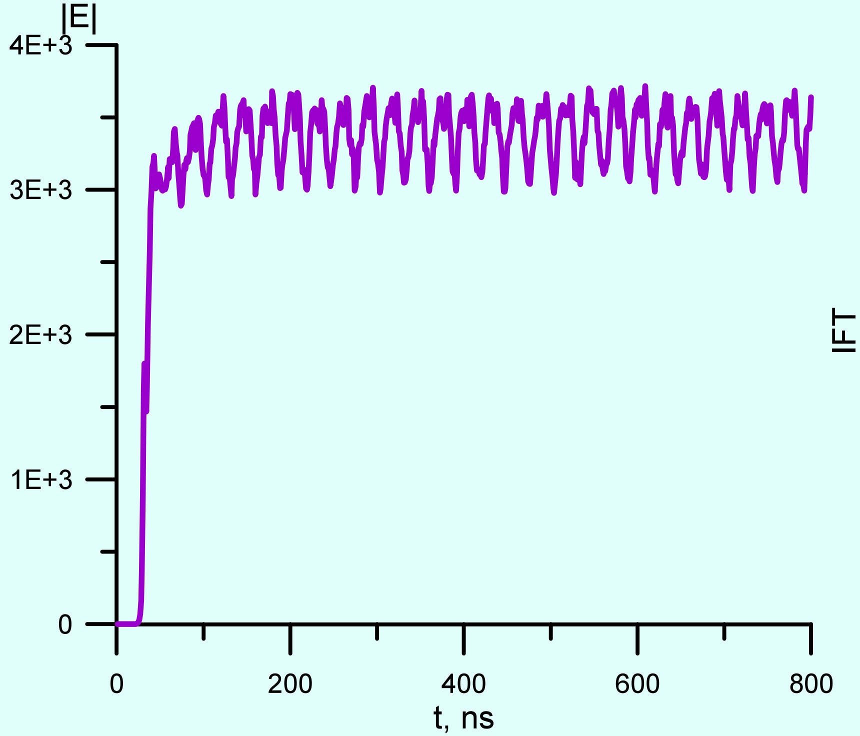


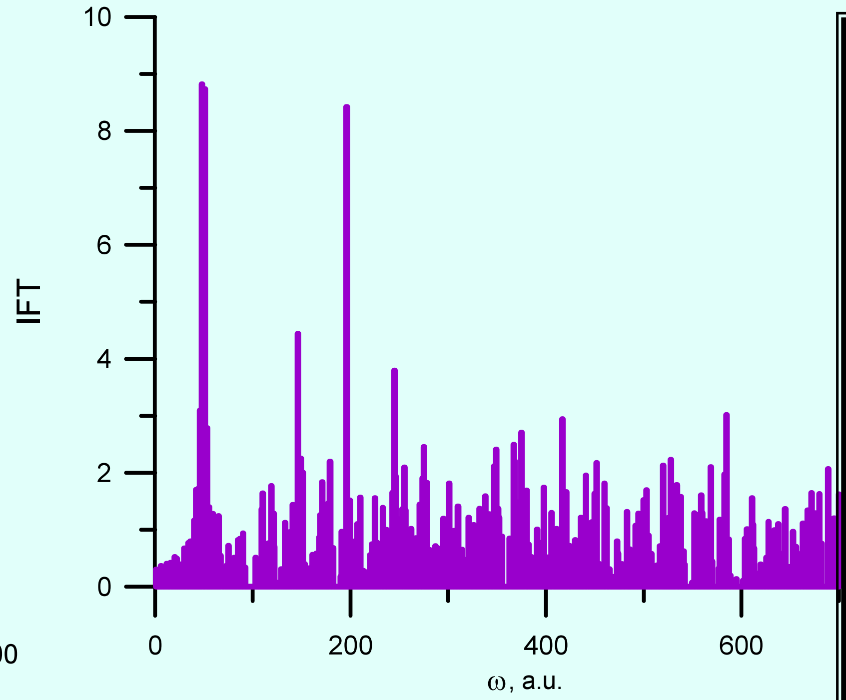
Quasiperiodicity is associated with the Hopf bifurcations which introduces a new frequency into the system. The ratios between the fundamental frequencies are incommensurate (Hahn, Lee, *Phys. Rev.E(1993),48, 2162*)



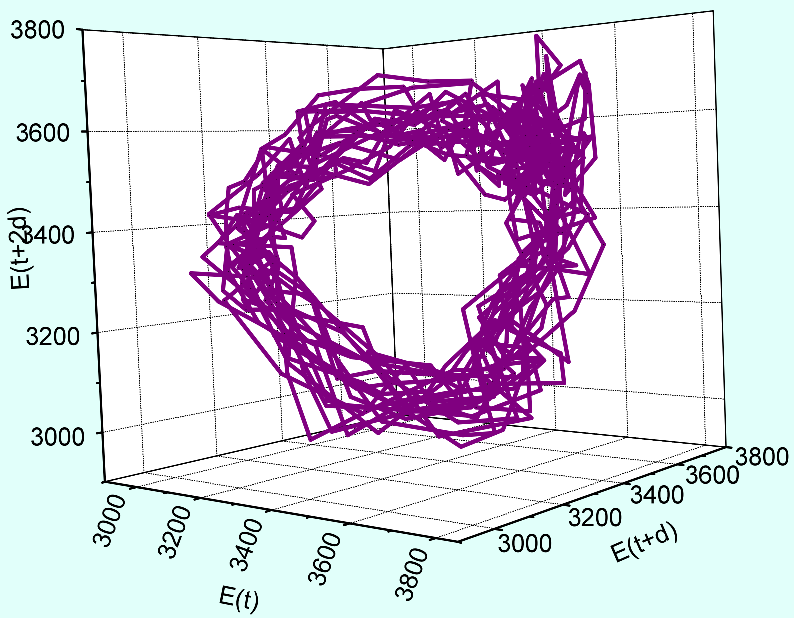


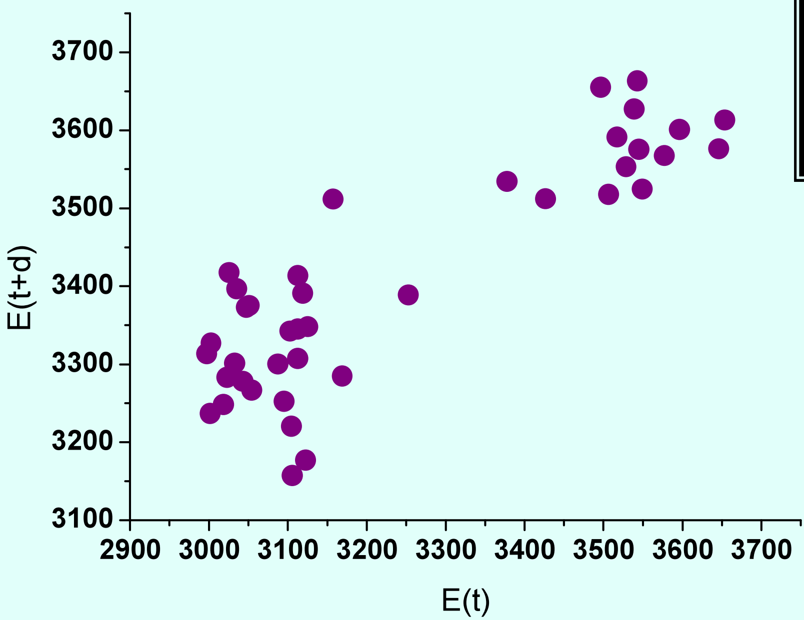
“Weak” chaotic regime



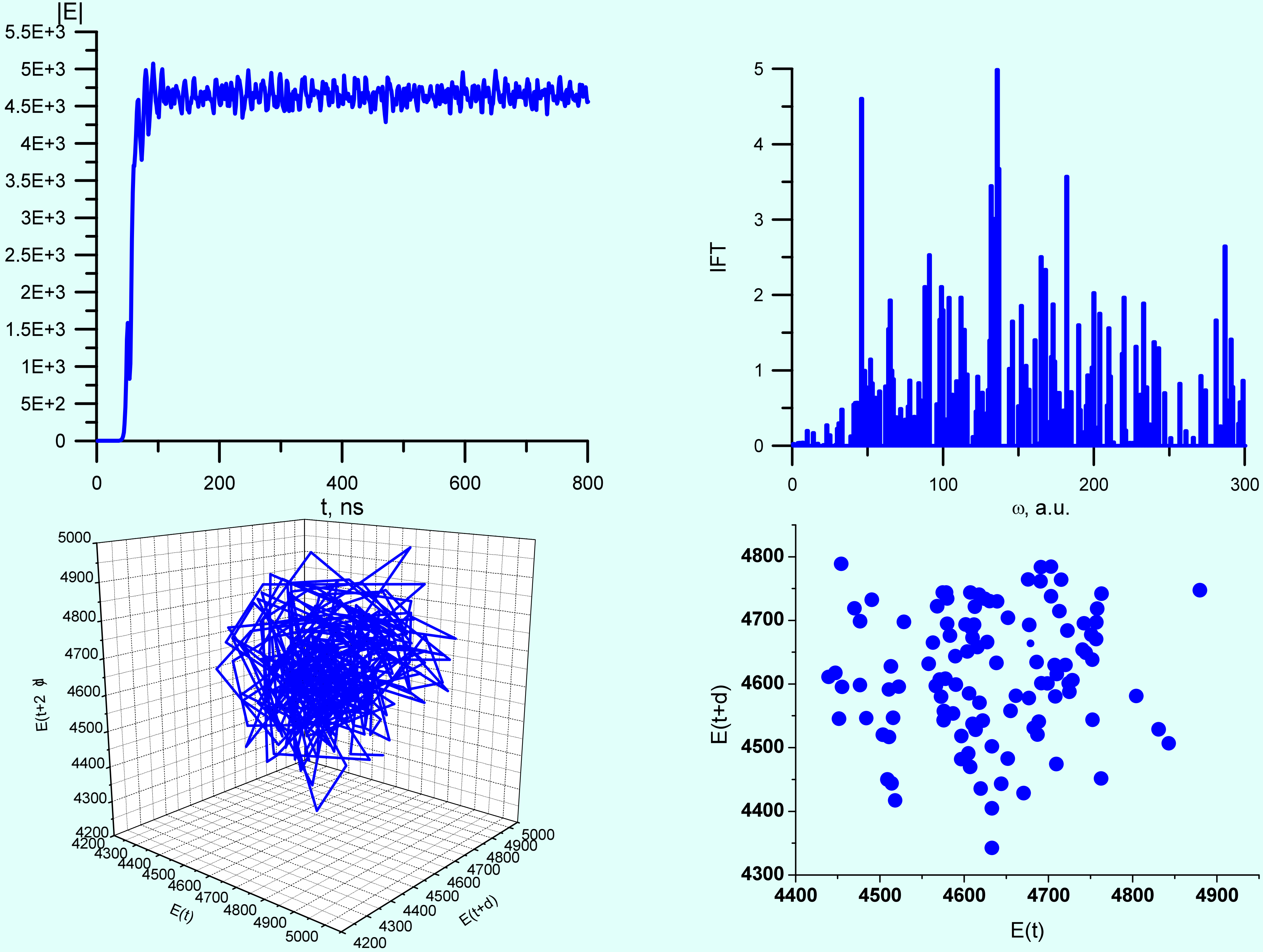


Dependence of amplitude in time seems as approximate repetition of equitype spikes close in dimensions per approximately equal time space.

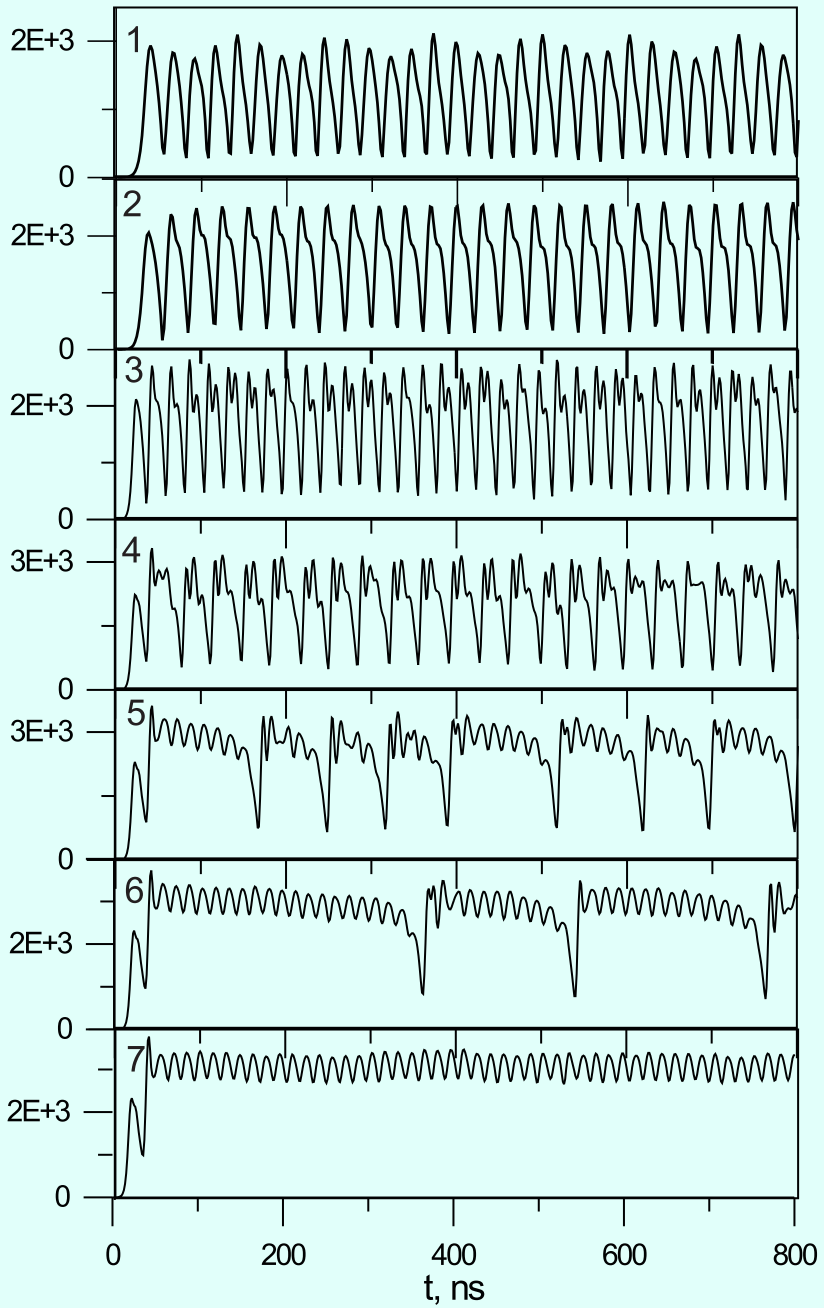




Chaotic self-oscillations (hyperchaos)

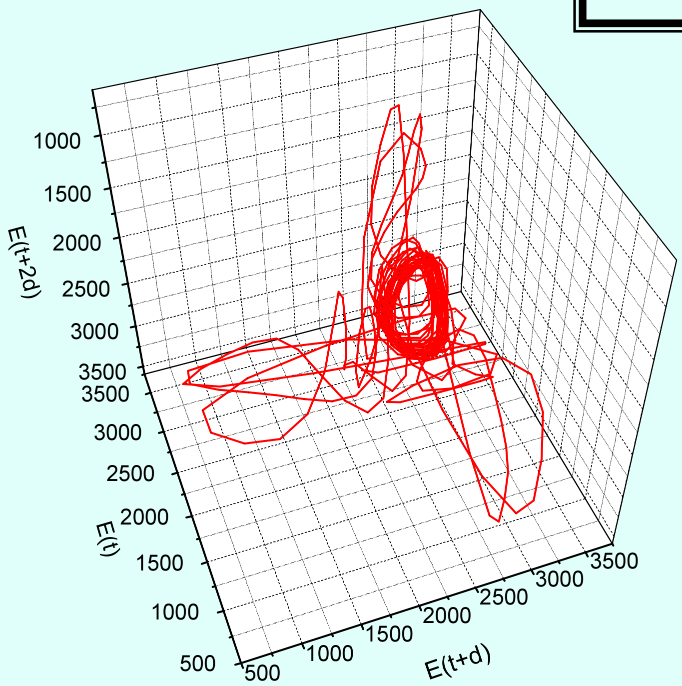


Quasiperiodicity and intermittency



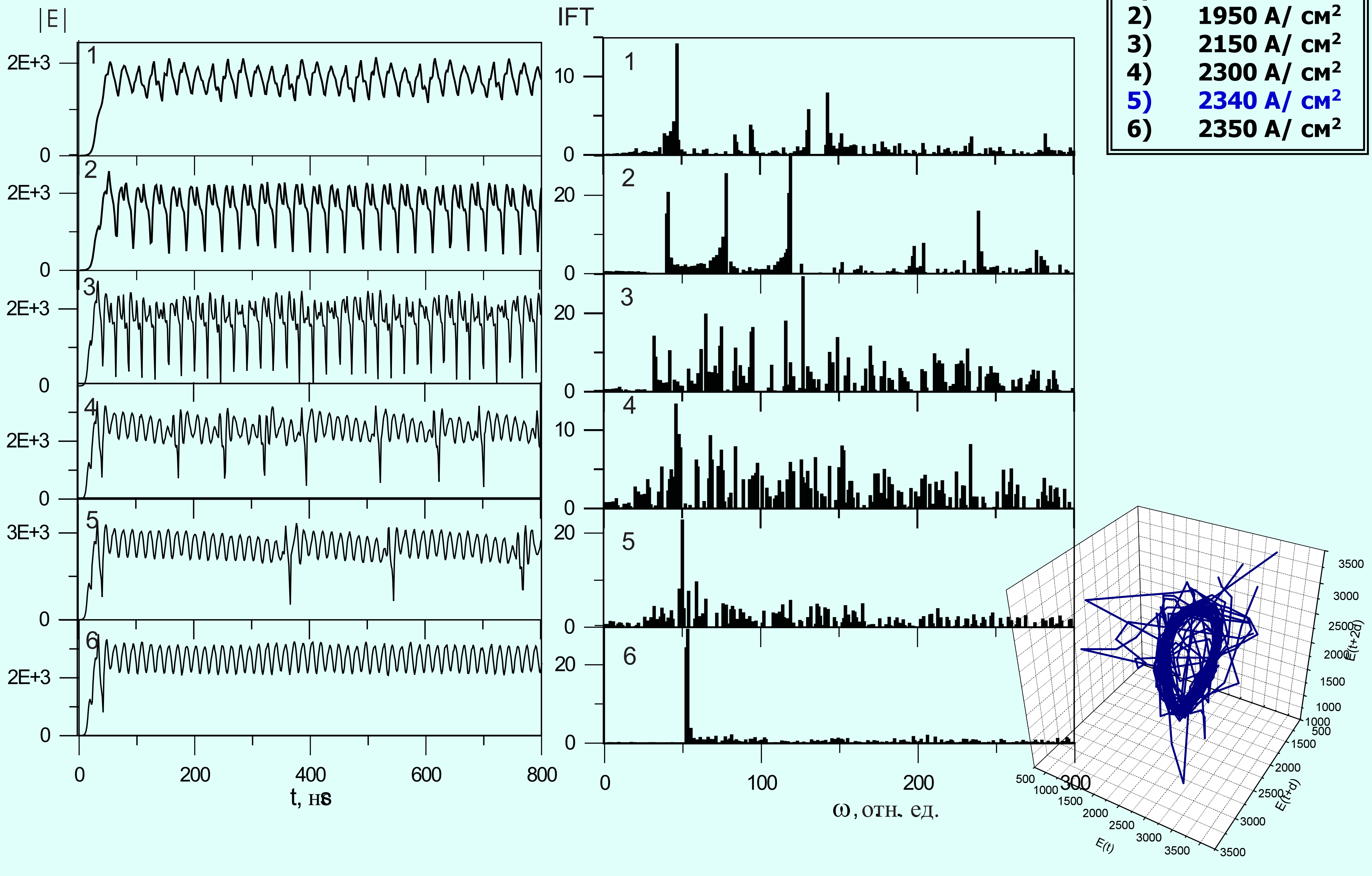
1) 1750 A/ см²
2) 1950 A/ см²
3) 2150 A/ см²
4) 2220 A/ см²
5) 2300 A/ см²
6) 2340 A/ см²
7) 2350 A/ см²

Intermittency is closely related to saddle-node bifurcations. This means the collision between stable and unstable points, that then disappears. **(Hahn, Lee, *Phys. Rev.E(1993),48, 2162*)**

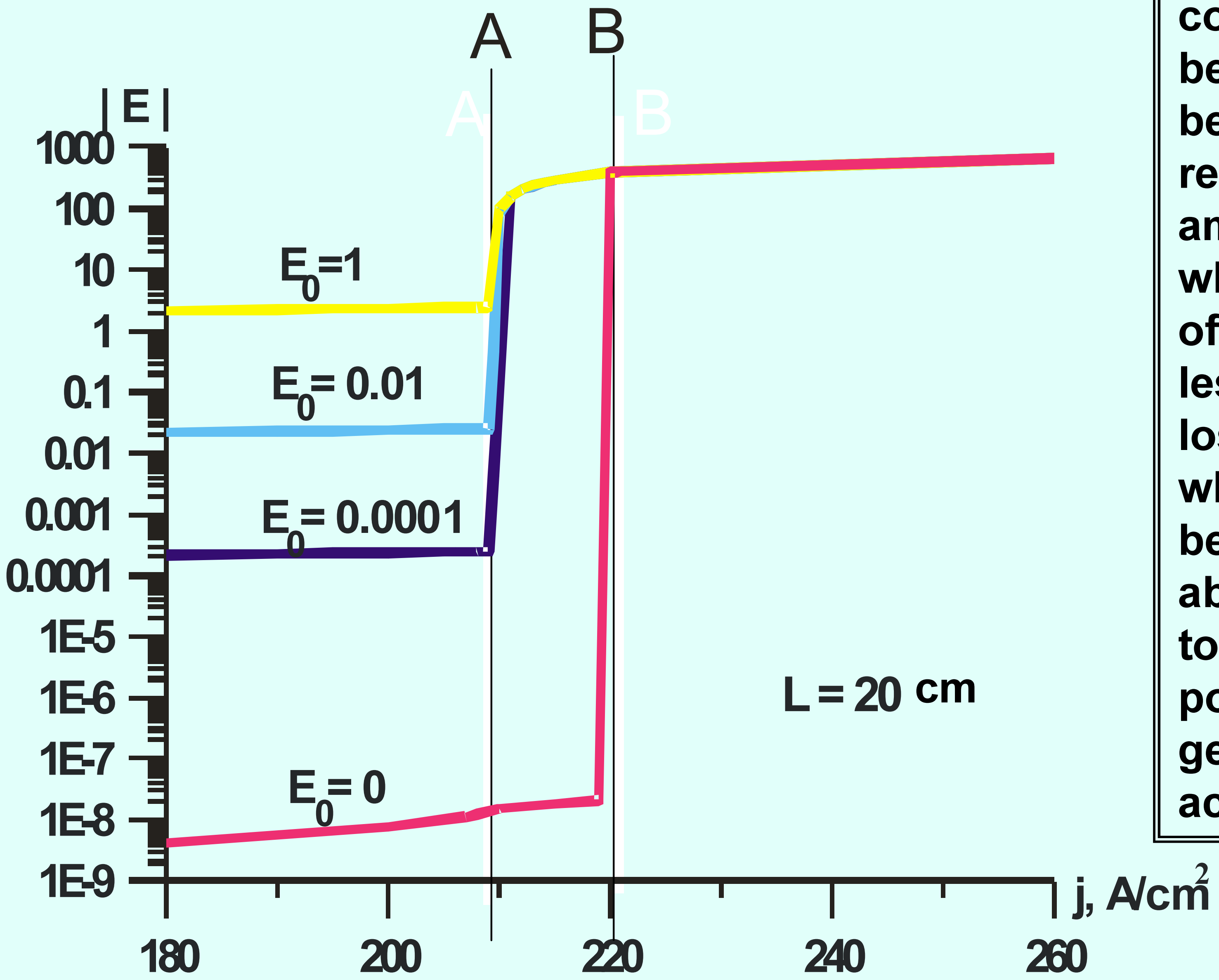


Quasiperiodicity and intermittency

1) 1750 A/ см^2
2) 1950 A/ см^2
3) 2150 A/ см^2
4) 2300 A/ см^2
5) 2340 A/ см^2
6) 2350 A/ см^2

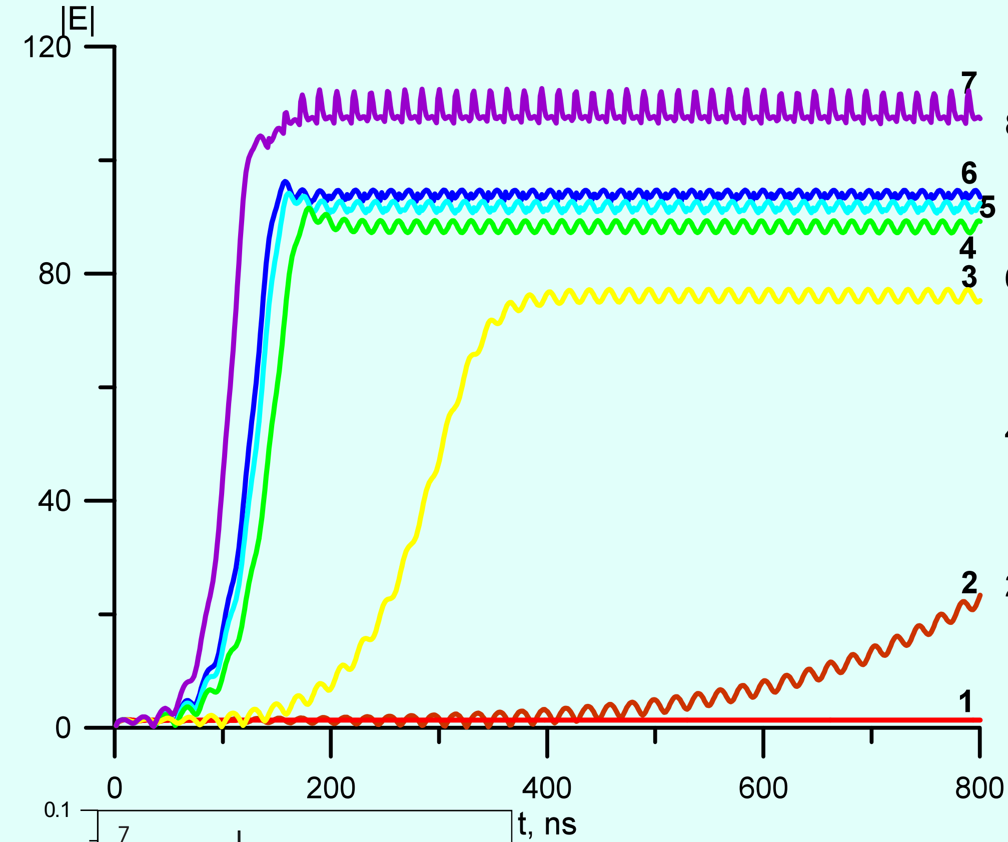


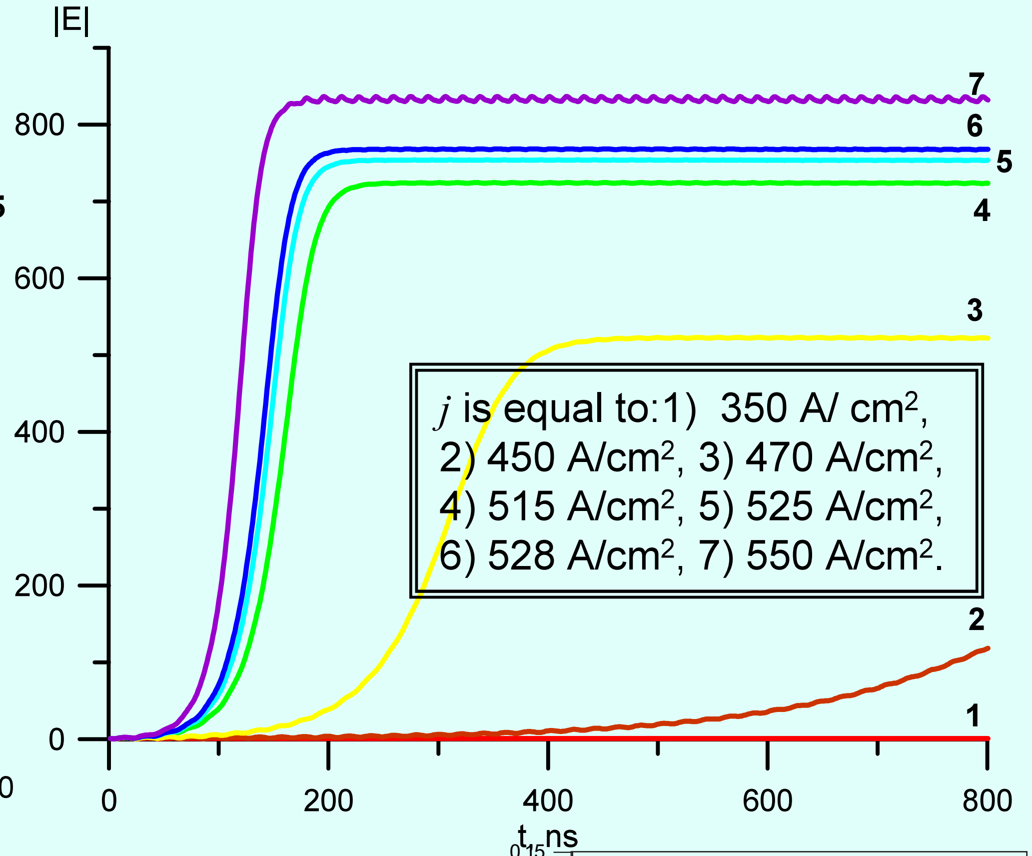
Beginning of amplification and generation regimes are first and second bifurcation points

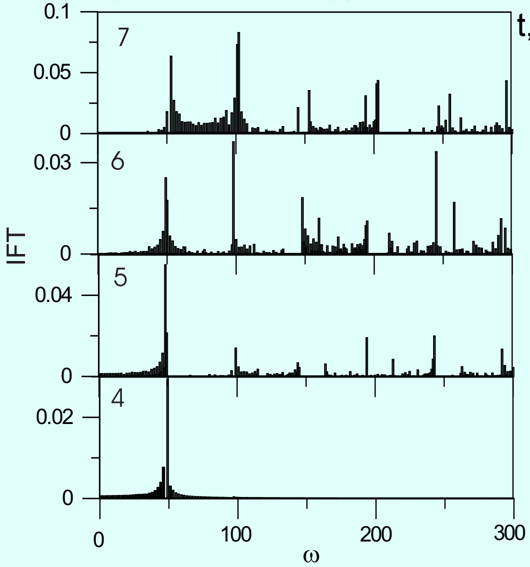


First threshold point corresponds to beginning of electron beam instability. Here regenerative amplification starts while the radiation gain of generating mode is less than radiation losses. Parameters at which radiation gain becomes equal to absorption correspond to the second threshold point after that generation progresses actively.

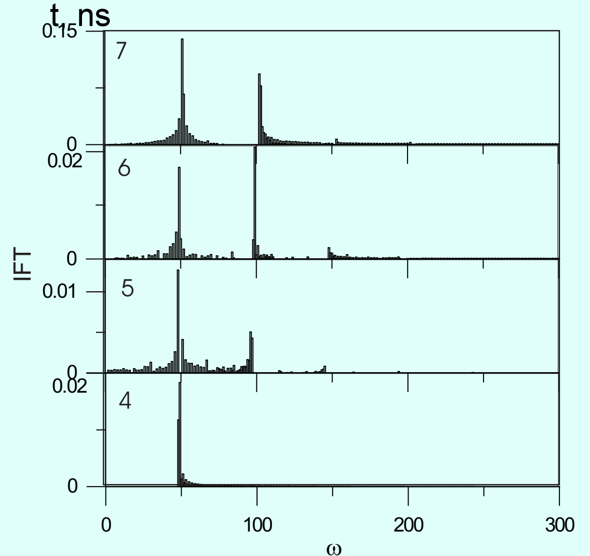
Amplification regime



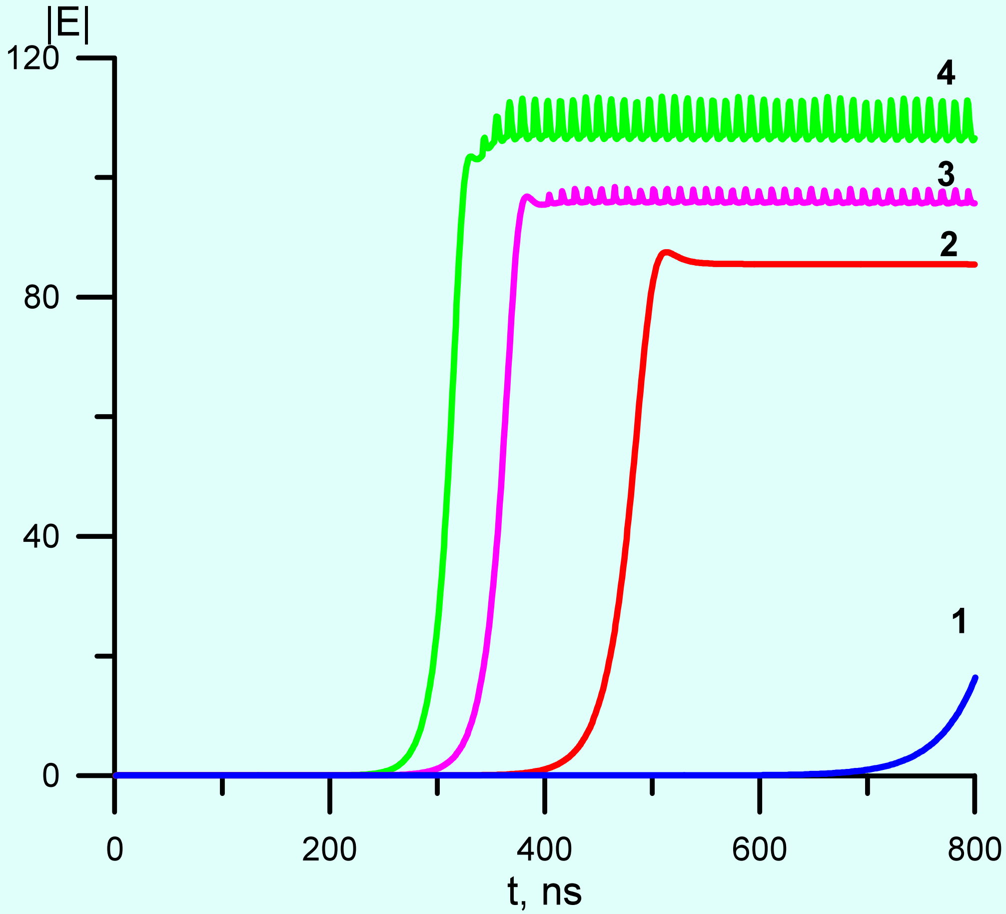


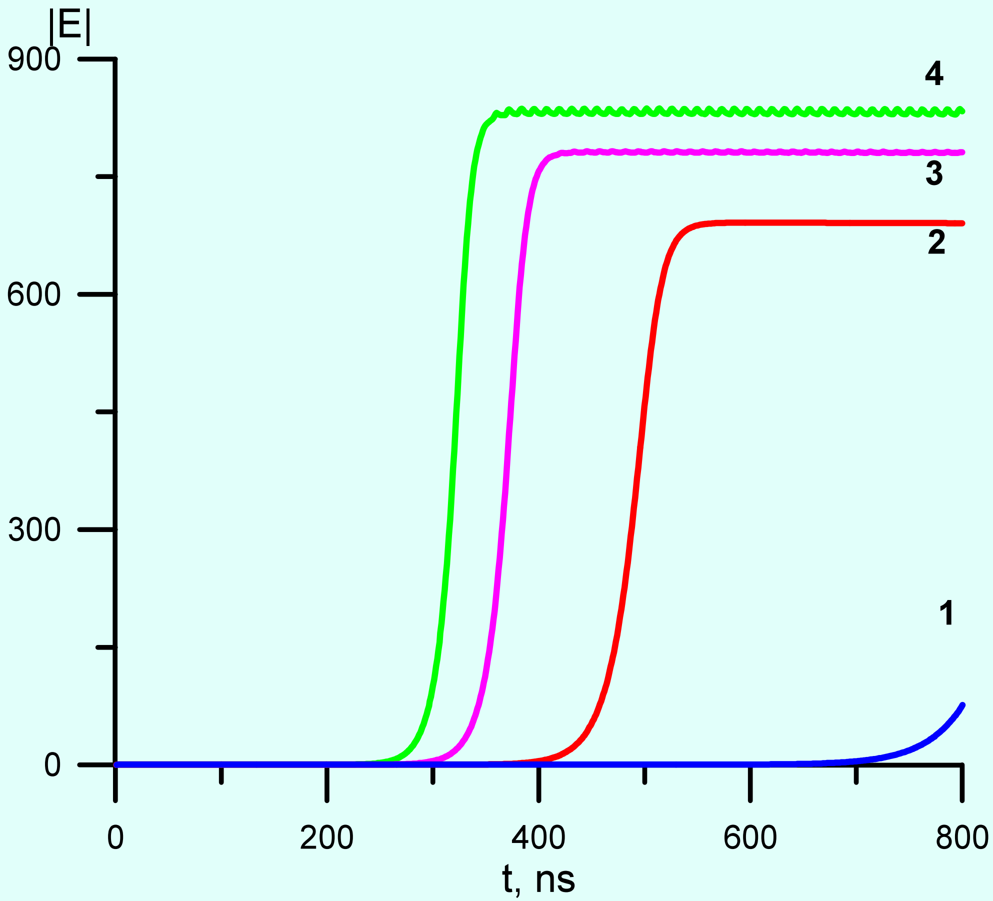


In simulations an important VFEL feature due to VDFB was shown. This is the initiation of quasiperiodic regimes at relatively small current near firsts threshold points.



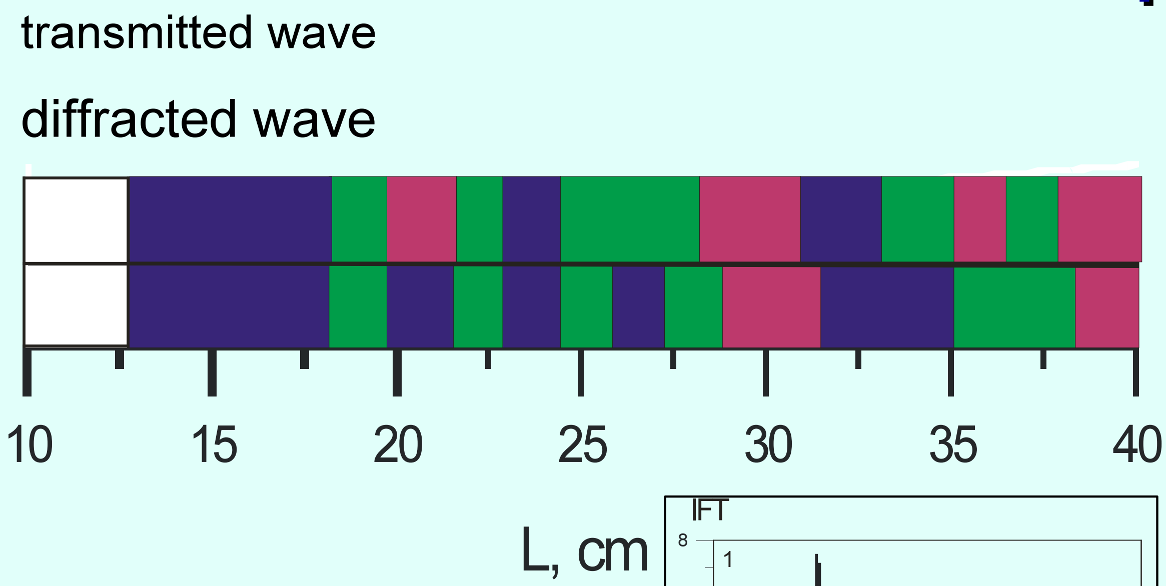
Generation regime

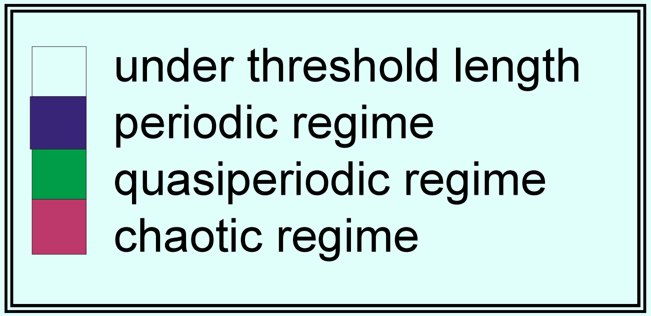


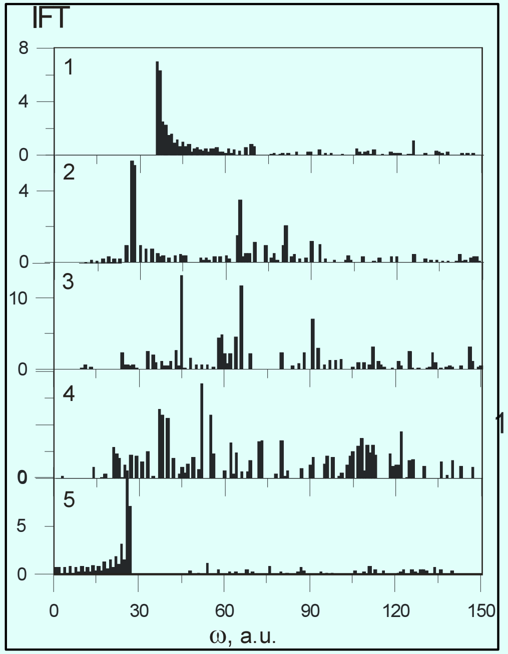


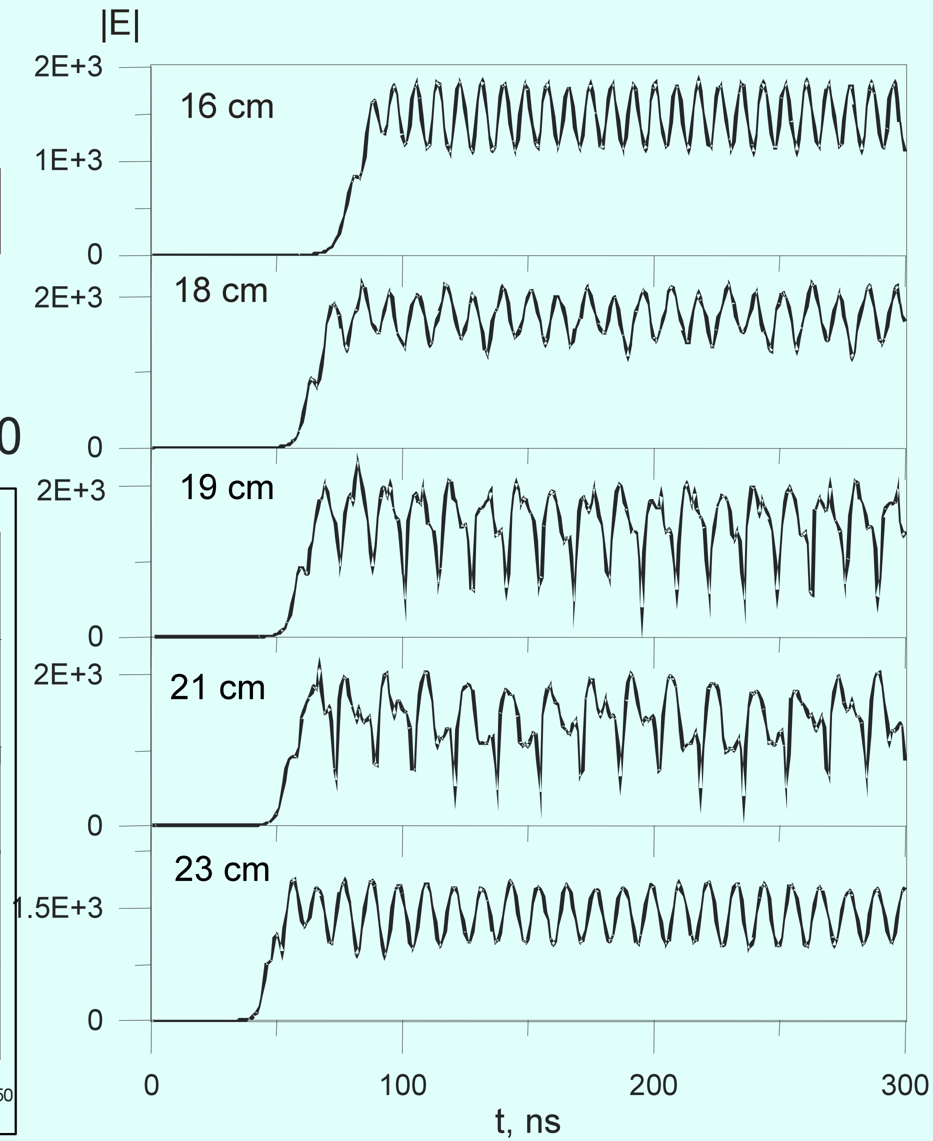
j is equal to:
1) 490 A/ cm^2, 2) 505 A/cm^2,
3) 530 A/cm^2, 4) 550 A/cm^2

Initiation of quasiperiodic regimes at relatively small resonator length near threshold point

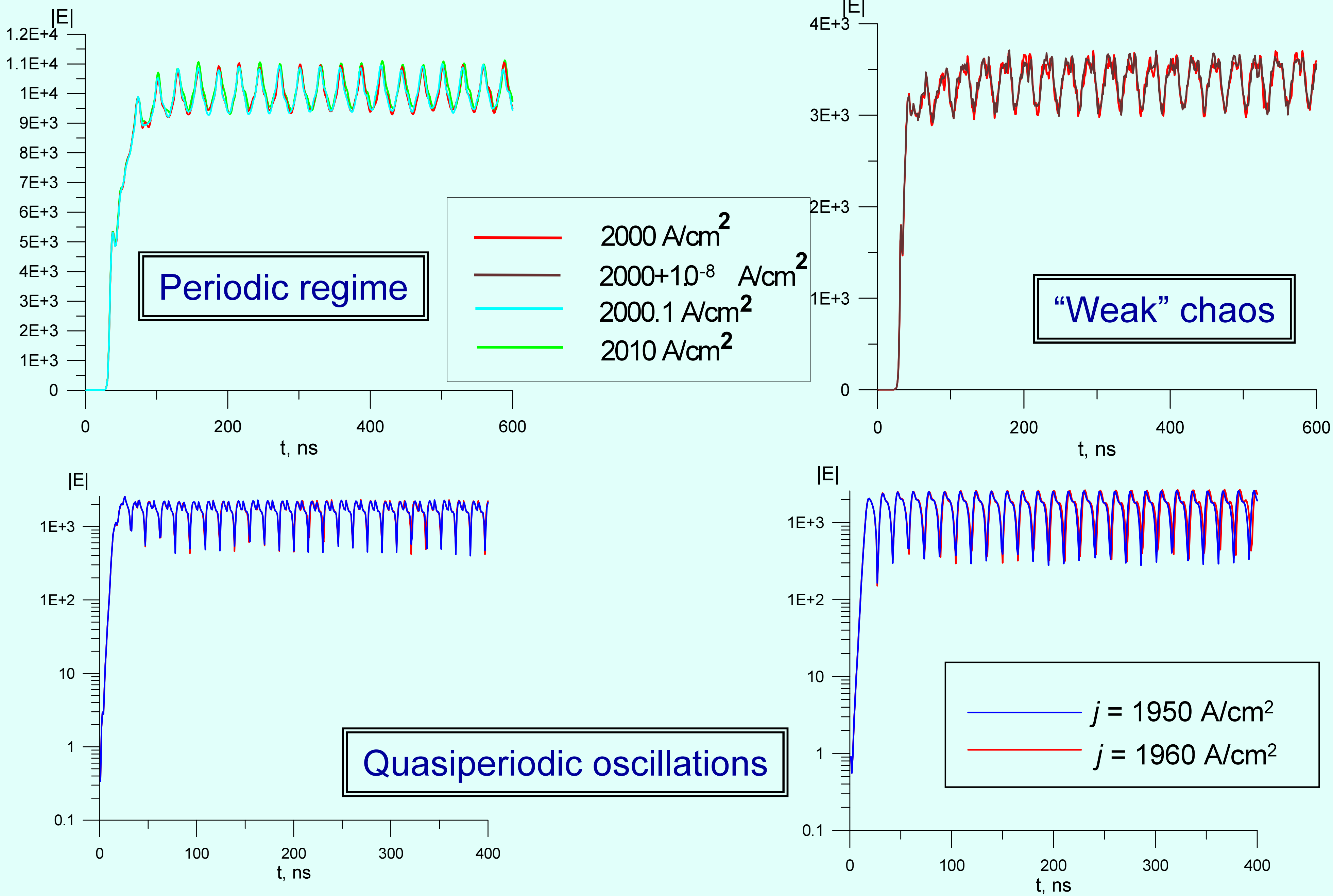




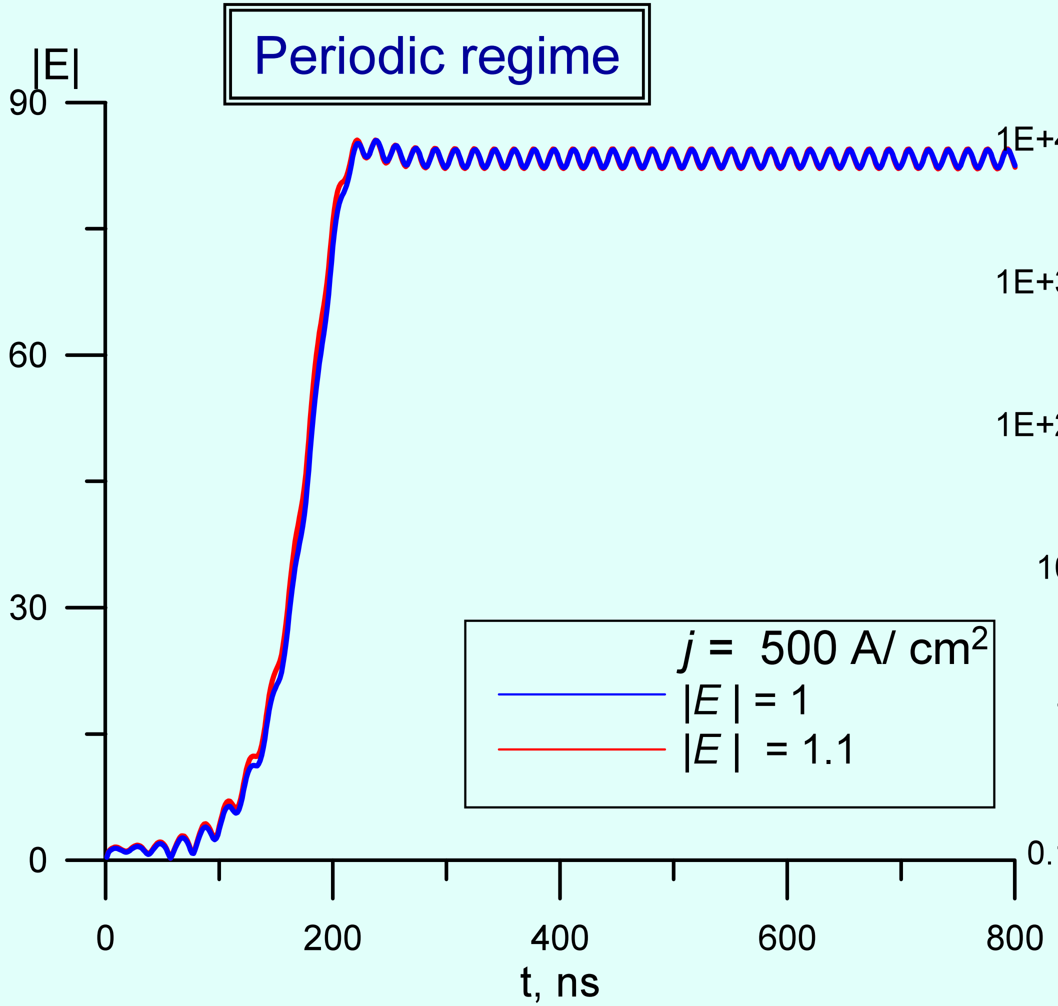


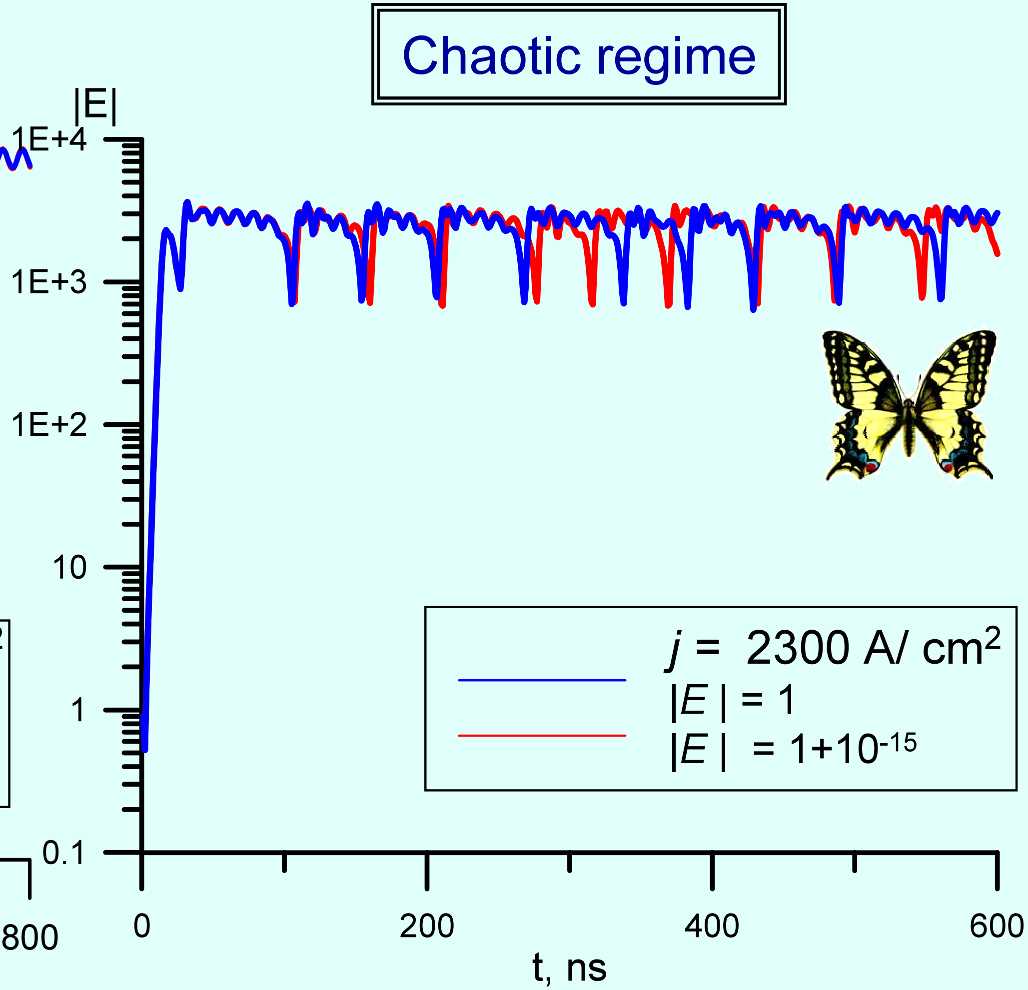


Sensibility to initial conditions for generator regime



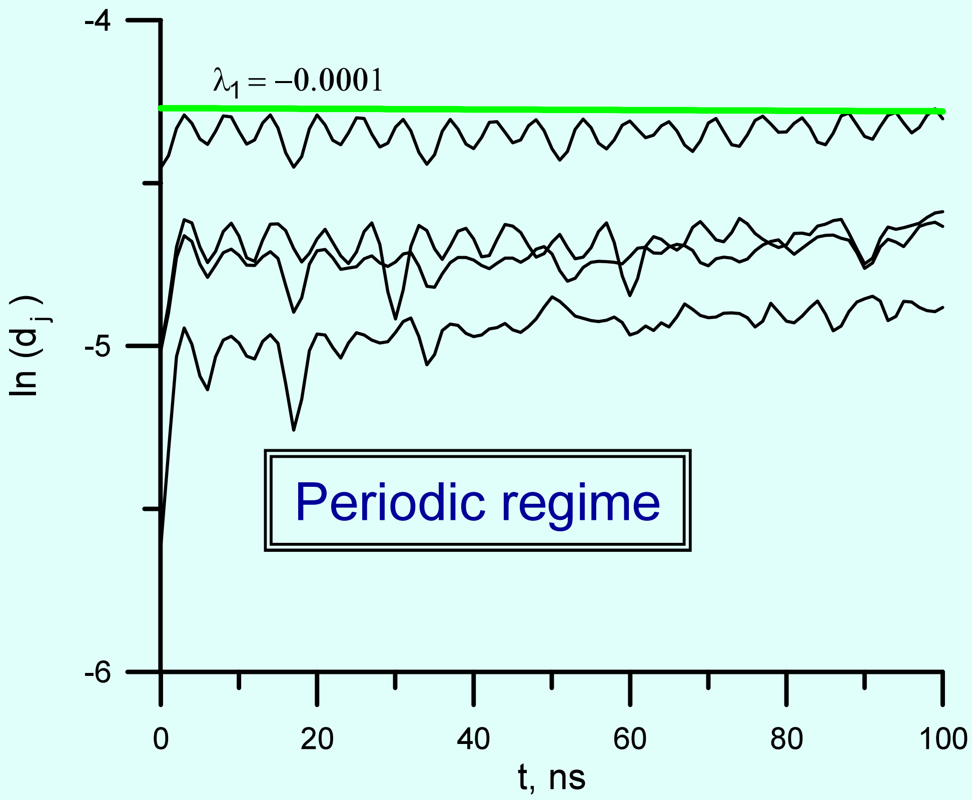
Sensibility to initial conditions for amplification regime

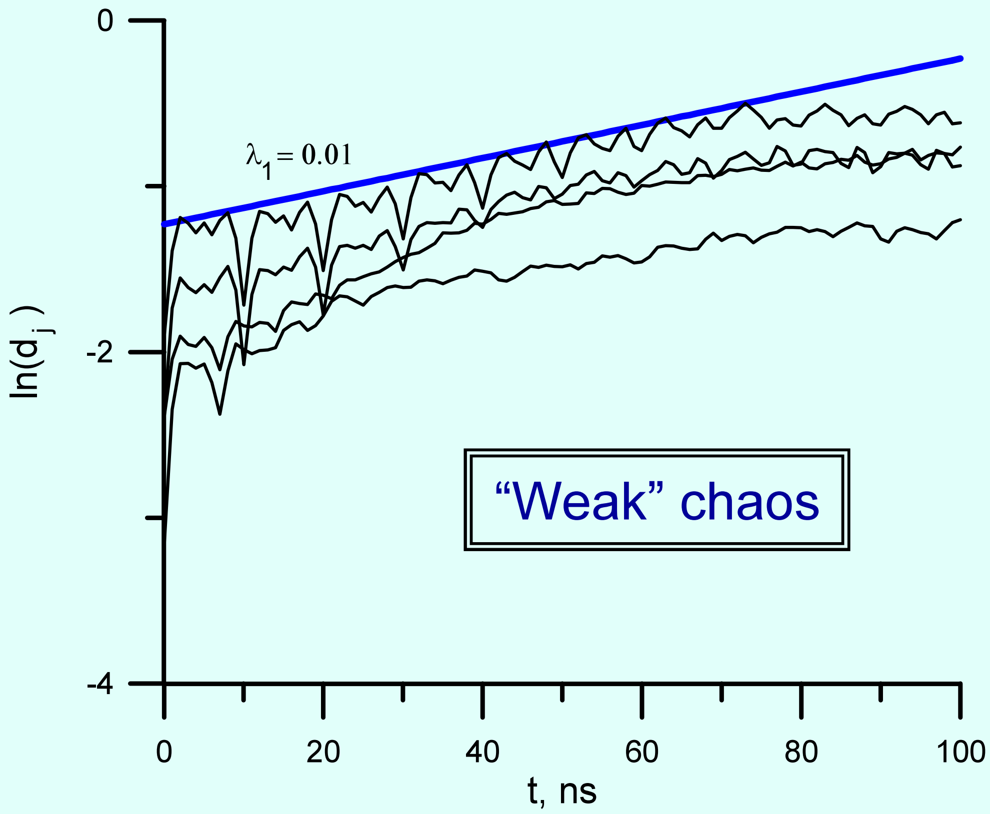




The largest Lyapunov exponent reconstructed with Rosenstein approach*

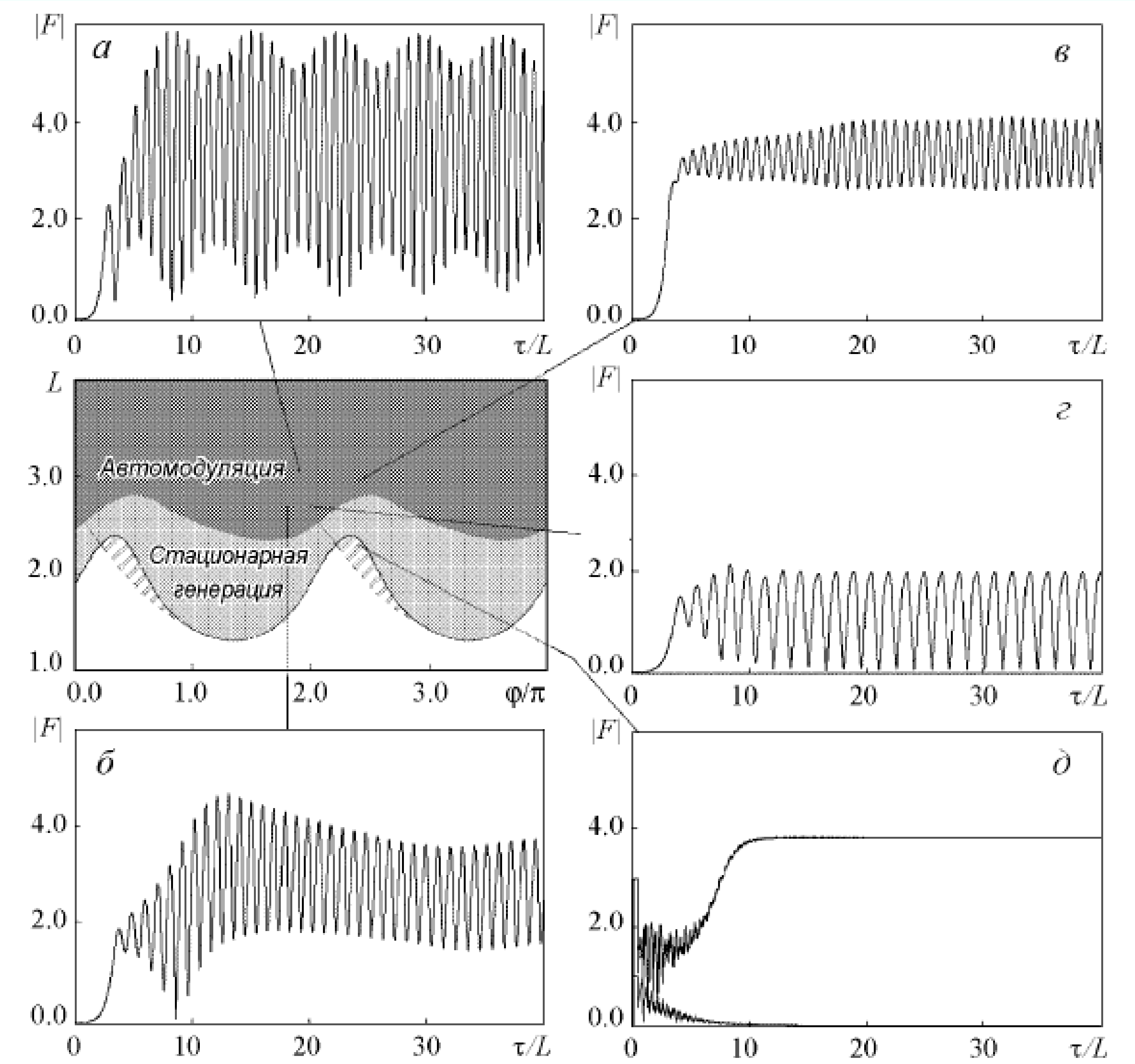
The largest Lyapunov exponent is a measurement of the stability of the underlying dynamics of time series. It specifies the mean velocity of divergence of neighboring points.





*M.T.Rosenstein et al. *Physica D65 (1993), 117-134*

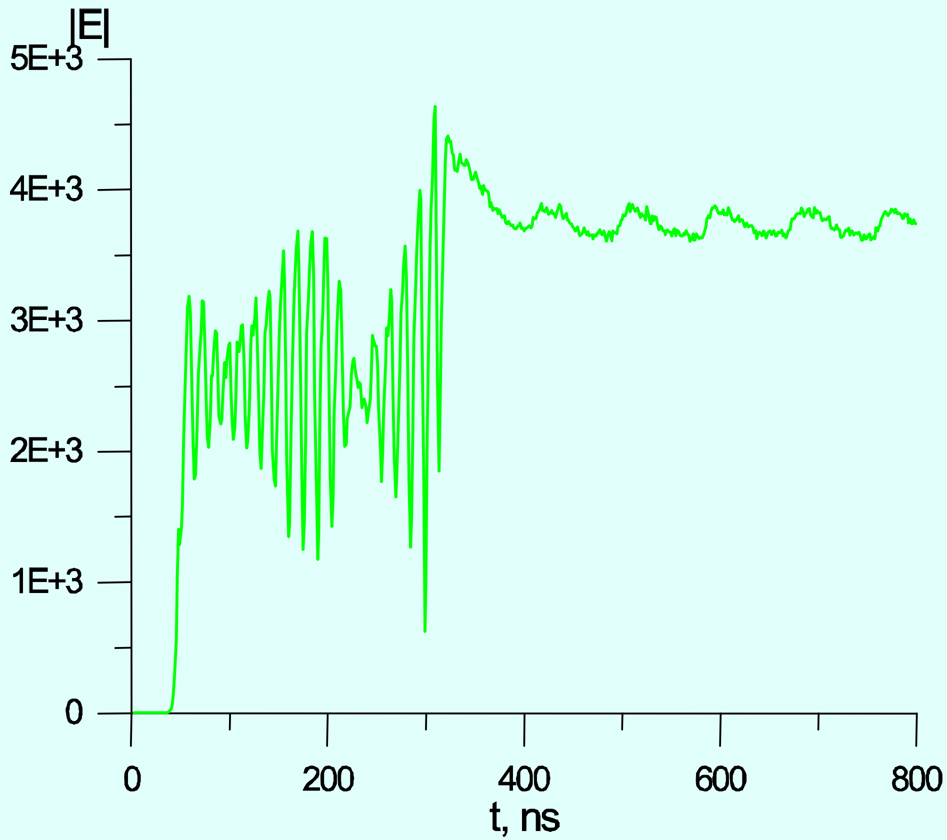
Map of BWT dynamical regimes with strong reflections*

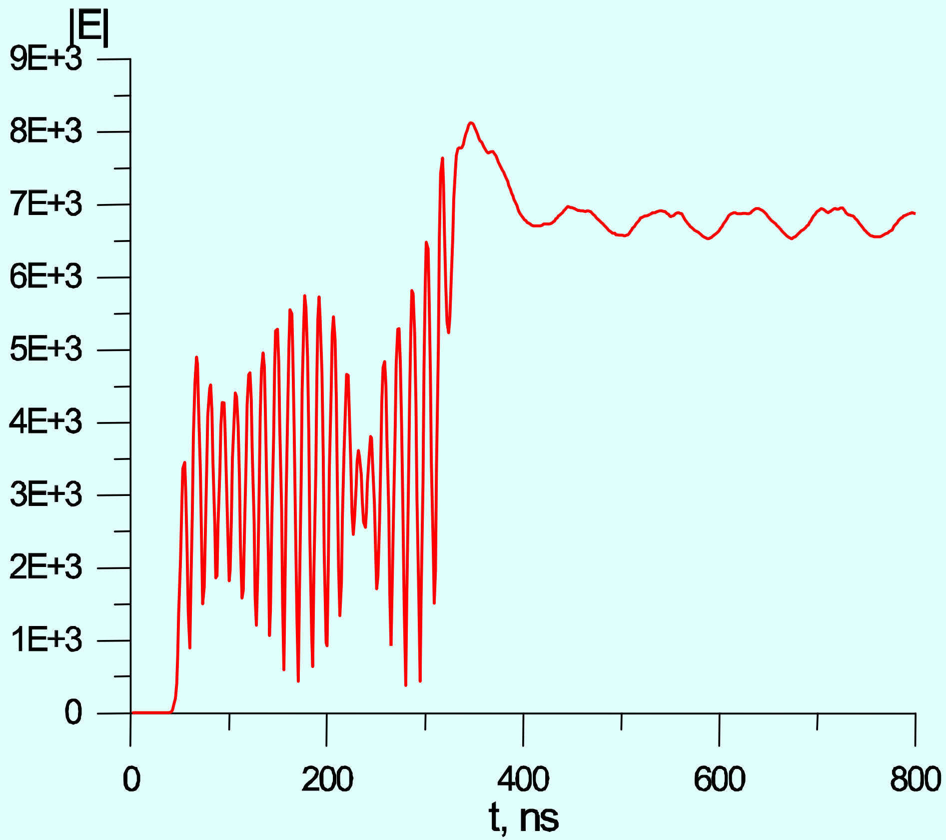


with large-scale and small-scale amplitude regimes

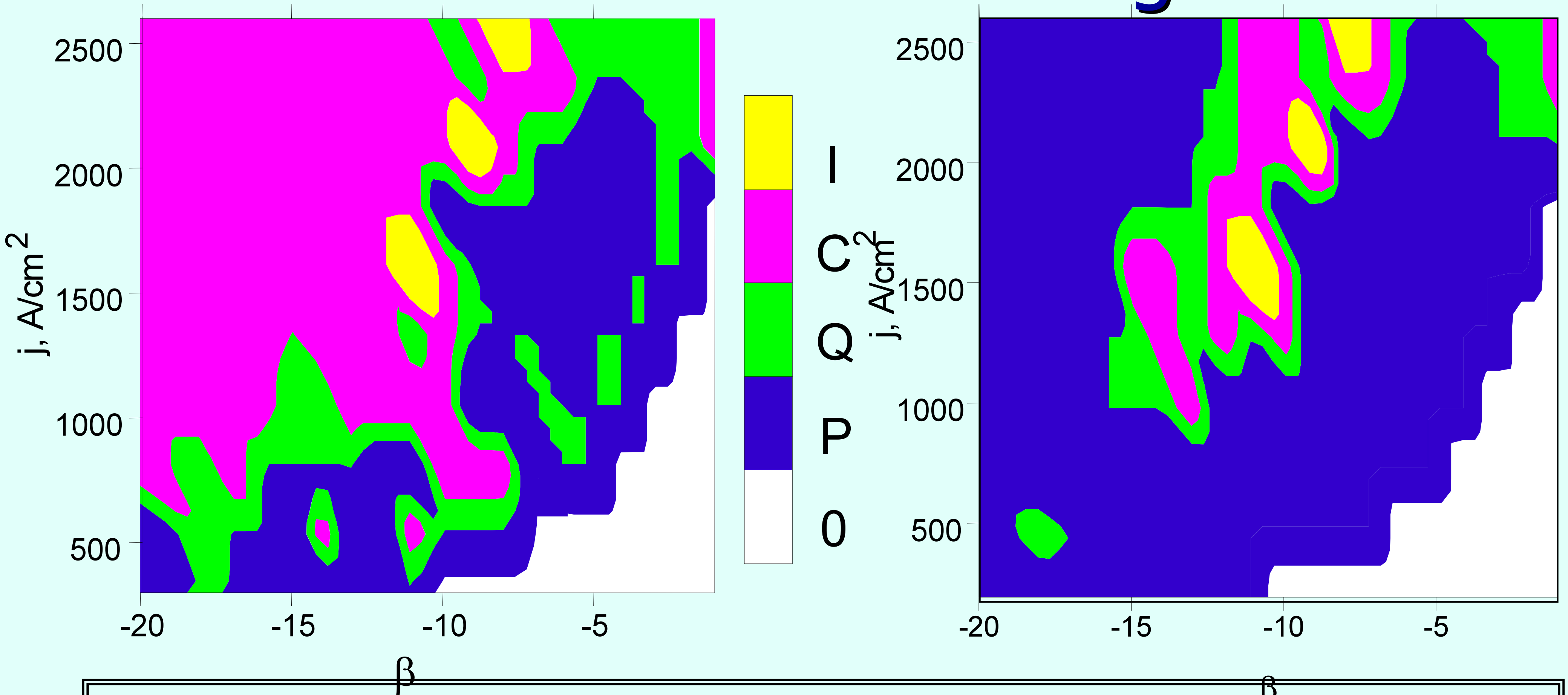
S.P.Kuznetsov.** ***Izvestia Vuzov "Applied Nonlinear Dynamics", 2006, v.14, 3-35

Domains with transition between large-scale and small-scale amplitudes





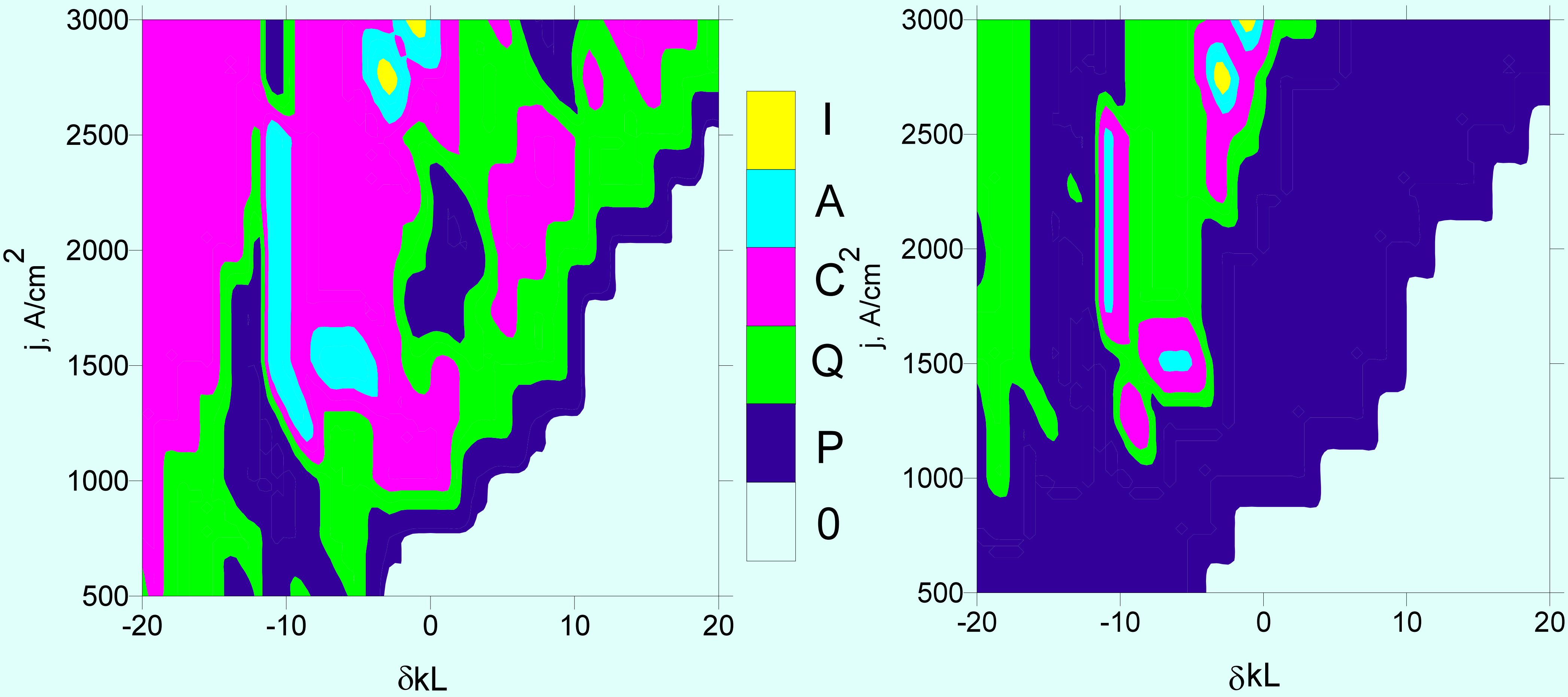
Root to chaotic lasing



0 depicts a domain under beam current threshold. P – periodic regimes, Q – quasiperiodicity, I – intermittency, C – chaos.

Larger number of principle frequencies for transmitted wave can be explained the fact that in VFEL simultaneous generation at several frequencies is available. Here electrons emit radiation namely in the direction of transmitted wave.

Another root to chaotic lasing



0 depicts a domain under beam current threshold. P – periodic regimes, Q –quasiperiodicity, A – domains with transition between large-scale and small-scale amplitudes, I – intermittency, C – chaos.

Conclusions

- The original software for VFEL simulation allows to obtain all main VFEL physical laws and dependencies.
- In simulation VFEL was considered as a dynamical system.
- Two-parameter analysis shows the complicated root to chaos in VFEL lasing.

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