Some aspects of numerical investigation of Volume Free Electron Laser nonlinear stage

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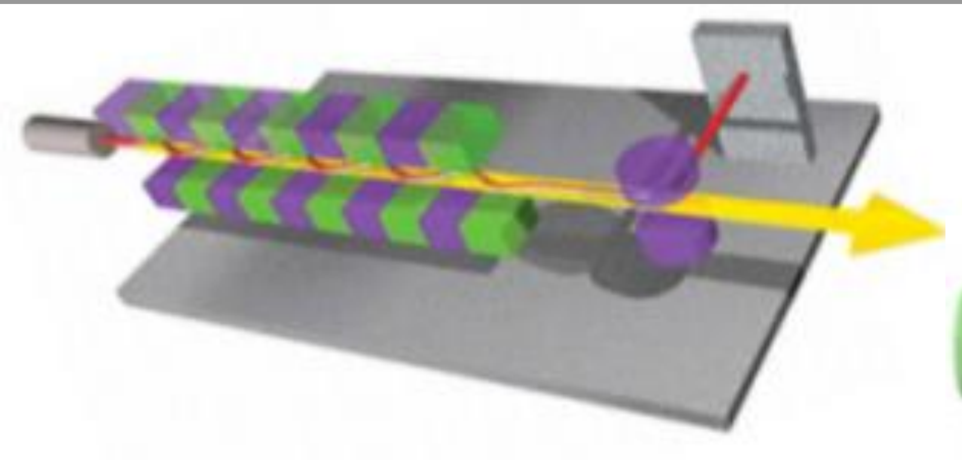
Outline

- What is Volume Free Electron Laser (VFEL)
- Physical and mathematical models considered
- What is new
- Some numerical results
- Conclusions

Vacuum electronic devices



FEL

TWT

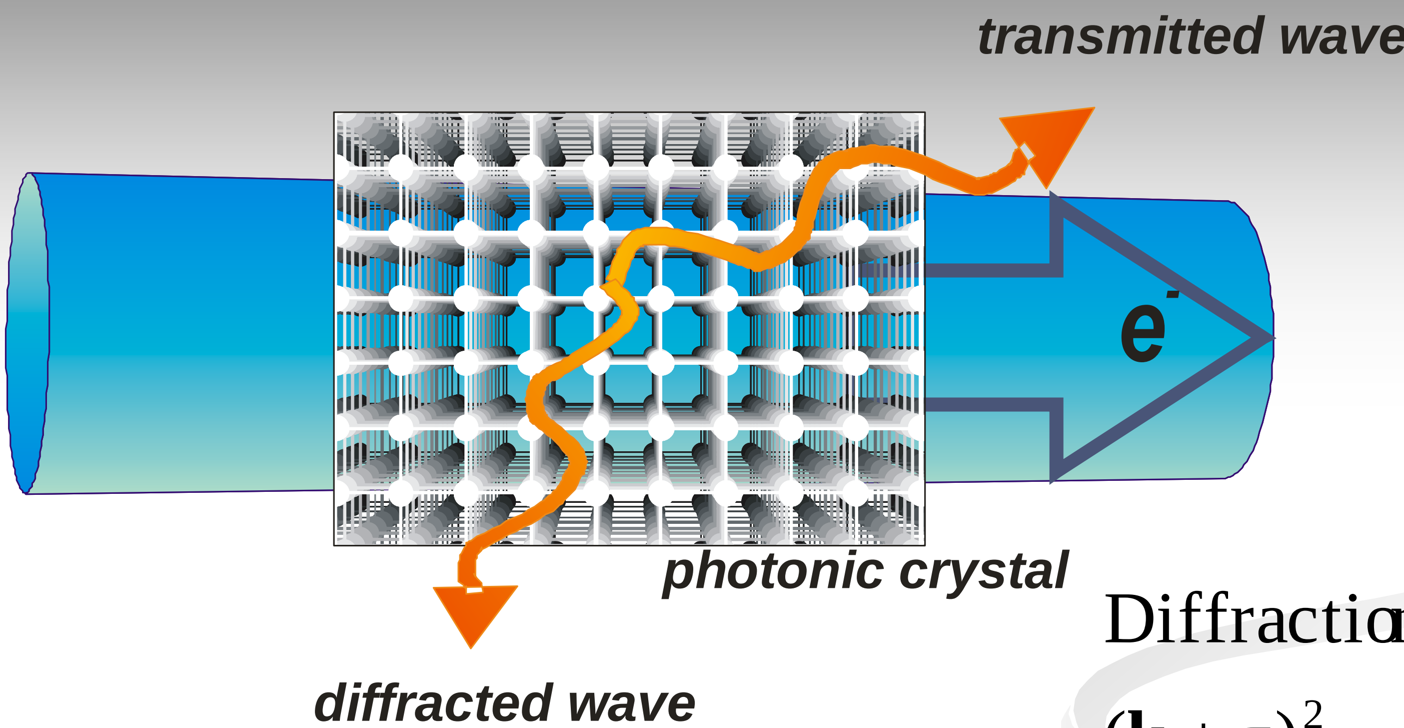
The new law of instability* for an electron beam passing through a spatially-periodic medium provides the following estimation on threshold current in degeneration points in dependence on s surplus waves appearing due to diffraction

$$j_{start} \sim 1 \big/ (kL)^{3+2s}$$

This law is universal and valid for all wavelength ranges regardless the spontaneous radiation mechanism

**V.G.Baryshevsky, I.D.Feranchuk, Phys.Lett. 102A (1984) 141,*

Volume FEL



$$j_{start} \sim \frac{1}{(kL)^{3+2s}}$$

Diffraction conditions

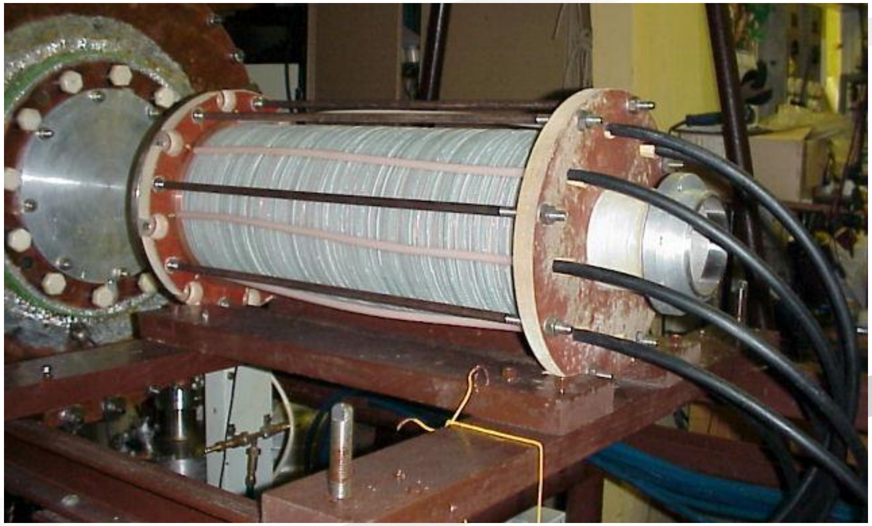
$$(\mathbf{k}+\boldsymbol{\tau})^2 \approx \mathbf{k}^2,$$

Synchronism conditions

$$|\omega - \mathbf{ku}| = \delta\omega \approx 0$$

So, threshold current can be significantly decreased when modes are degenerated in multiwave diffraction geometry

VFEL setup (50-500 keV)*



*** V.G. Baryshevsky et al., Nucl. Instr. Meth. B 252 (2006) 86**

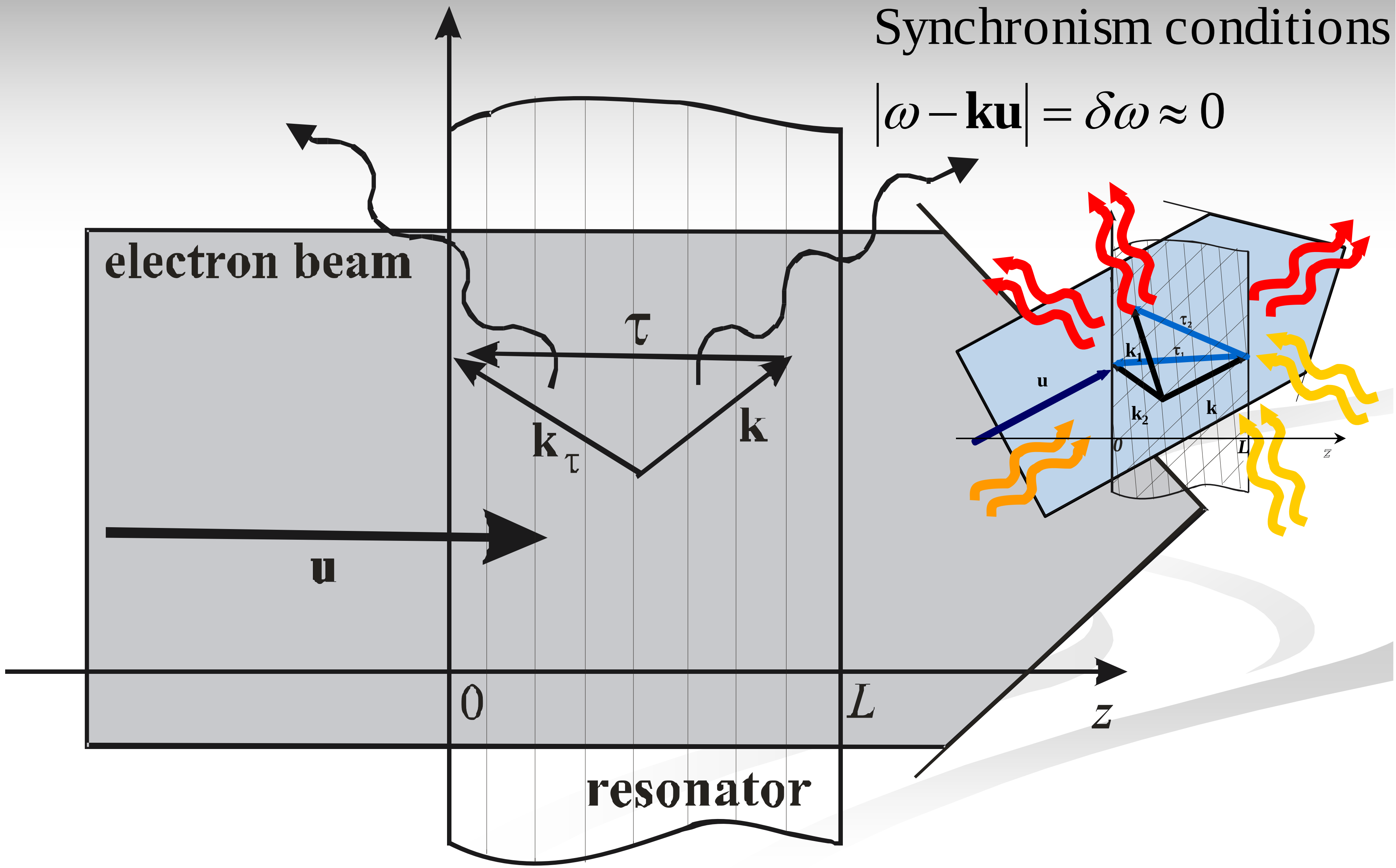
Two-wave VFEL

Diffraction conditions

$$2\mathbf{k}\boldsymbol{\tau}+\boldsymbol{\tau}^2\approx 0,$$

Synchronism conditions

$$\left|\omega-\mathbf{k}\mathbf{u}\right|=\delta\omega\approx 0$$



System for two-wave VFEL:

$$\frac{\partial E}{\partial t}+\gamma_0 c\frac{\partial E}{\partial z}+0.5i\omega lE-0.5i\omega\chi_\tau E_\tau=$$

$$=2\pi j\Phi\int_0^{2\pi}\frac{2\pi-p}{8\pi^2}\left(e^{-i\theta(t,z,p)}+e^{-i\theta(t,z,-p)}\right)dp,$$

$$\frac{\partial E_\tau}{\partial t}+\gamma_1 c\frac{\partial E_\tau}{\partial z}-0.5i\omega\chi_{-\tau}E+0.5i\omega l_1 E_\tau=0$$

$l_i=\dfrac{k_i^2c^2-\omega^2\varepsilon_0}{\omega^2}$ **are system parameters,** $\Phi=\sqrt{l_0+\chi_0-1/(u/c\gamma)^2}$

$l=l_0+\delta,\quad \delta$ **- detuning from synchronism condition**

$\gamma_{0,1}$ **are direction cosines,** $\beta=\gamma_0/\gamma_1$ **is an asymmetry factor**

χ_0, $\chi_{\pm1}$ **are Fourier components of the dielectric susceptibility of the target**

Equations for electron beam

$$\frac{d^2\theta(t,z,p)}{dz^2} = \frac{e\Phi}{m\gamma^3\omega^2}\left(k - \frac{d\theta(t,z,p)}{dz}\right)^3 \mathrm{Re}\big(E(t - z/u, z)\big)\times$$

$$\times\exp(i\theta(t,z,p)),$$

$$\frac{d\theta(t,0,p)}{dz} = k - \omega/u, \quad \theta(t,0,p) = p,$$

$$t > 0, \quad z \in [0,L], \quad p \in [-2\pi, 2\pi]$$

$\theta(t,z,p)$ is an electron phase in a wave

We use the method of averaging over initial phases of electron entrance in the resonator.

Energy conservation law

Using the Chu's kinetic power theorem for coupled electromagnetic waves we obtain

$$\frac{\partial W}{\partial t} + P + P_\tau = C\eta,$$

$$\eta = \int_0^{2\pi} \frac{2\pi - p}{8\pi^2} \frac{2u^2 - v^2(z = L, p) - v^2(z = L, -p)}{u^2} dp$$ is the electron efficiency

$$W = EE^* + E_\tau E_\tau^* = |E|^2 + |E_\tau|^2$$ is the electromagnetic energy stored in the resonator

$$P = C_1 |E(L)|^2 \text{ and } P_\tau = C_2 |E_\tau(0)|^2$$

represent radiation losses associated with the transmitted wave and diffracted wave respectively

System of equations for BWT, TWT etc. *

$$\partial^2\theta/\partial\zeta^2 = -\mathrm{Re}\left[F\exp(i\theta)\right],\ \ \partial F/\partial\tau - \partial F/\partial\zeta = \tilde{I},\ \ \tilde{I} = -\frac{1}{\pi}\int_0^{2\pi} e^{-i\theta}\, d\theta_0,$$

$$\theta|_{\zeta=0} = \theta_0,\quad \partial\theta/\partial\zeta|_{\zeta=0} = 0,\quad F|_{\zeta=L} = 0,$$

System is versatile in the sense that they remain the same within some normalization for a wide range of electronic devices (FEL, BWT, TWB etc).

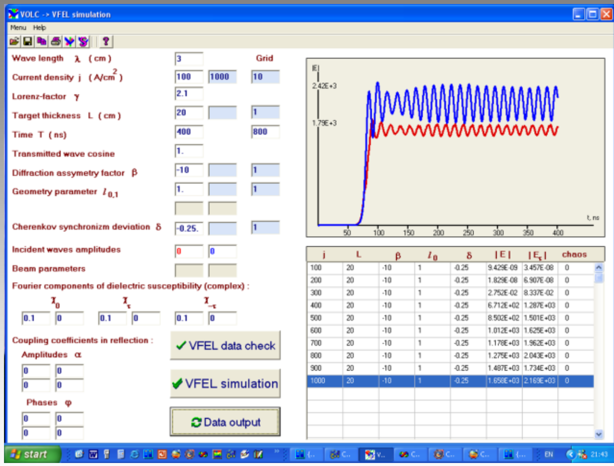
N.S.Ginzburg, S.P.Kuznetsov, T.N.Fedoseeva.** ***Izvestija VUZov - Radiophysics, 21 (1978), 1037 (in Russian).

Right-hand side of our system is more complicated than cited here, because it takes into account as initial phase of an electron not only the moment of time t_0 but also transverse spatial coordinate of an electron entrance in the resonator at $z = 0$.

What is new?

Investigation of chaotic behaviour of relativistic beam in three-dimensional periodical structure under ***volume (non-one-dimensional) multi-wave distributed feedback (VDFB)***

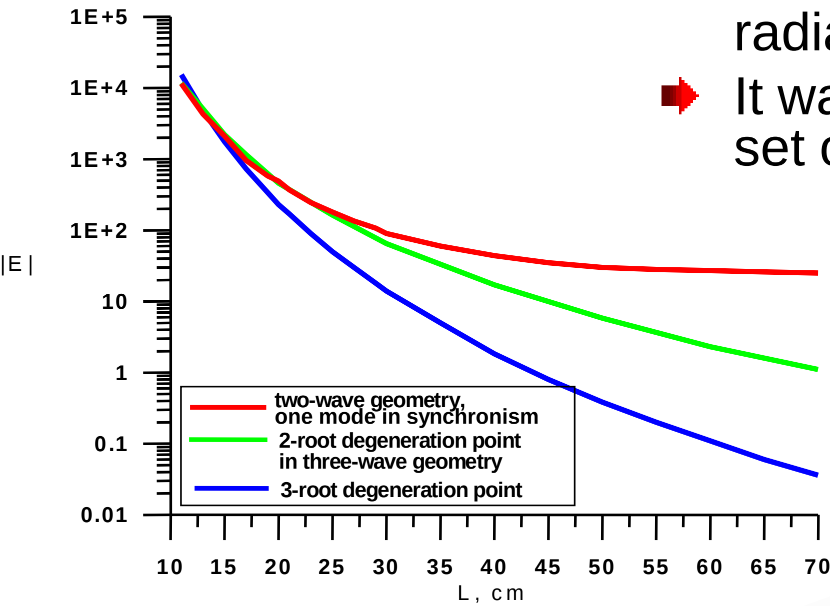
Investigation of chaos in VFEL is important in the light of its experimental development.

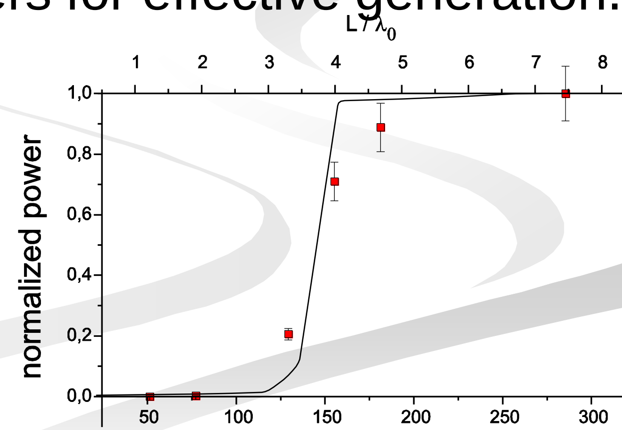


Main numerical results

- It was obtained numerically all main VFEL physical laws.
- It was demonstrated generation thresholds subject to beam current and target length.
- It was investigated:

  width of the zone of amplification subject to beam current for two-and three-wave geometries;
  SASE (Self Amplified Spontaneous Emission).
- It was obtained dependence of electromagnetic radiation on *L* for experimental setup*.
- It was demonstrated that there exists an optimal set of VFEL parameters for effective generation.





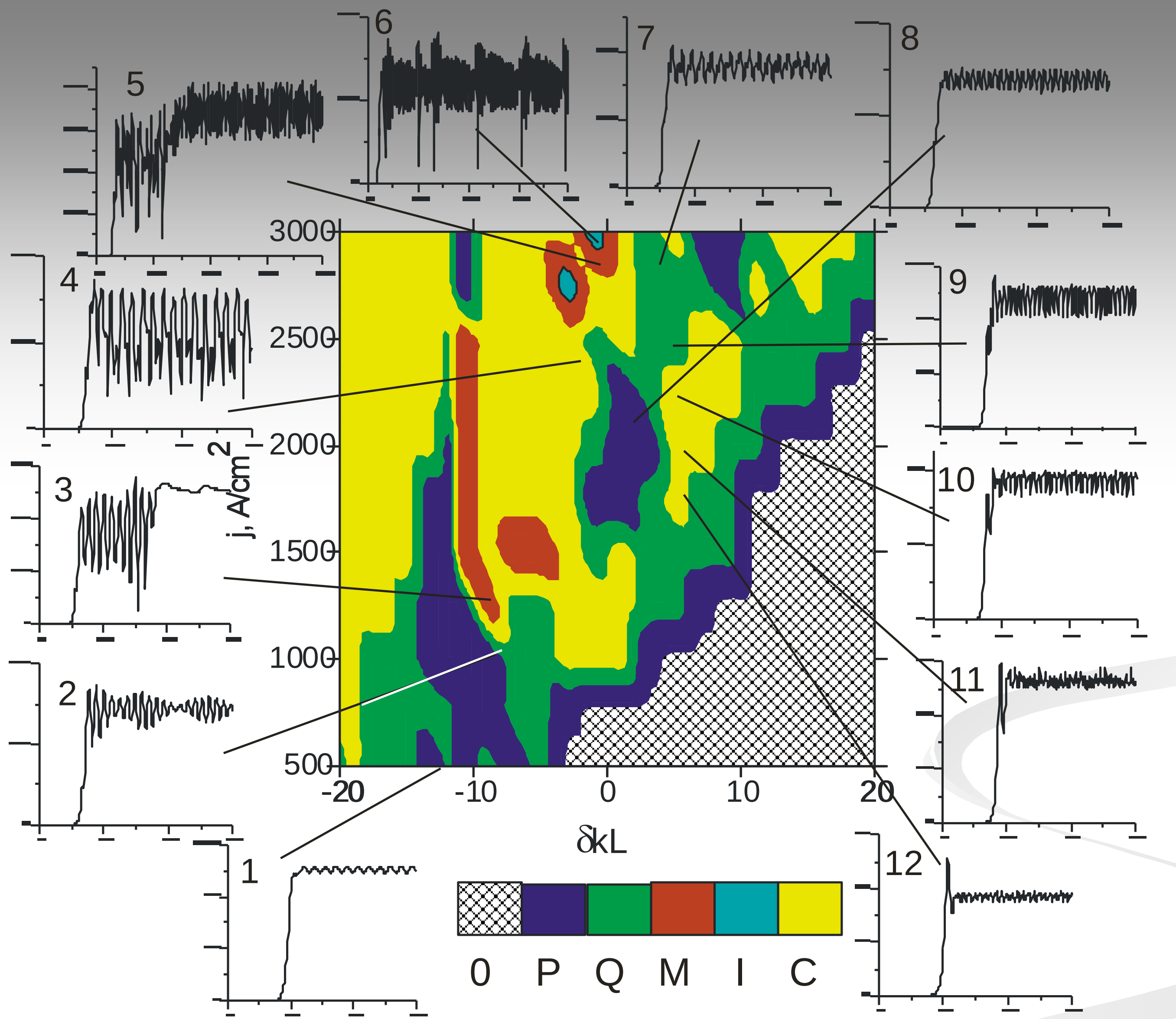
V.G.Baryshevsky, N.A.Belous, A.Gurinovich et al., Proc. FEL06, p.331

Investigation of chaos in VFEL

Analytical investigation of chaos in the system considered seems to be impossible because of its nonlinearity. Electron beam moving through periodical structure leads to a diversity of features of generation dynamics that is due to non-local nature of interaction between electron beam and electromagnetic field under VDFB.

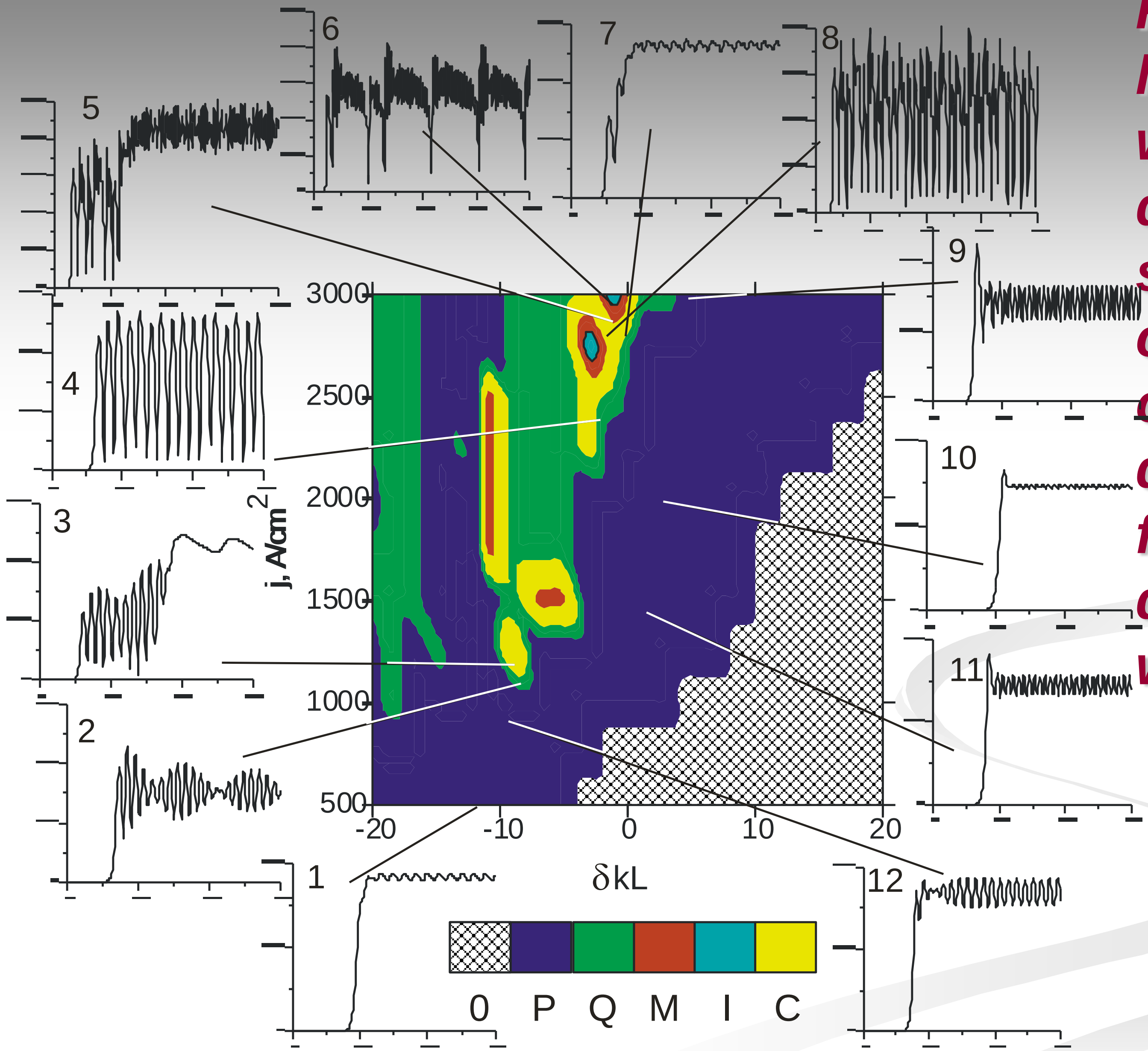
- A gallery of different chaotic regimes for VFEL laser intensity with corresponding phase space portraits, attractors and Poincare maps was proposed.
- It was demonstrated the following transition between different regimes "period doubling - chaos", "intermittency - chaos", "quasiperiodicity - chaos“, “periodicity - transition chaos - chaos”.
- It was demonstrated sensibility to initial conditions for different regimes.
- It was investigated the largest Lyapunov exponents for periodic and chaotic regimes.

Root to chaotic lasing with respect to detuning from synchronism condition δ and current density j for transmitted wave

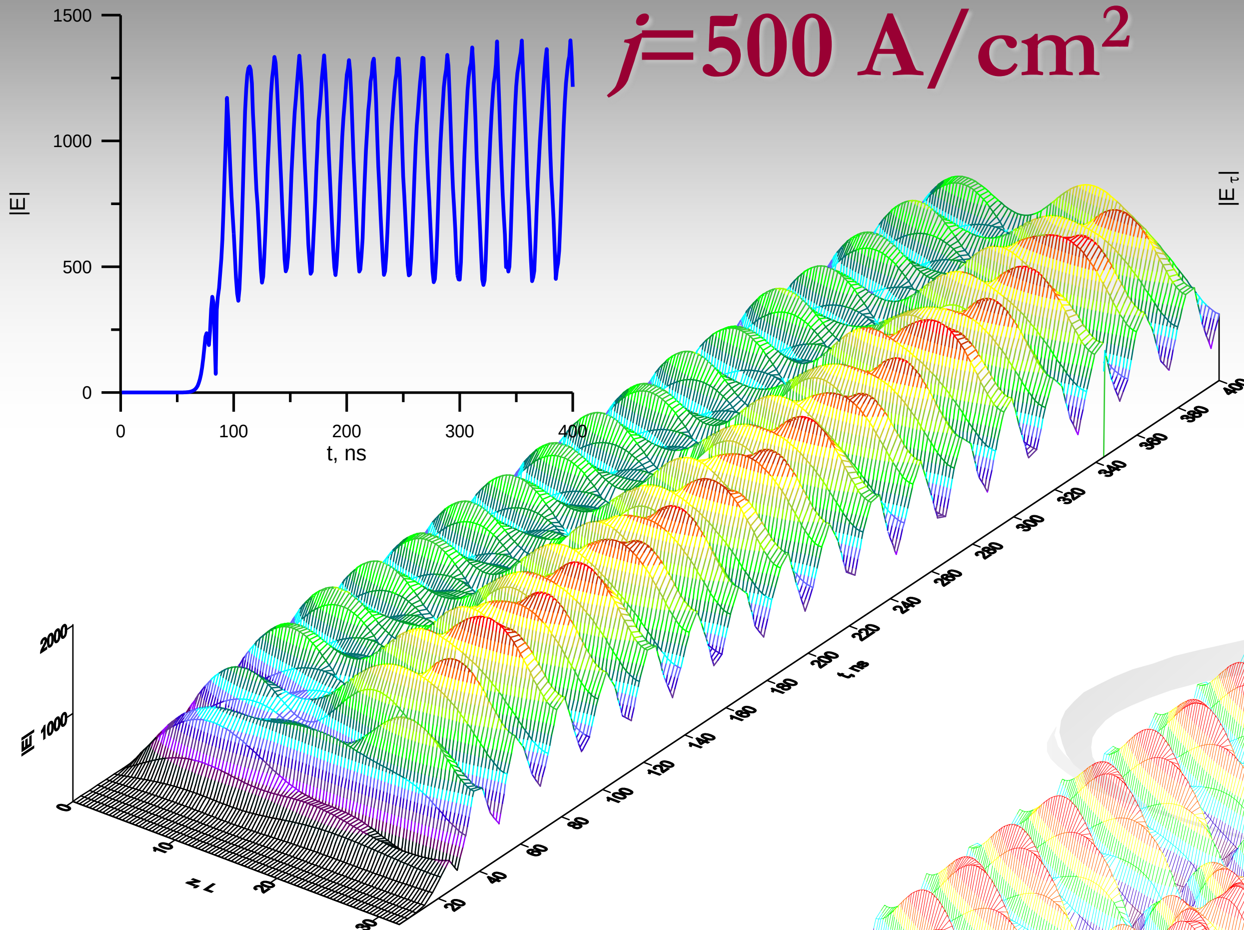


0 depicts a domain under beam current threshold. P – periodic regimes, Q –quasiperiodicity, M – domains with transition between large-scale and small-scale amplitudes, I – intermittency, C – chaos.

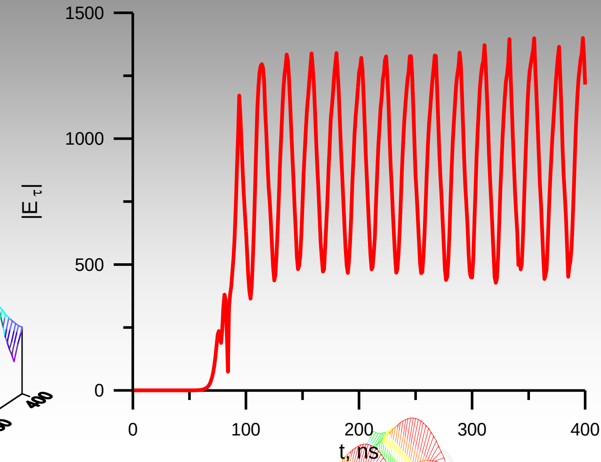
Root to chaotic lasing with respect to detuning from synchronism condition δ and current density j for diffracted wave

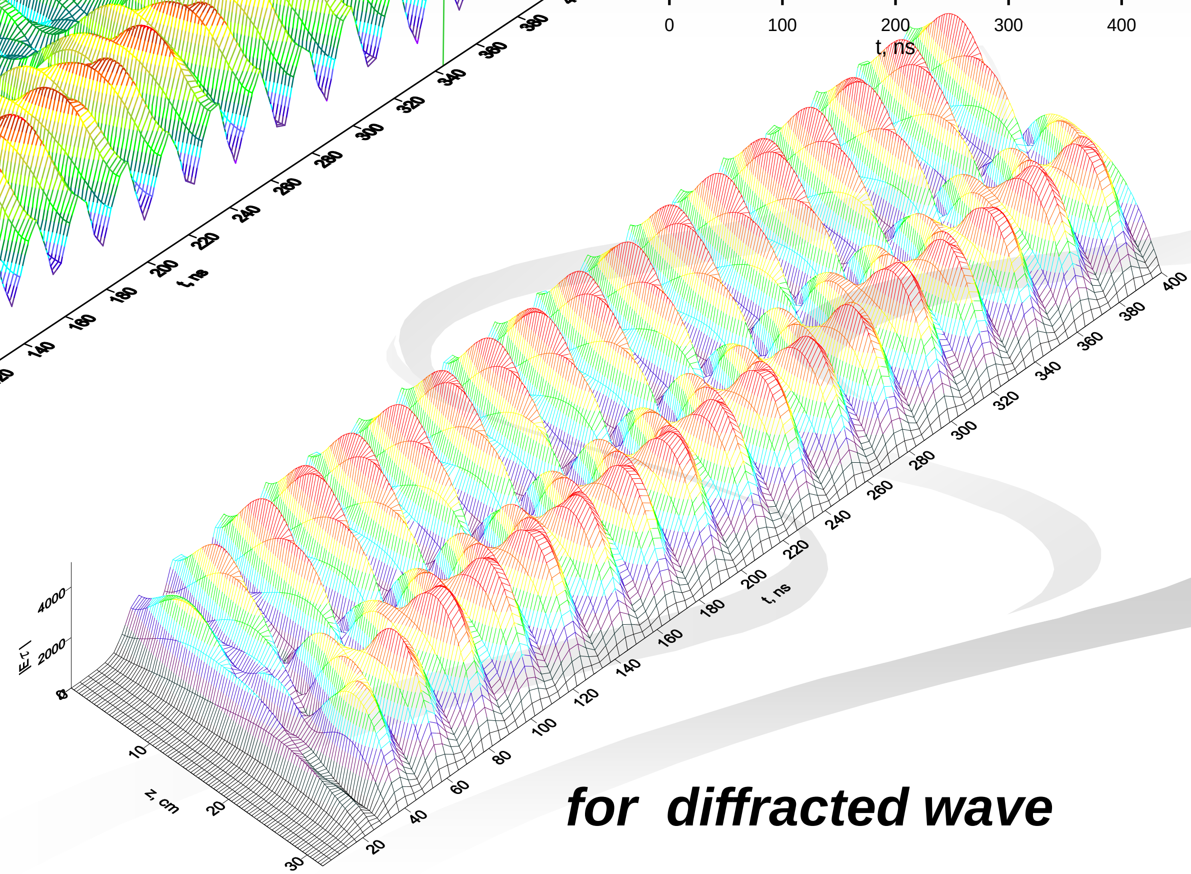


Space-temporal dynamics for L=32 cm and j=500 A/cm^2



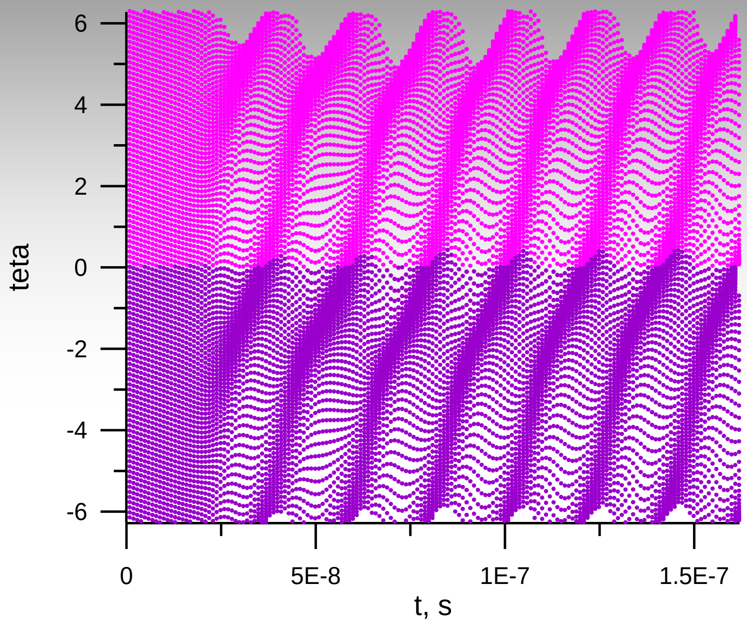
for transmitted wave



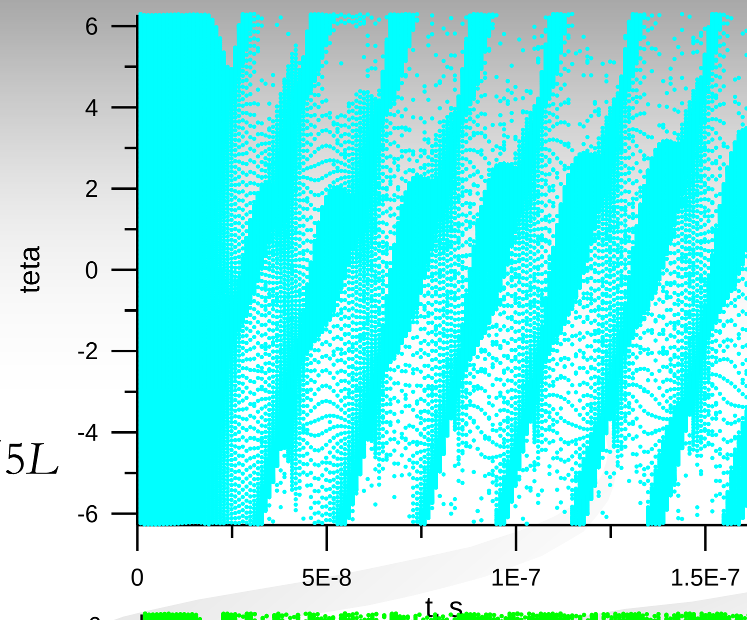


for diffracted wave

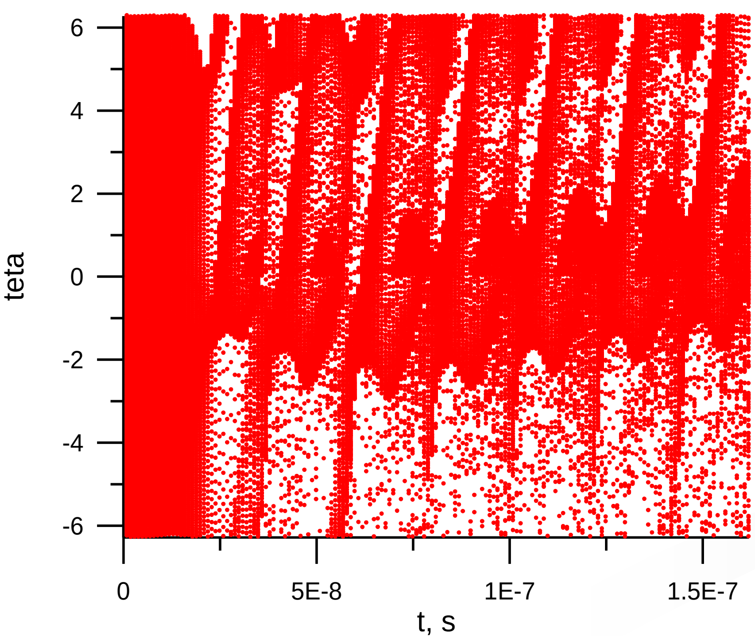
Phase temporal dependencies



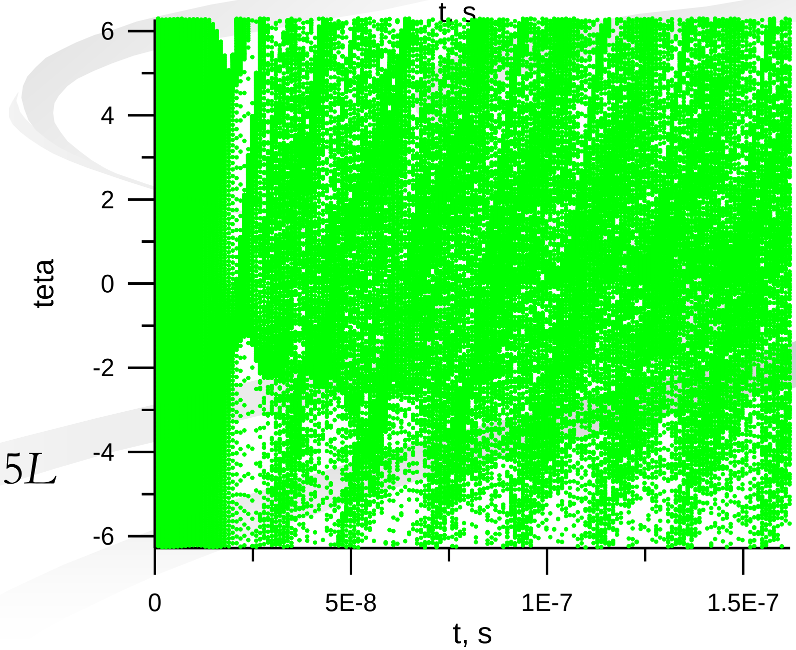
$z = 1/5L$



$z = 2/5L$

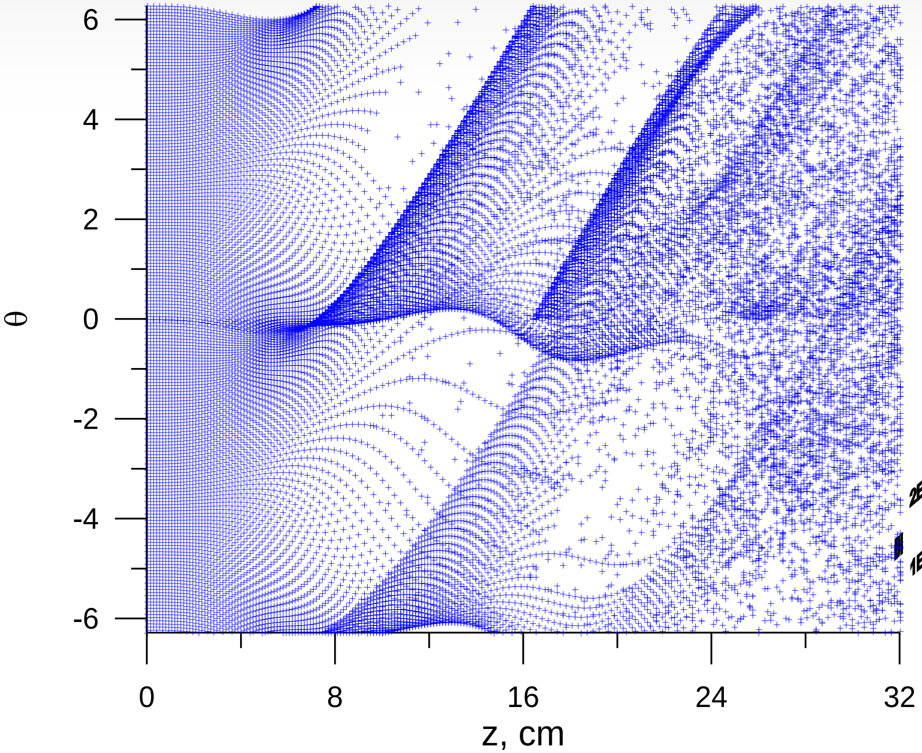


$z = 3/5L$

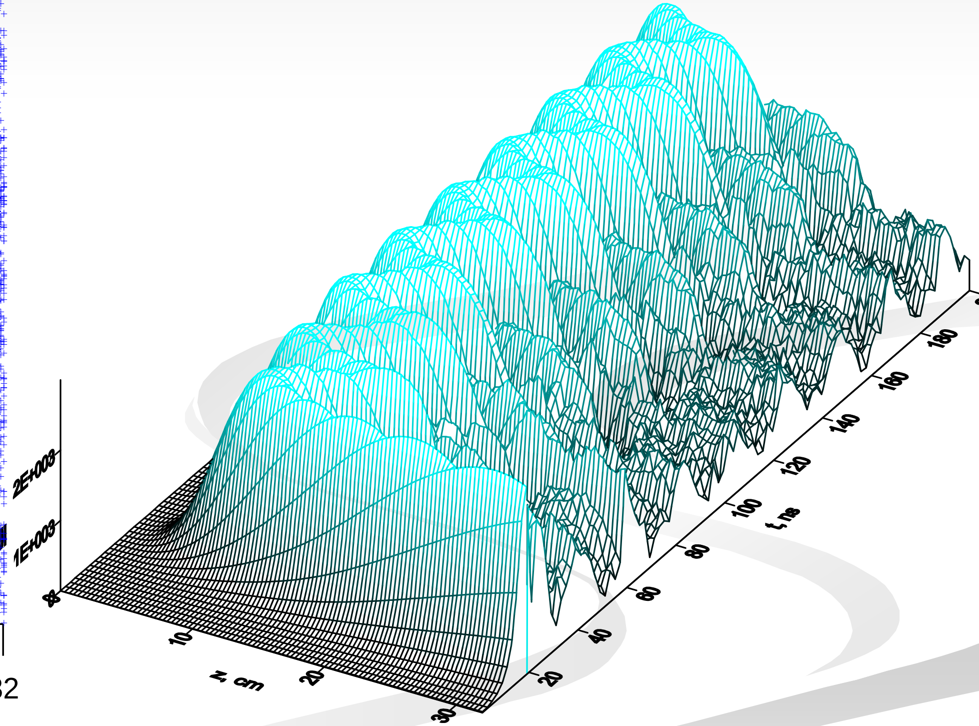


$z = 4/5L$

Spatial dependence of phase θ



Spatio-temporal dynamics $\partial j/\partial t$



Conclusions

- The original software for VFEL simulation allows to obtain all main VFEL physical laws and dependencies.
- In simulation VFEL was considered as a dynamical system.
- Numerical analysis shows the complicated root to chaos in VFEL lasing including effects of periodicity and quasiperiodicity near generation thresholds considered.

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