# Generation of radiation in Volume Free Electron Lasers and problems of mathematical modeling of nonlinear processes in such generators

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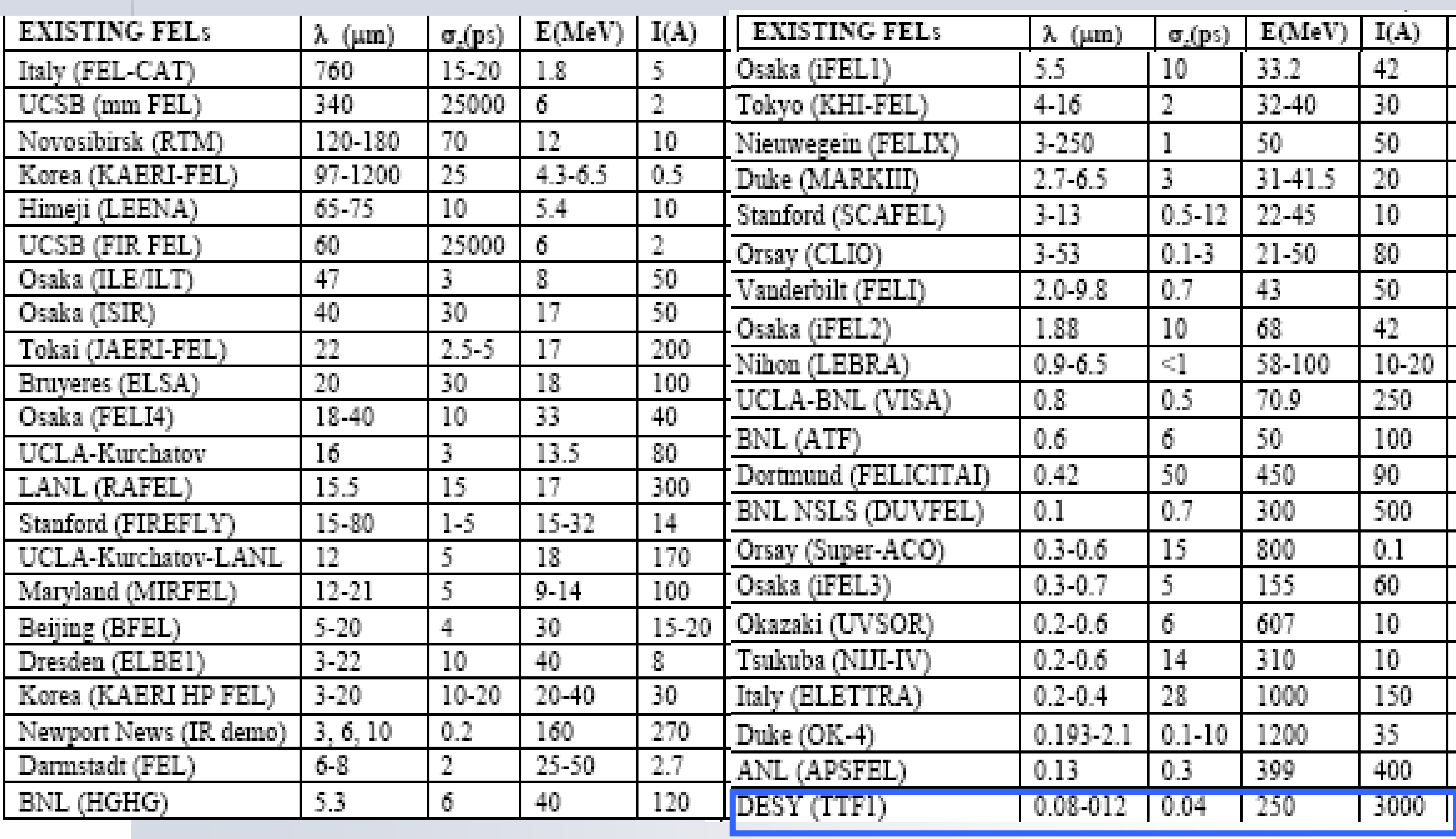
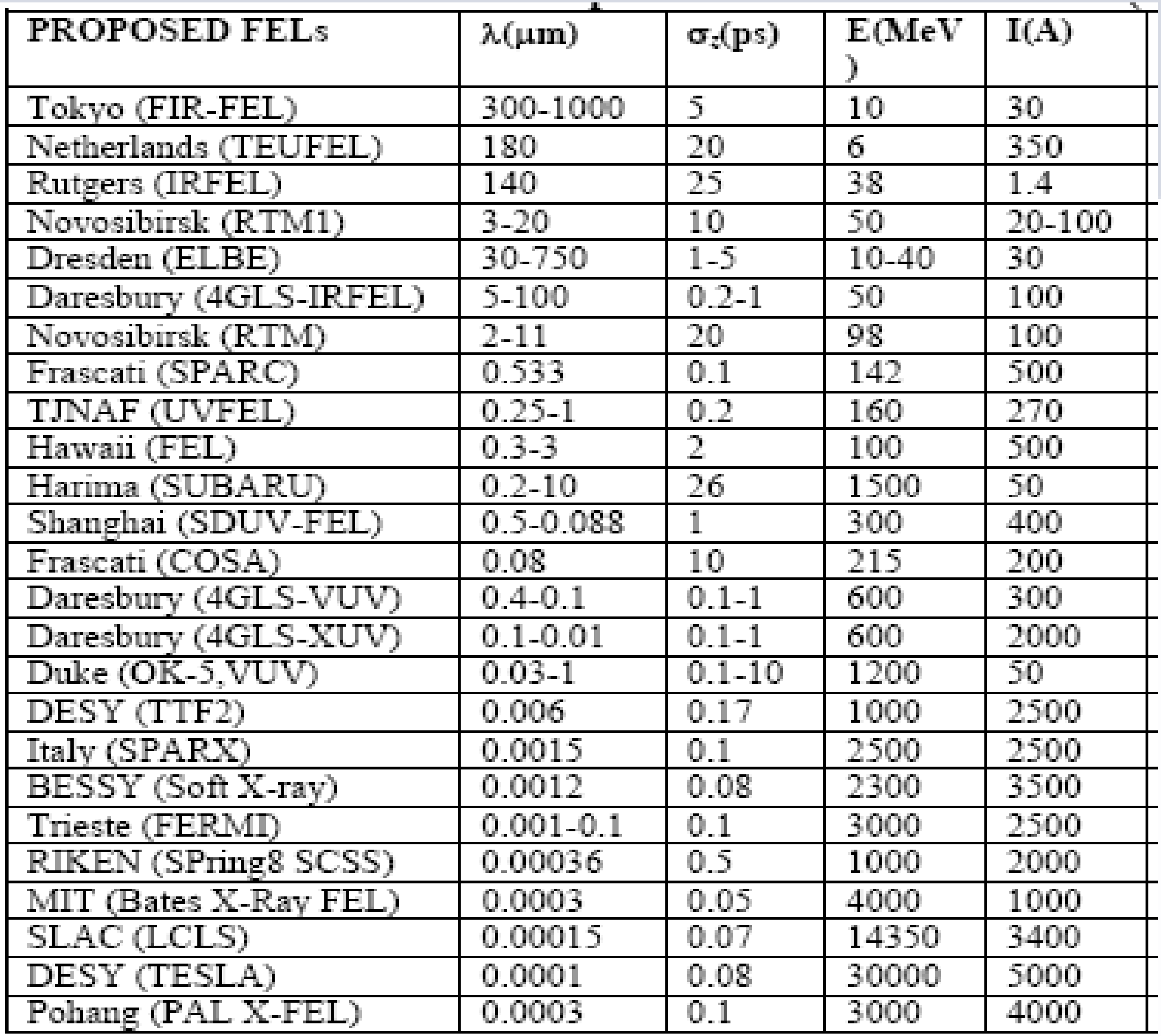
# Table 1: Free Electron Lasers (2004)

## (W.B.Colson et al. *Proc. 2004 FEL Conference*, 706-710)



| EXISTING FELs | λ (μm) | $\sigma_z$(ps) | E(MeV) | I(A) |
|---|---|---|---|---|
| Italy (FEL-CAT) | 760 | 15-20 | 1.8 | 5 |
| UCSB (mm FEL) | 340 | 25000 | 6 | 2 |
| Novosibirsk (RTM) | 120-180 | 70 | 12 | 10 |
| Korea (KAERI-FEL) | 97-1200 | 25 | 4.3-6.5 | 0.5 |
| Himeji (LEENA) | 65-75 | 10 | 5.4 | 10 |
| UCSB (FIR FEL) | 60 | 25000 | 6 | 2 |
| Osaka (ILE/ILT) | 47 | 3 | 8 | 50 |
| Osaka (ISIR) | 40 | 30 | 17 | 50 |
| Tokai (JAERI-FEL) | 22 | 2.5-5 | 17 | 200 |
| Bruyeres (ELSA) | 20 | 30 | 18 | 100 |
| Osaka (FELI4) | 18-40 | 10 | 33 | 40 |
| UCLA-Kurchatov | 16 | 3 | 13.5 | 80 |
| LANL (RAFEL) | 15.5 | 15 | 17 | 300 |
| Stanford (FIREFLY) | 15-80 | 1-5 | 15-32 | 14 |
| UCLA-Kurchatov-LANL | 12 | 5 | 18 | 170 |
| Maryland (MIRFEL) | 12-21 | 5 | 9-14 | 100 |
| Beijing (BFEL) | 5-20 | 4 | 30 | 15-20 |
| Dresden (ELBE1) | 3-22 | 10 | 40 | 8 |
| Korea (KAERI HP FEL) | 3-20 | 10-20 | 20-40 | 30 |
| Newport News (IR demo) | 3, 6, 10 | 0.2 | 160 | 270 |
| Darmstadt (FEL) | 6-8 | 2 | 25-50 | 2.7 |
| BNL (HGHG) | 5.3 | 6 | 40 | 120 |
| Osaka (iFEL1) | 5.5 | 10 | 33.2 | 42 |
| Tokyo (KHI-FEL) | 4-16 | 2 | 32-40 | 30 |
| Nieuwegein (FELIX) | 3-250 | 1 | 50 | 50 |
| Duke (MARKIII) | 2.7-6.5 | 3 | 31-41.5 | 20 |
| Stanford (SCAFEL) | 3-13 | 0.5-12 | 22-45 | 10 |
| Orsay (CLIO) | 3-53 | 0.1-3 | 21-50 | 80 |
| Vanderbilt (FELI) | 2.0-9.8 | 0.7 | 43 | 50 |
| Osaka (iFEL2) | 1.88 | 10 | 68 | 42 |
| Nihon (LEBRA) | 0.9-6.5 | <1 | 58-100 | 10-20 |
| UCLA-BNL (VISA) | 0.8 | 0.5 | 70.9 | 250 |
| BNL (ATF) | 0.6 | 6 | 50 | 100 |
| Dortmund (FELICITAI) | 0.42 | 50 | 450 | 90 |
| BNL NSLS (DUVFEL) | 0.1 | 0.7 | 300 | 500 |
| Orsay (Super-ACO) | 0.3-0.6 | 15 | 800 | 0.1 |
| Osaka (iFEL3) | 0.3-0.7 | 5 | 155 | 60 |
| Okazaki (UVSOR) | 0.2-0.6 | 6 | 607 | 10 |
| Tsukuba (NIJI-IV) | 0.2-0.6 | 14 | 310 | 10 |
| Italy (ELETTRA) | 0.2-0.4 | 28 | 1000 | 150 |
| Duke (OK-4) | 0.193-2.1 | 0.1-10 | 1200 | 35 |
| ANL (APSFEL) | 0.13 | 0.3 | 399 | 400 |
| DESY (TTF1) | 0.08-012 | 0.04 | 250 | 3000 |

# Table 2: Proposed Free Electron Lasers (2004)



| PROPOSED FELs | λ(μm) | $\sigma_z$(ps) | E(MeV) | I(A) |
|---|---|---|---|---|
| Tokyo (FIR-FEL) | 300-1000 | 5 | 10 | 30 |
| Netherlands (TEUFEL) | 180 | 20 | 6 | 350 |
| Rutgers (IRFEL) | 140 | 25 | 38 | 1.4 |
| Novosibirsk (RTM1) | 3-20 | 10 | 50 | 20-100 |
| Dresden (ELBE) | 30-750 | 1-5 | 10-40 | 30 |
| Daresbury (4GLS-IRFEL) | 5-100 | 0.2-1 | 50 | 100 |
| Novosibirsk (RTM) | 2-11 | 20 | 98 | 100 |
| Frascati (SPARC) | 0.533 | 0.1 | 142 | 500 |
| TJNAF (UVFEL) | 0.25-1 | 0.2 | 160 | 270 |
| Hawaii (FEL) | 0.3-3 | 2 | 100 | 500 |
| Harima (SUBARU) | 0.2-10 | 26 | 1500 | 50 |
| Shanghai (SDUV-FEL) | 0.5-0.088 | 1 | 300 | 400 |
| Frascati (COSA) | 0.08 | 10 | 215 | 200 |
| Daresbury (4GLS-VUV) | 0.4-0.1 | 0.1-1 | 600 | 300 |
| Daresbury (4GLS-XUV) | 0.1-0.01 | 0.1-1 | 600 | 2000 |
| Duke (OK-5,VUV) | 0.03-1 | 0.1-10 | 1200 | 50 |
| DESY (TTF2) | 0.006 | 0.17 | 1000 | 2500 |
| Italy (SPARX) | 0.0015 | 0.1 | 2500 | 2500 |
| BESSY (Soft X-ray) | 0.0012 | 0.08 | 2300 | 3500 |
| Trieste (FERMI) | 0.001-0.1 | 0.1 | 3000 | 2500 |
| RIKEN (SPring8 SCSS) | 0.00036 | 0.5 | 1000 | 2000 |
| MIT (Bates X-Ray FEL) | 0.0003 | 0.05 | 4000 | 1000 |
| SLAC (LCLS) | 0.00015 | 0.07 | 14350 | 3400 |
| DESY (TESLA) | 0.0001 | 0.08 | 30000 | 5000 |
| Pohang (PAL X-FEL) | 0.0003 | 0.1 | 3000 | 4000 |

# First lasing of Volume FEL (VFEL) in wavelength range $\lambda$~ 4 - 6 mm (2001)

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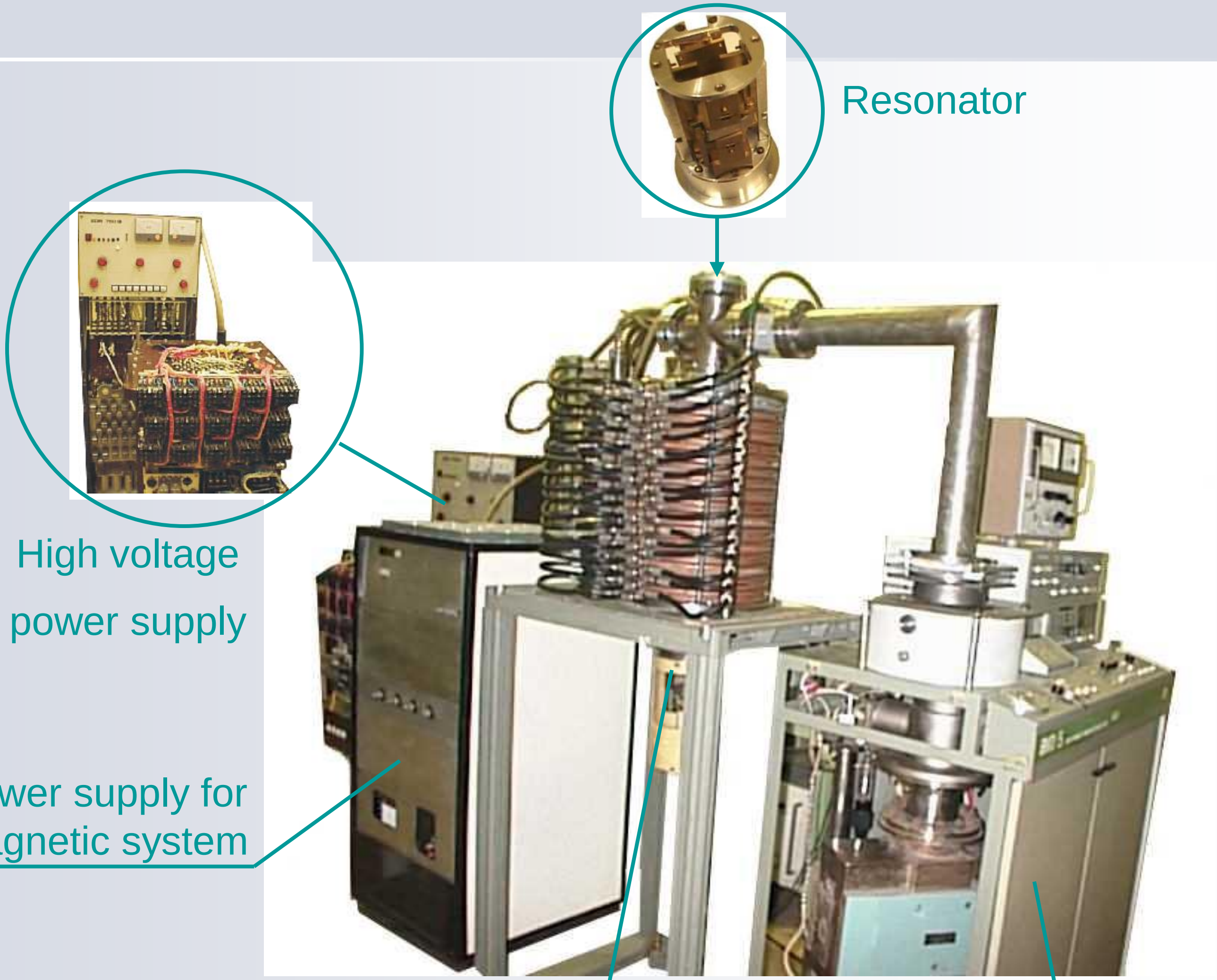
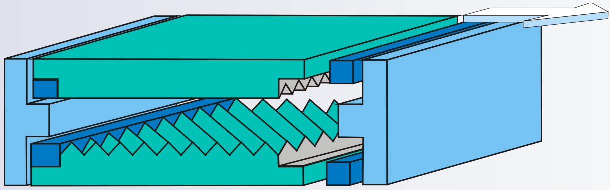
# VFEL-10 keV (2001)

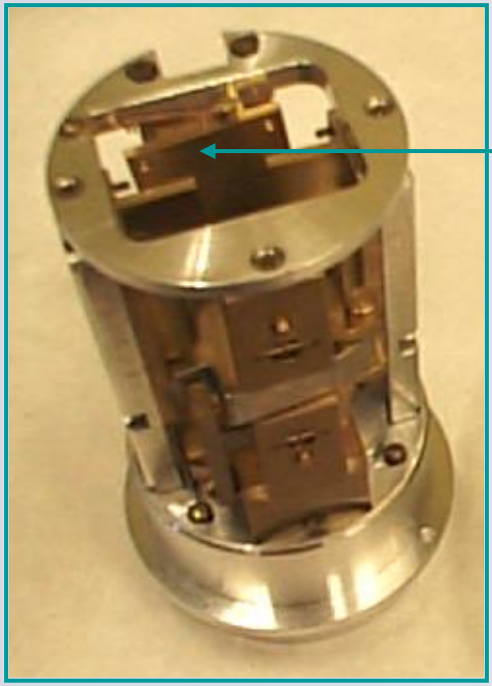
# General view



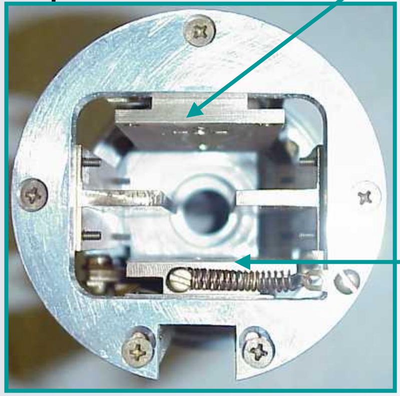
# Resonator



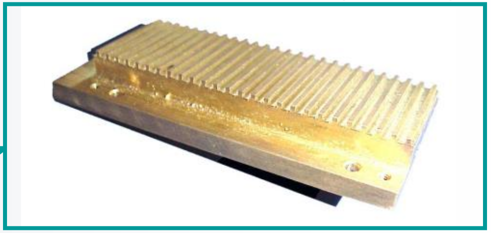
side view

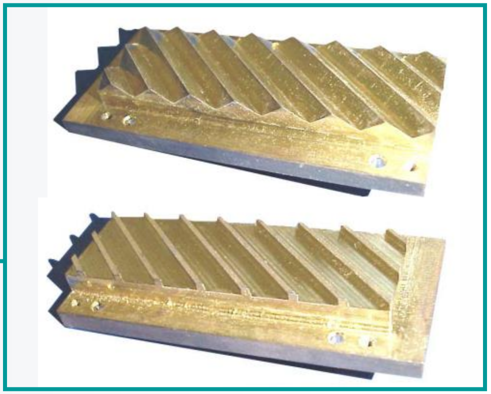


top view



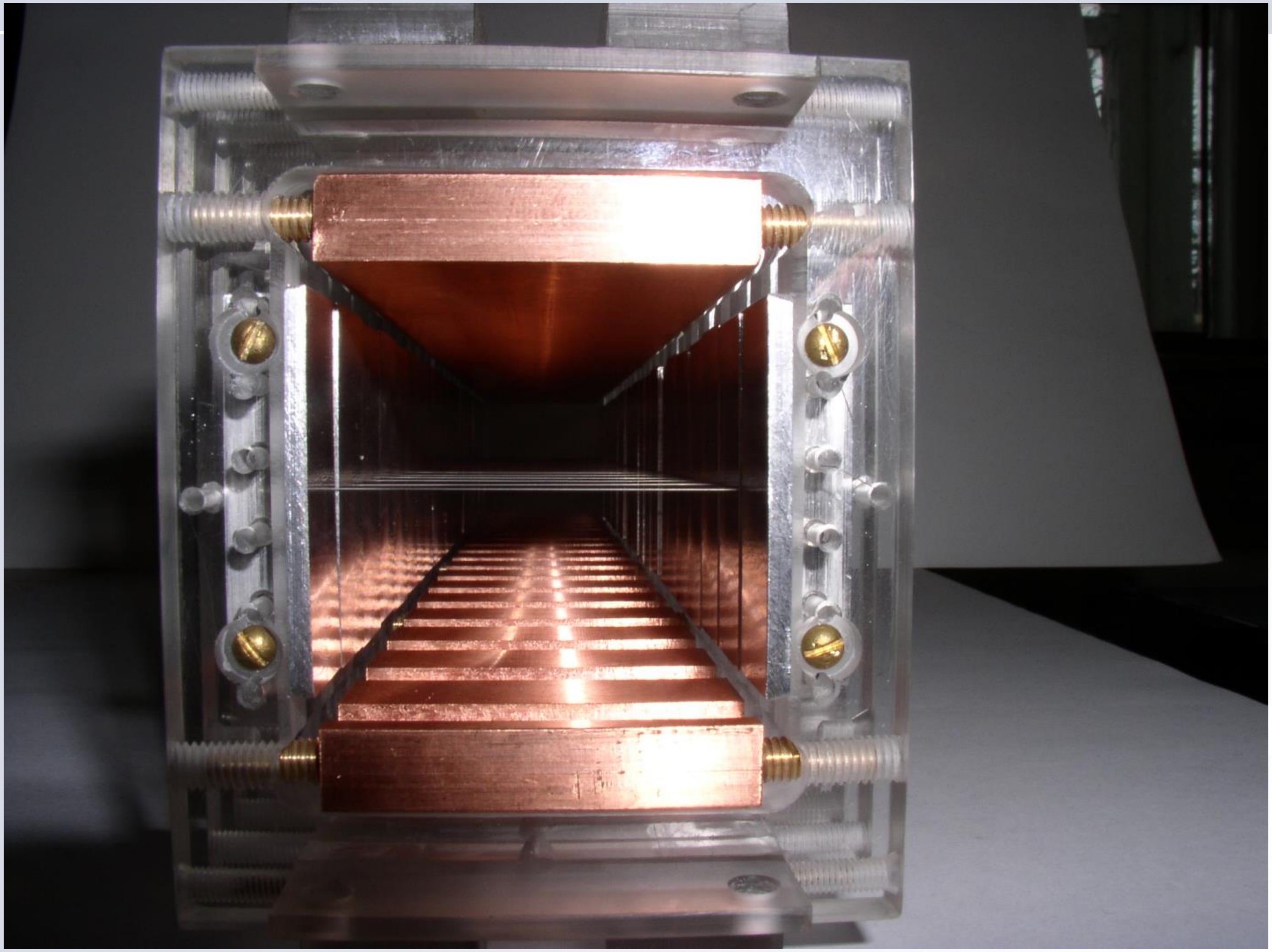
diffraction gratings





# VFEL-250 keV (2003)

# “Grid” volume resonator



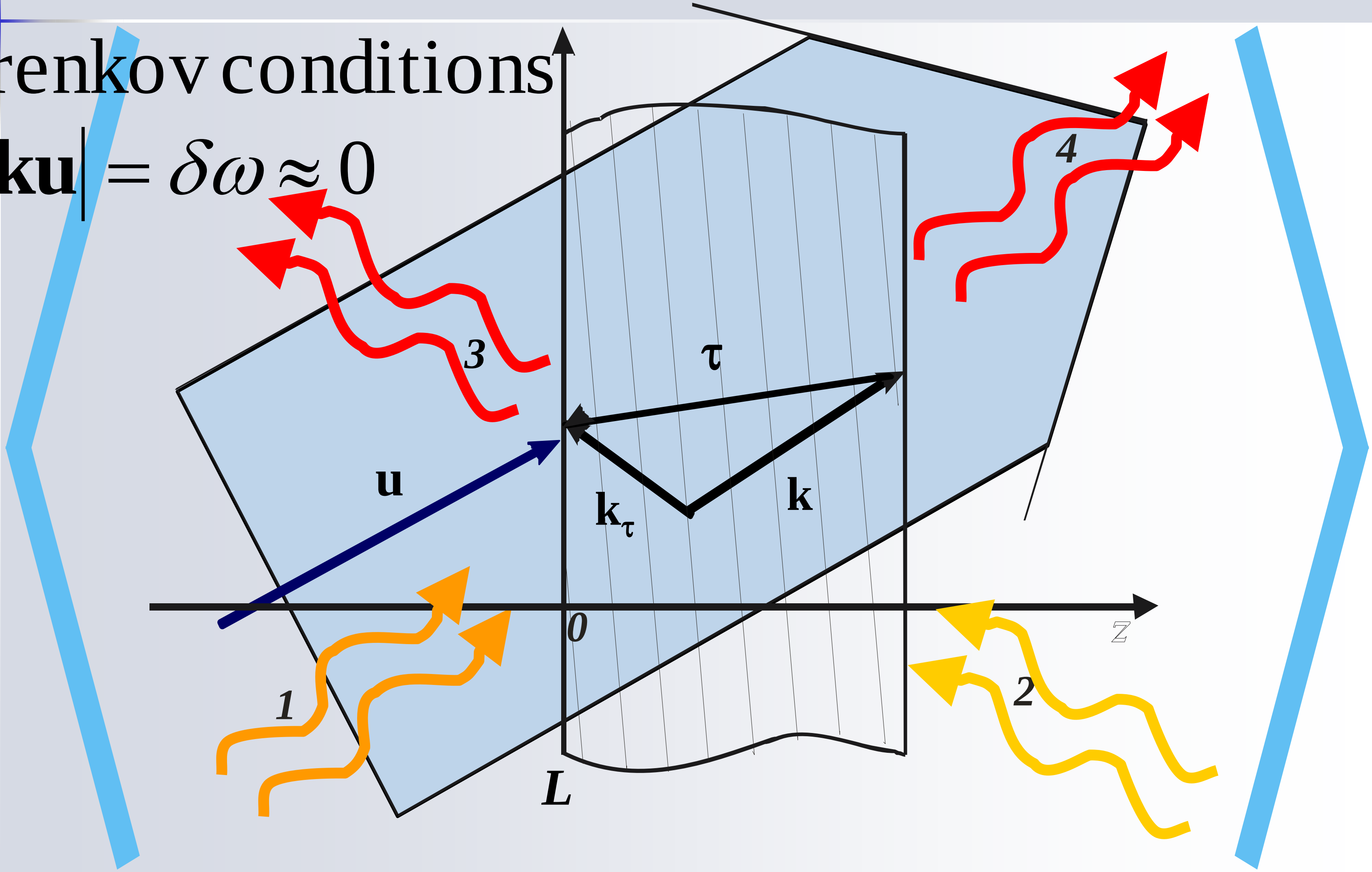
# VFEL in Bragg geometry

Bragg conditions

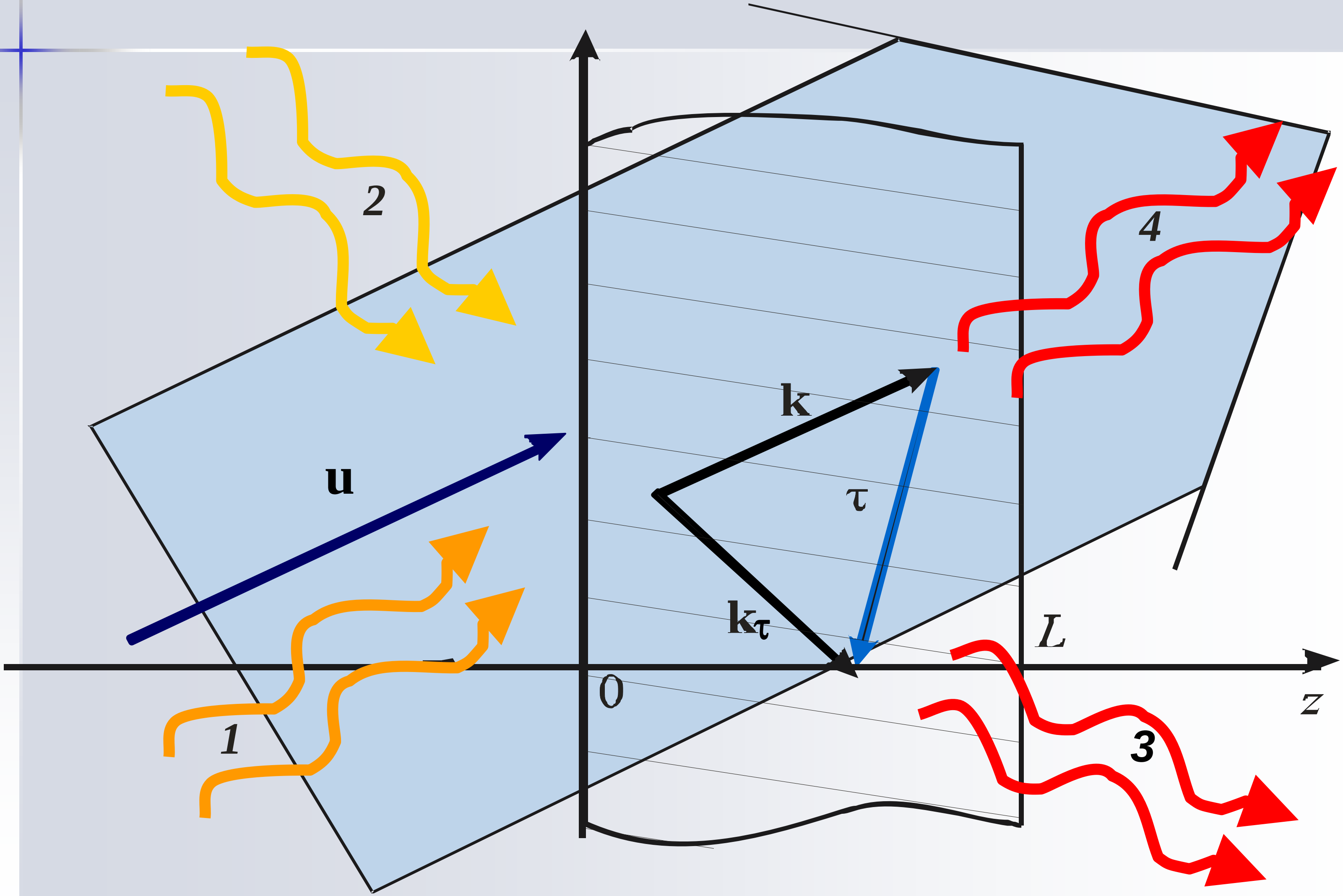
$$2\mathbf{k}\boldsymbol{\tau} + \boldsymbol{\tau}^2 \approx 0,$$

Cherenkov conditions

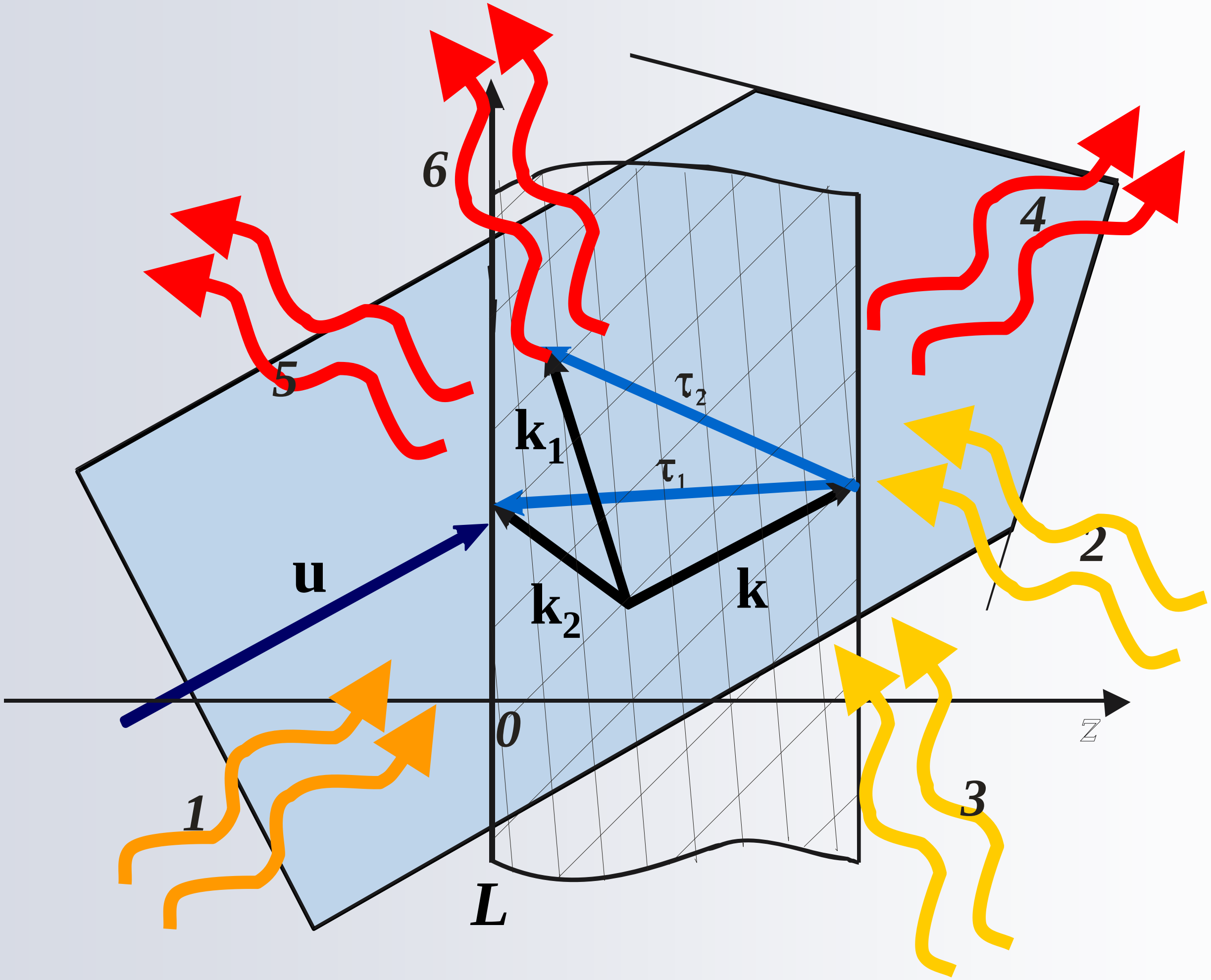
$$\left|\omega - \mathbf{ku}\right| = \delta\omega \approx 0$$



# Laue geometry

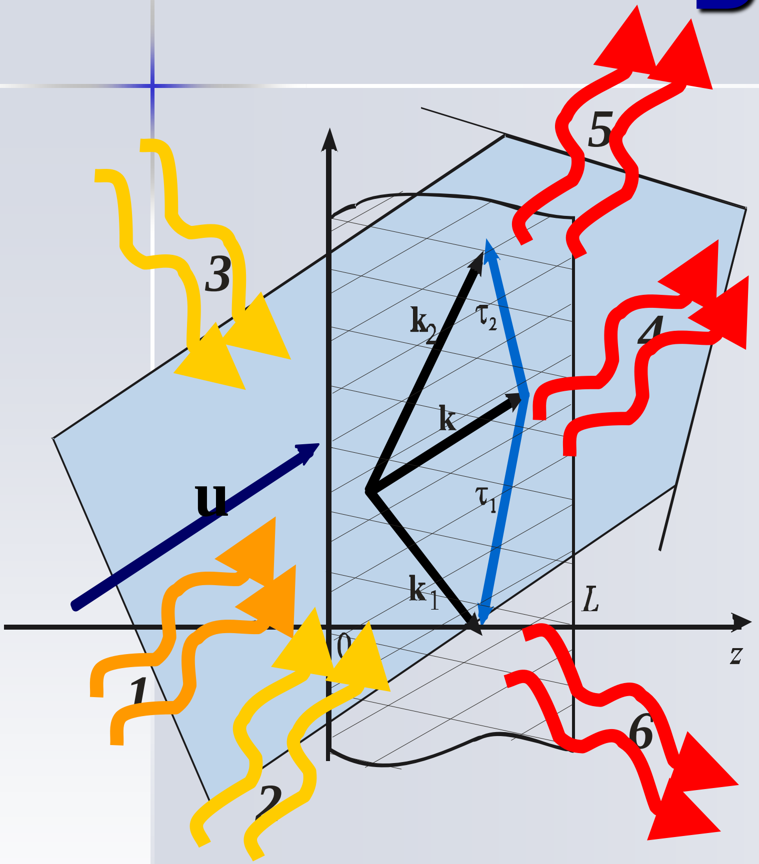


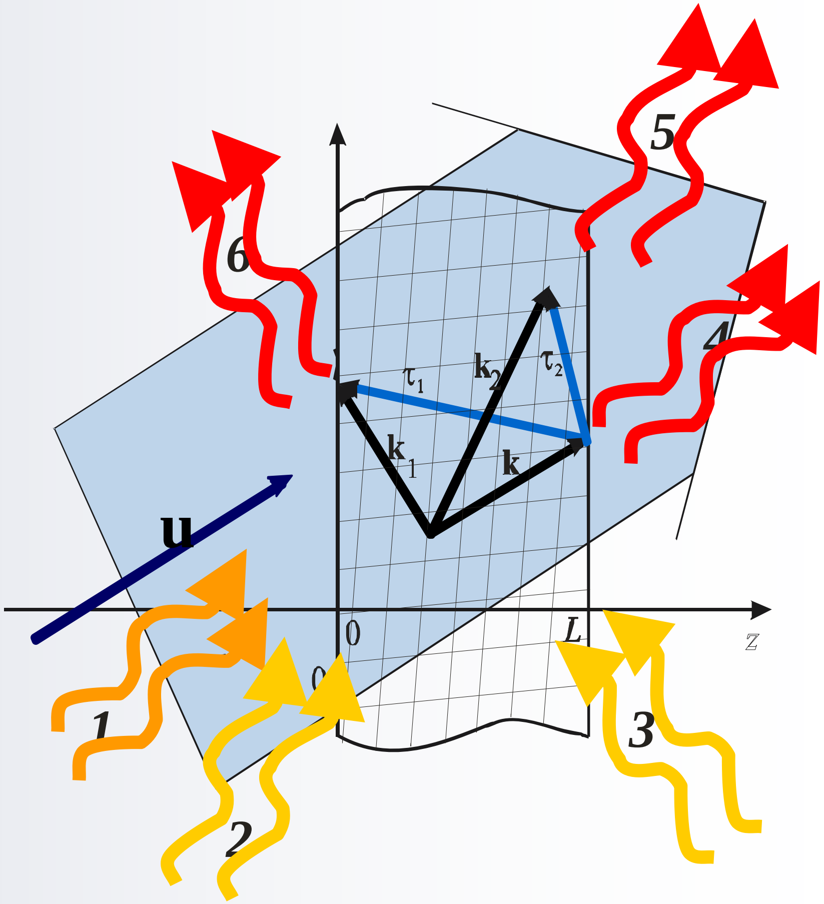
# Three-wave VFEL, Bragg-Bragg geometry



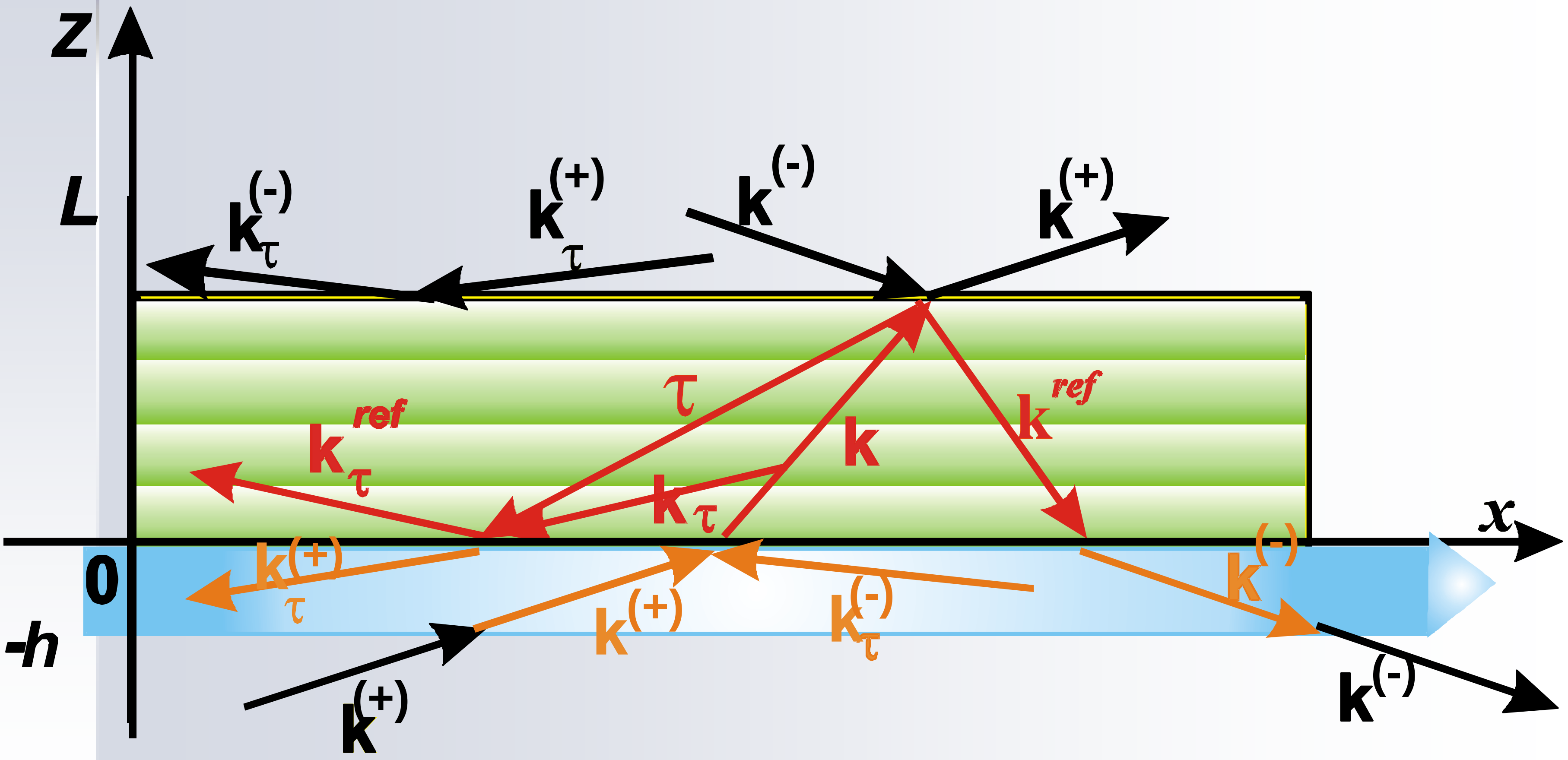
# Laue-Laue geometry

# Bragg-Laue geometry





# Surface VFEL



# Volume distributed feedback

- If **one mode** is in synchronism, the threshold current ***j***:

$$j \sim \frac{1}{(kL)^3}$$

- If **two modes are** in synchronism, the threshold current ***j***:

$$j \sim \frac{1}{(kL)^5}$$

- If **n modes are** in synchronism, the threshold current ***j***:

$$j \sim \frac{1}{(kL)^{3+2(n-1)}}$$

We assume $kL \gg 1$

# Main equations

$$\Delta\mathbf{E} - \nabla(\nabla\mathbf{E}) - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t} = \frac{\partial\mathbf{j}_b}{\partial t}$$

$$\mathbf{E} = \mathbf{e}_\sigma\left(Ee^{i(\mathbf{kr}-\omega t)} + E_\tau e^{i(\mathbf{k}_\tau\mathbf{r}-\omega t)}\right),$$

$$\mathbf{j}_b = \mathbf{e}_\sigma j e^{i(\mathbf{kr}-\omega t)},$$

In the common $n$ – wave case:

$$\mathbf{E} = \mathbf{e}\sum_{i=1}^{n} E_i e^{i(\mathbf{k}_i\mathbf{r}-\omega t)}$$

# System for three-wave VFEL:

$$\frac{\partial E_0}{\partial t}+\gamma_0 c\frac{\partial E_0}{\partial z}+0.5i\omega l E_0-0.5i\omega\chi_1 E_1-0.5i\omega\chi_2 E_2=$$

$$=2\pi j\Phi\int_0^{2\pi}\frac{2\pi-p}{8\pi^2}\left(e^{-i\theta(t,z,p)}+e^{-i\theta(t,z,-p)}\right)dp,$$

$$\frac{\partial E_1}{\partial t}+\gamma_1 c\frac{\partial E_1}{\partial z}-0.5i\omega\chi_{-1}E_0+0.5i\omega l_1 E_1-0.5i\omega\chi_{2-1}E_2=0,$$

$$\frac{\partial E_2}{\partial t}+\gamma_2 c\frac{\partial E_2}{\partial z}-0.5i\omega\chi_{-2}E_0-0.5i\omega\chi_{1-2}E_1+0.5i\omega l_2 E_2=0$$

# System parameters:

$$l_i = \frac{k_i^2 c^2 - \omega^2 \varepsilon_0}{\omega^2}, \quad i = 0, 1, 2$$

$$l = l_0 + \delta, \quad \delta$$ — **detuning from exact Cherenkov condition**

# Simple initial and boundary conditions:

$$E_0\big|_{z=0} = E_0^0,$$

$$E_1\big|_{z=L_1} = E_1^0$$

$$E_2\big|_{z=L_2} = E_2^0,$$

$$E_j\big|_{t=0} = 0$$

# System for n-wave VFEL:

$$\frac{\partial \mathbf{E}}{\partial t} + \mathbf{A}\frac{\partial \mathbf{E}}{\partial z} + \mathbf{BE} = \mathbf{F}(I),$$

$$\mathbf{E} = (E_j)^T,\ j = 1,\dots,n$$

# Boundary conditions including mirrors:

$$E_i\Big|_{z=L_i} = E_i^0 + \sum_{j\neq i} \alpha_j E_j\Big|_{z=L_j} \exp(i\varphi_j);$$

$$i = 1, \ldots, n$$

## Common boundary conditions:

$$\mathbf{B}\frac{\partial \mathbf{E}}{\partial t} + \sum_{i=2}^{n} \mathbf{P}_i \frac{\partial \mathbf{E}}{\partial x_i} + \mathbf{QE} = \mathbf{G}(\mathbf{j}, \mathbf{E}^0)$$

# Equations for electron beam

$$\frac{d^2\theta}{dt^2} = \frac{e}{m\gamma^3}(\mathbf{e}_\sigma \mathbf{n})\,\mathrm{Re}\Big\{E\exp\Big(i(\mathbf{k}_\perp \mathbf{r}_\perp + k_z z - \omega t)\Big)\Big\},$$

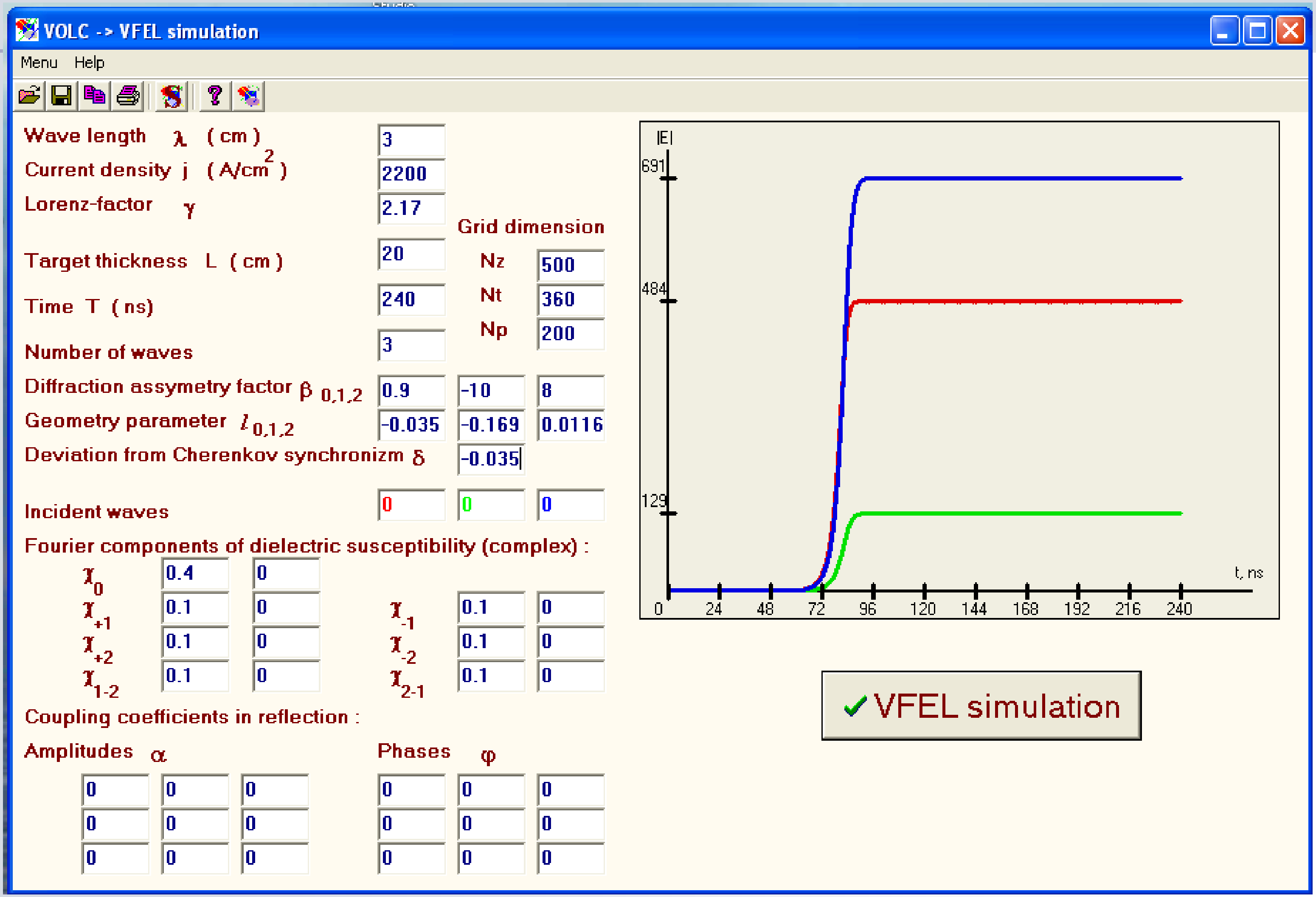
$$\theta(t, t_0, \mathbf{r}_\perp) = \mathbf{k}_\perp \mathbf{r}_\perp + k_z z - \omega t(z, t_0) -$$

electronphase in a wave

$$\frac{d\theta(t,0,p)}{dz} = k - \omega / u, \quad \theta(t,0,p) = p,$$

$$t > 0, \quad z \in [0, L], \quad p \in [-2\pi, 2\pi]$$

# Code VOLC - VFEL simulation



Wave length $\lambda$ ( cm ) 3

Current density j ( A/cm$^2$ ) 2200

Lorenz-factor $\gamma$ 2.17

Target thickness L ( cm ) 20

Time T ( ns) 240

Number of waves 3

Grid dimension

Nz 500

Nt 360

Np 200

Diffraction assymetry factor $\beta_{0,1,2}$ 0.9 -10 8

Geometry parameter $l_{0,1,2}$ -0.035 -0.169 0.0116

Deviation from Cherenkov synchronizm $\delta$ -0.035

Incident waves 0 0 0

Fourier components of dielectric susceptibility (complex) :

$\chi_0$ 0.4 0

$\chi_{+1}$ 0.1 0

$\chi_{+2}$ 0.1 0

$\chi_{1-2}$ 0.1 0

$\chi_{-1}$ 0.1 0

$\chi_{-2}$ 0.1 0

$\chi_{2-1}$ 0.1 0

Coupling coefficients in reflection :

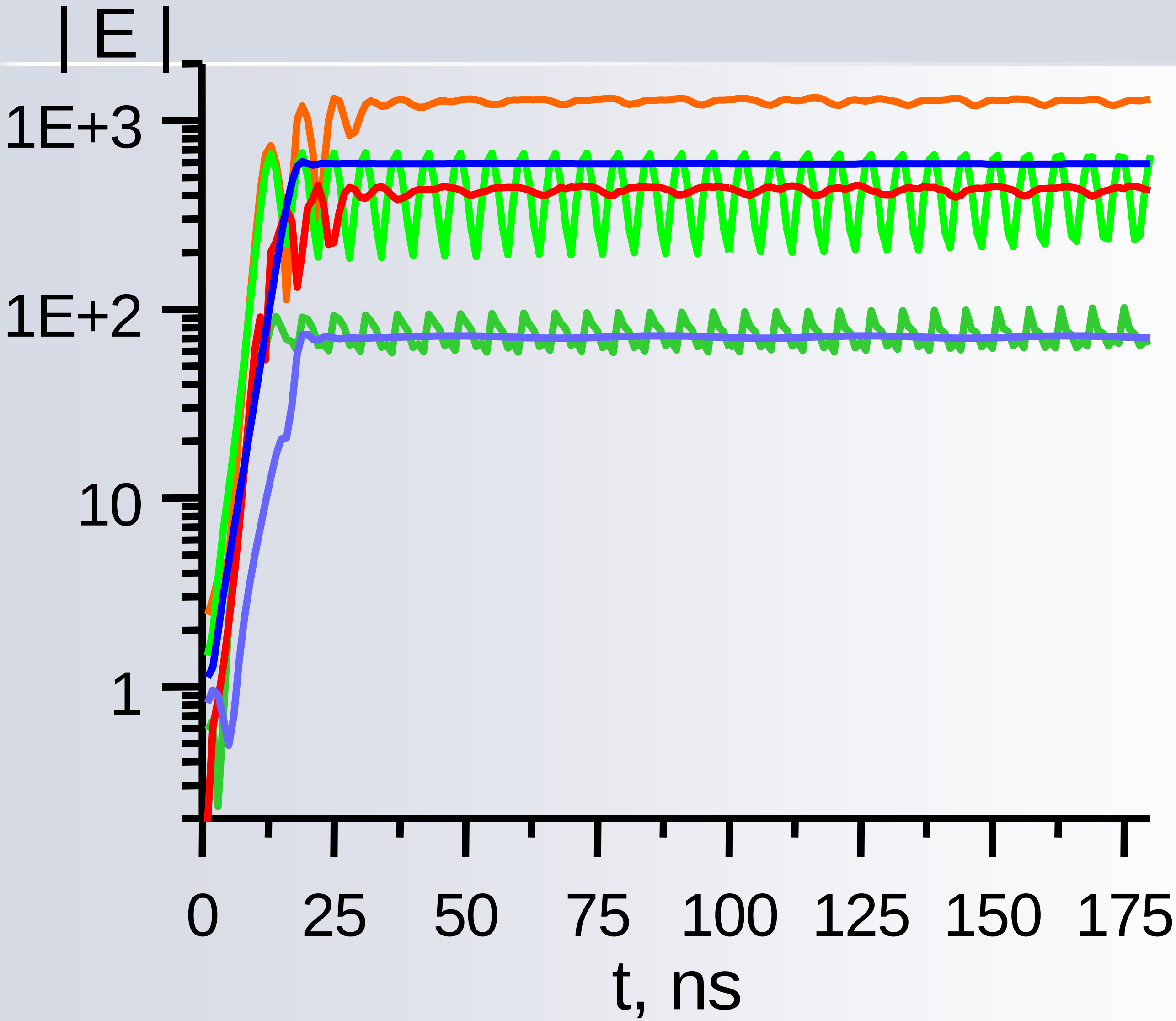
Amplitudes $\alpha$

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

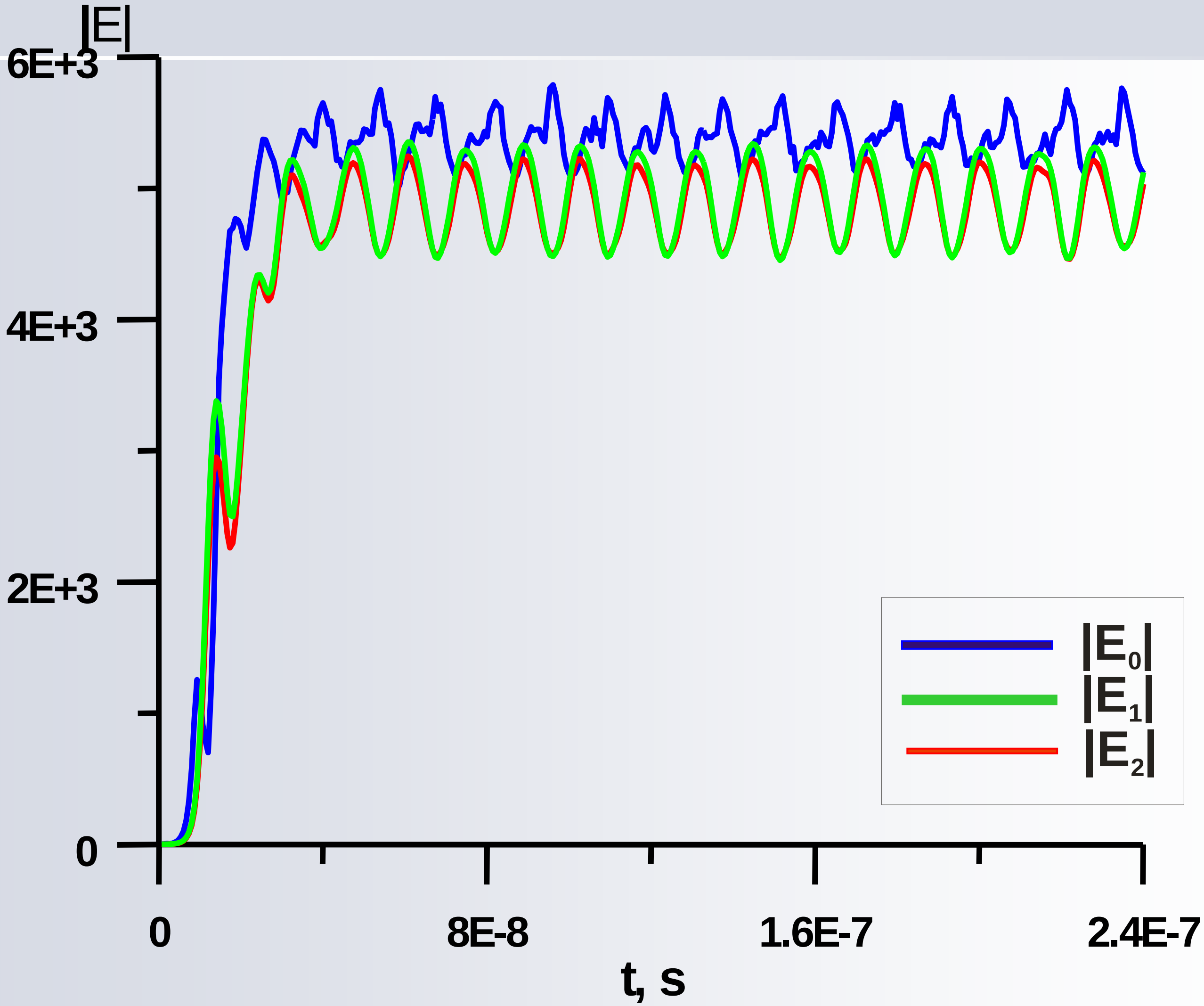
Phases $\varphi$

| | | |
|---|---|---|
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |

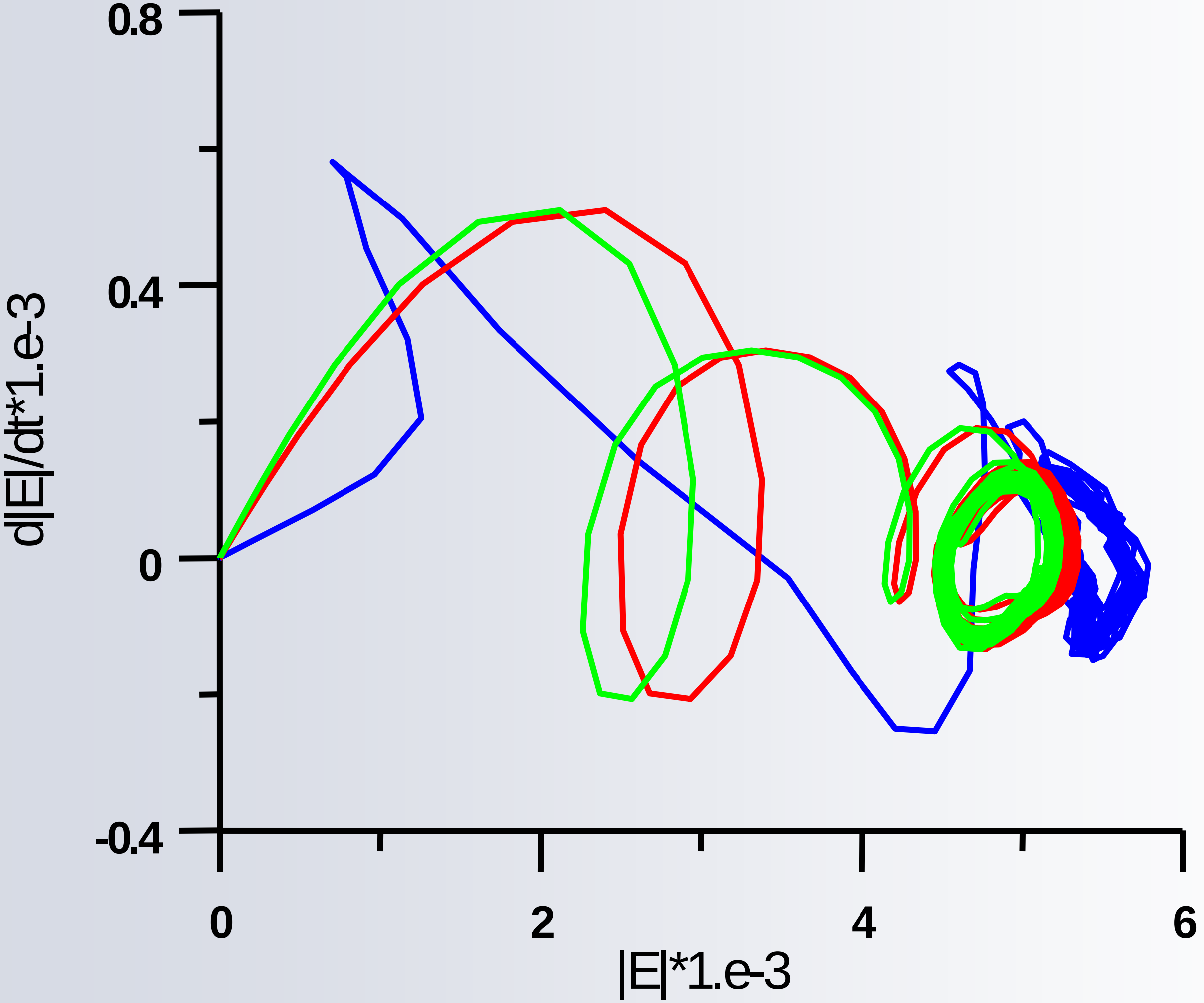
# Establishment of nonstationary solution in two-wave geometry



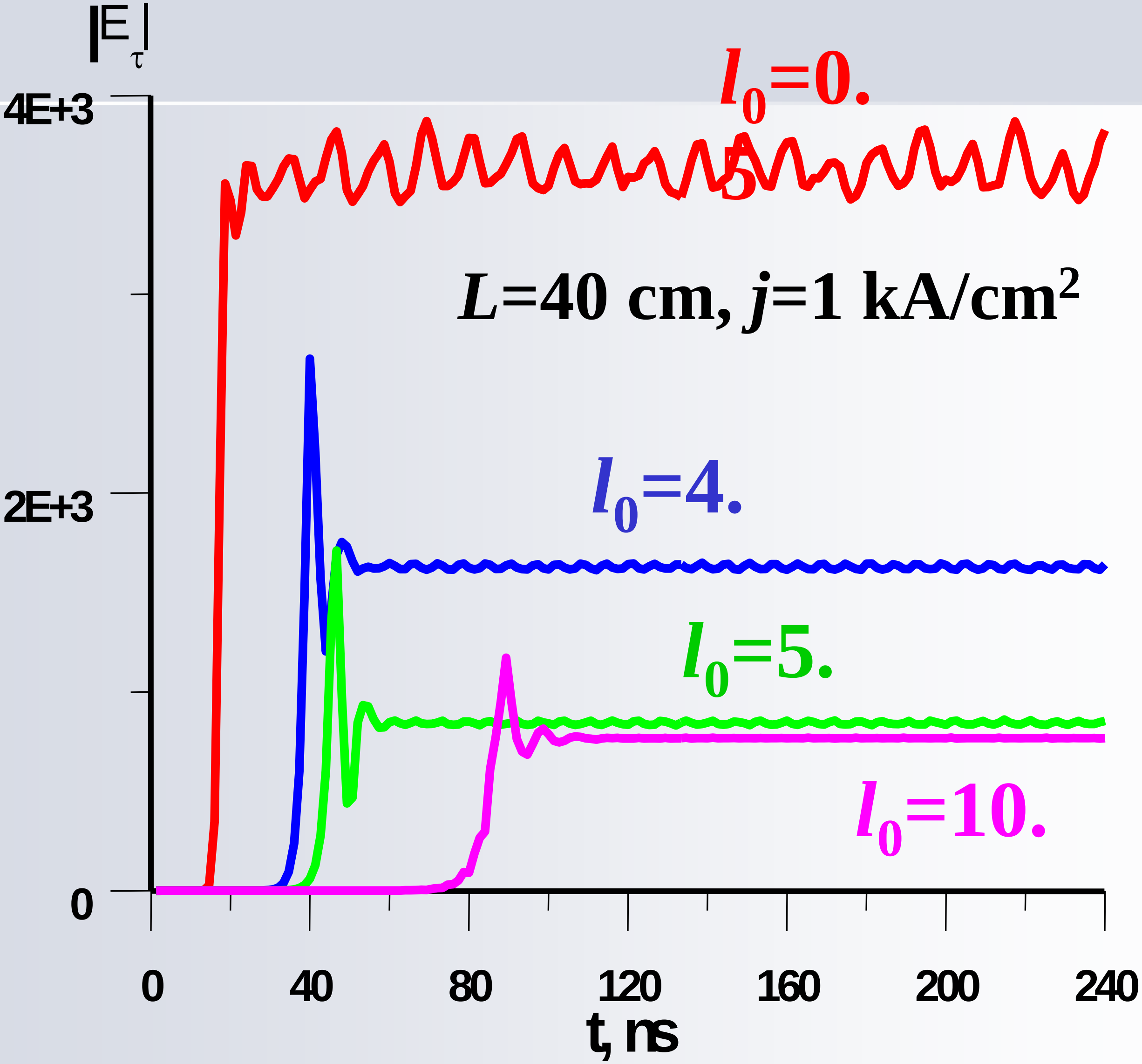
# Periodic regime of VFEL intensity in three-wave geometry:



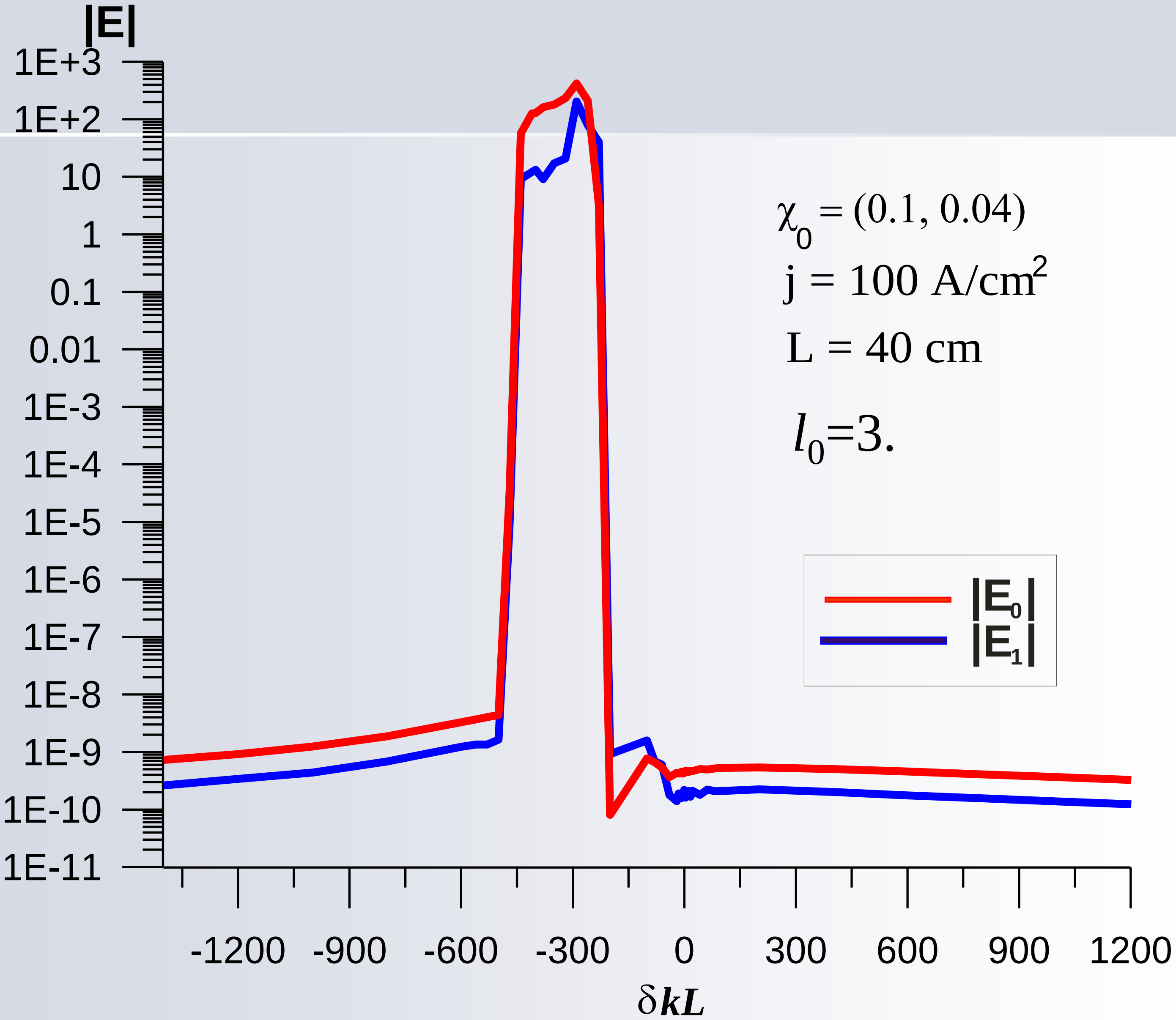
# Phase space portrait



# Simulation of BWT



# Simulation of Smith-Purcell radiation



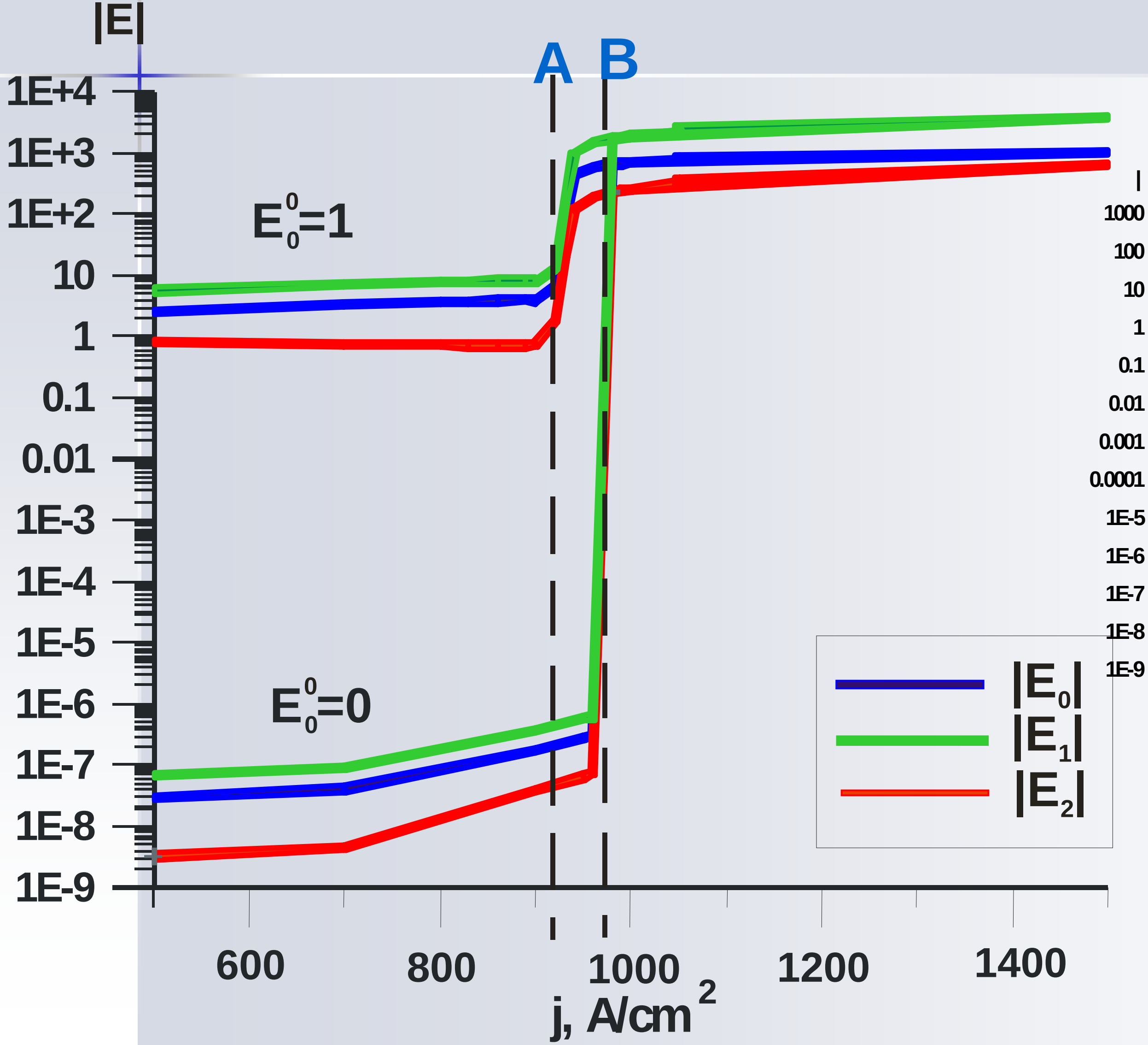
$\chi_0 = (0.1, 0.04)$

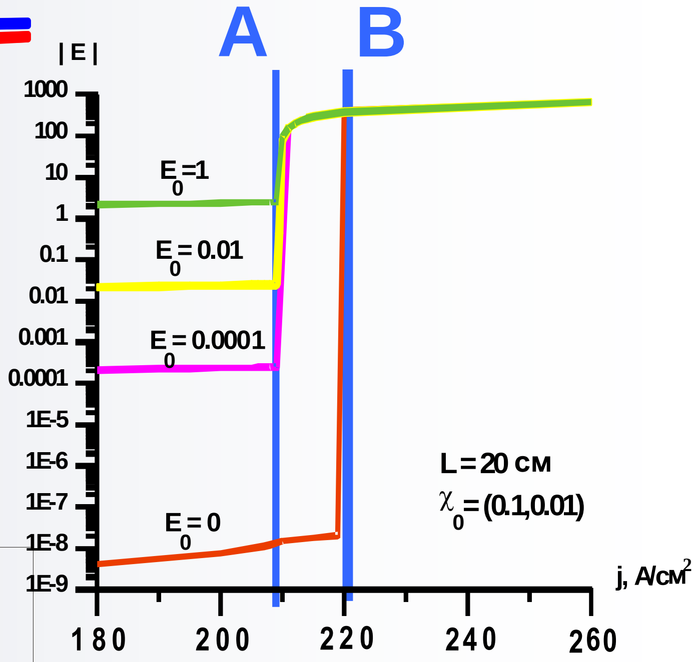
$j = 100\ \mathrm{A/cm}^2$

$L = 40\ \mathrm{cm}$

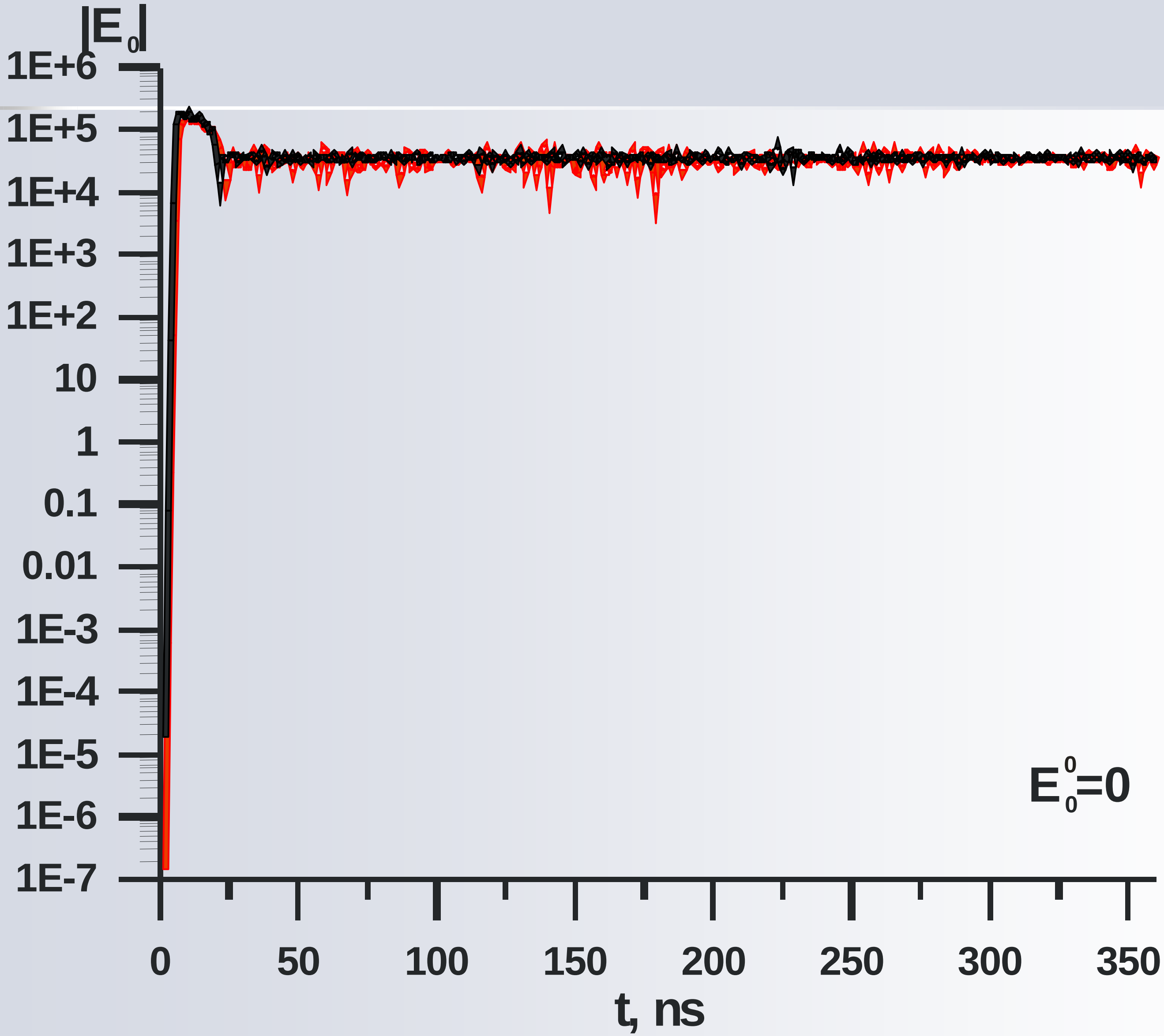
$l_0 = 3.$

# Amplification and oscillation regimes in three-wave geometry





# SASE regime simulation:



# Dispersion equation:

$$l_0 l_1 l_2 - l_0 r_{12} - l_1 r_2 - l_2 r_1 - \chi_1 \chi_{-2} \chi_{2-1} - \chi_2 \chi_{-1} \chi_{1-2} = 0$$

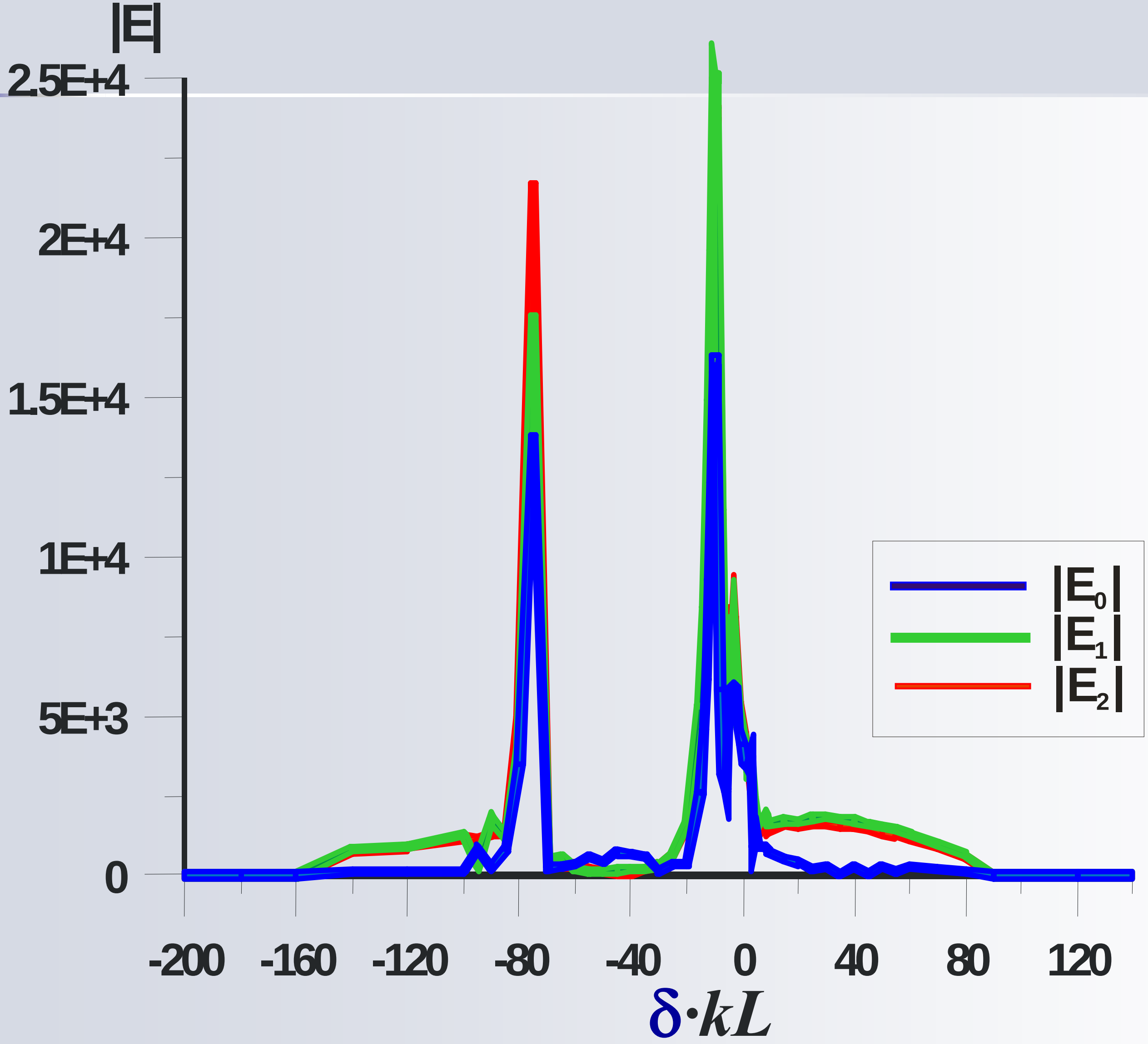
## Two-root degeneration case:

$$\beta_1 \beta_2 l_1 l_2 + (\beta_1 l_1 + \beta_2 l_2) l_0 - \beta_1 \beta_2 r_{12} - \beta_1 r_1 - \beta_2 r_2 = 0$$

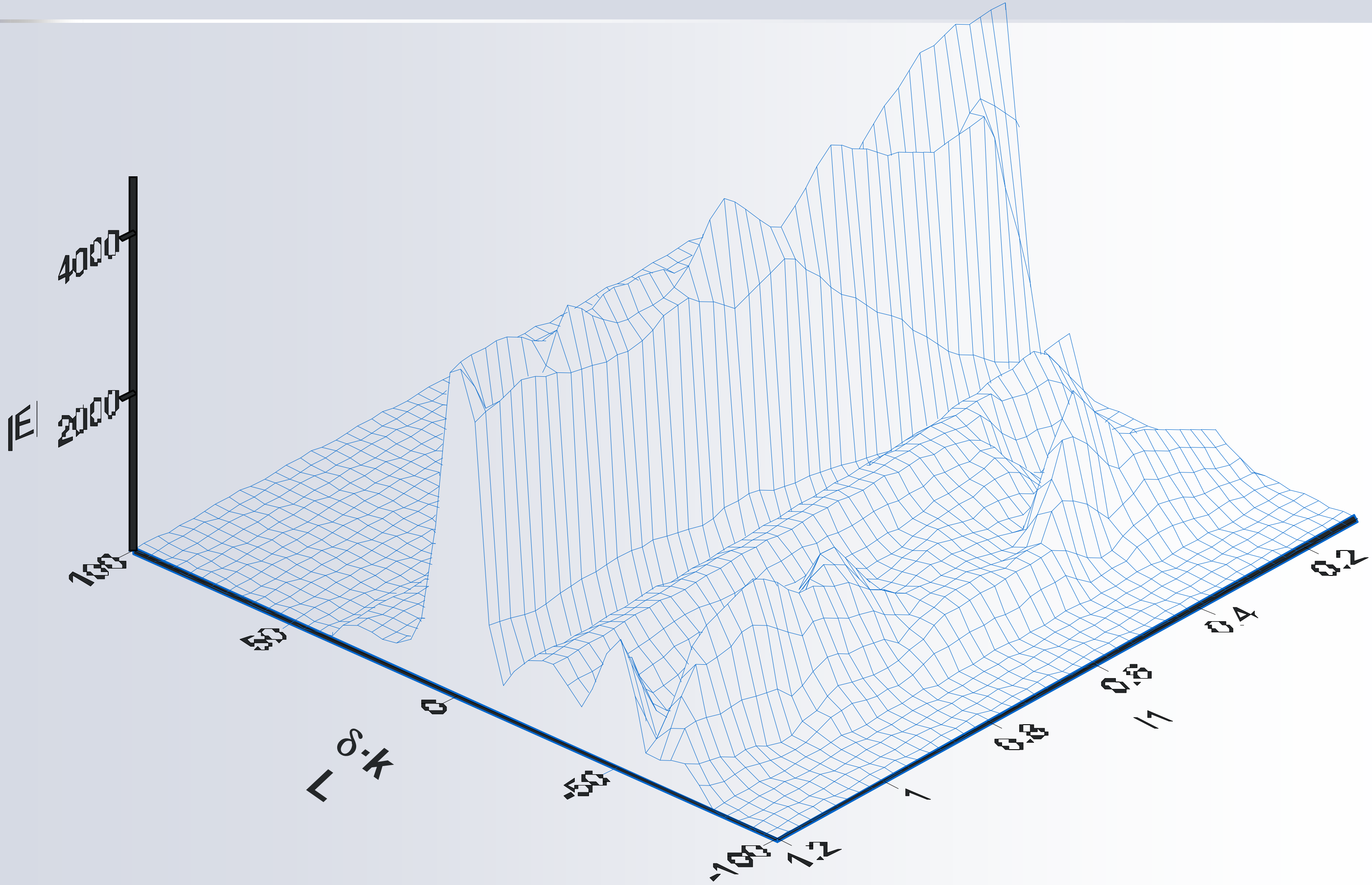
## Three-root degeneration case:

$$\beta_1 l_1 + \beta_2 l_2 + l_0 = 0$$

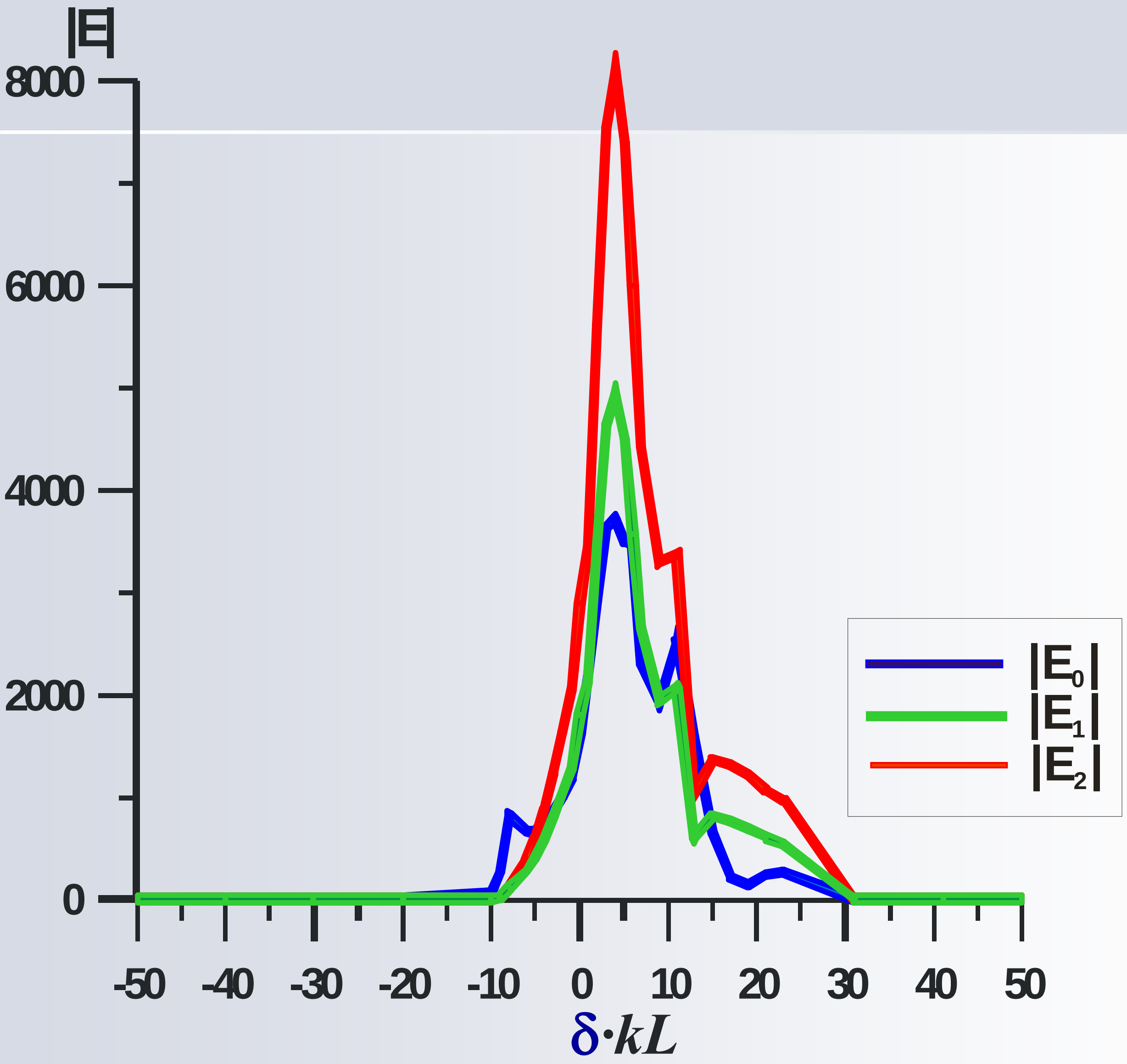
# One-mode synchronism, dependence on detuning from exact Cherenkov condition $\delta$



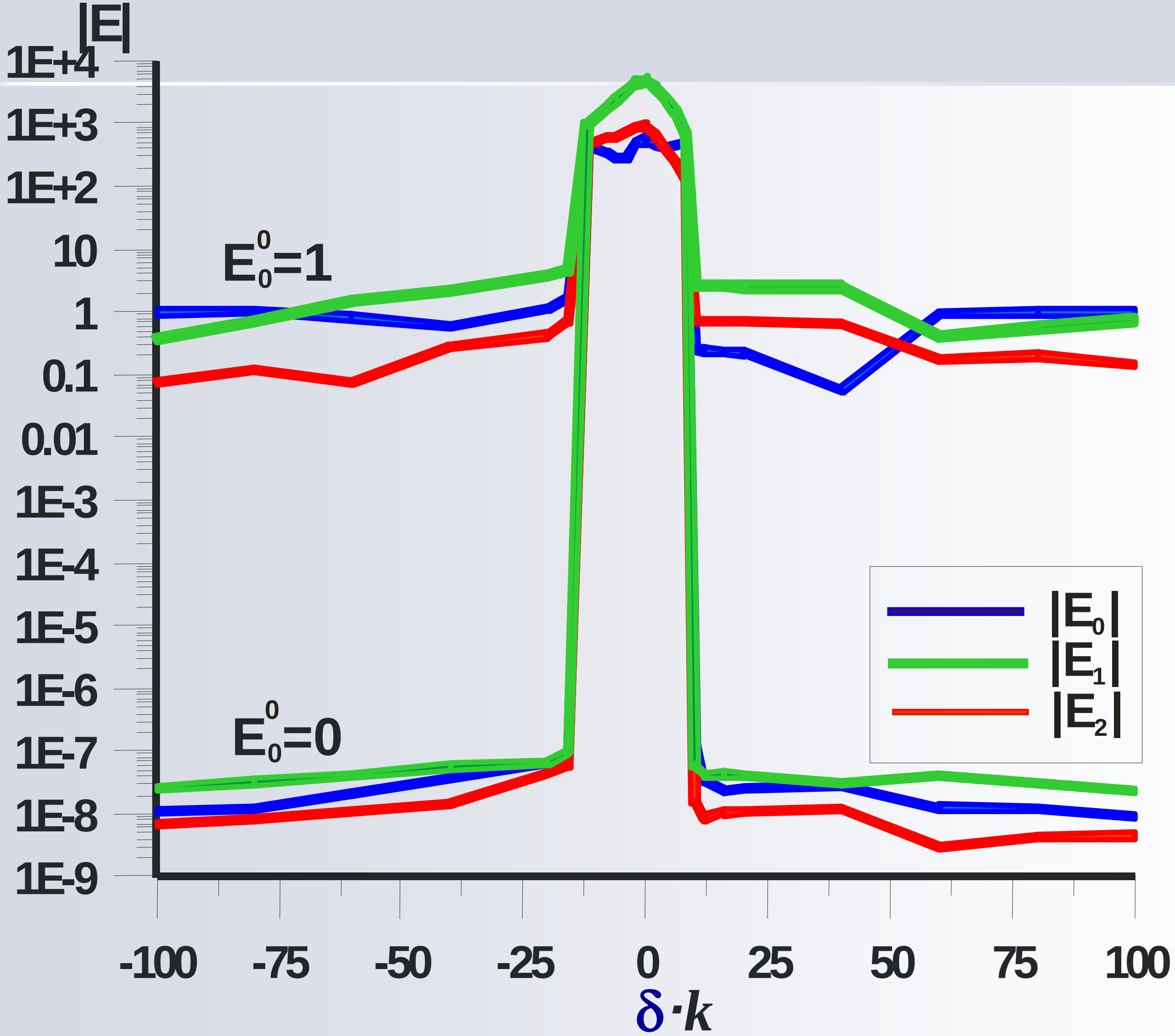
# One-mode synchronism, dependence on $\delta$ and system parameter $l_1$



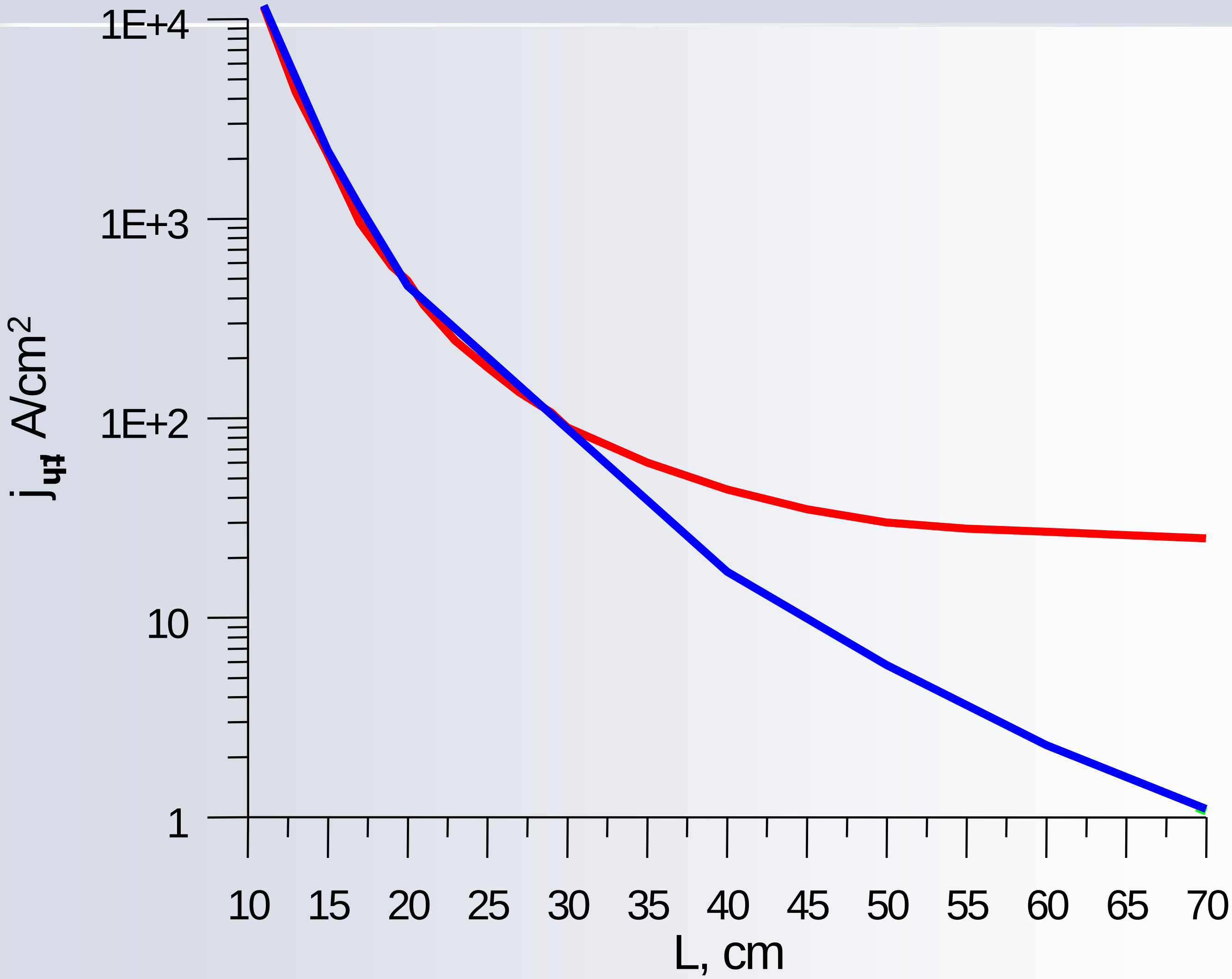
# Two-root degeneration case



# Three-root degeneration case



# Current threshold for two- and three-wave geometry in dependance on *L*



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# References (VFEL simulation)

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