Numerical Analysis of Lasing Dynamics in Volume Free Electron Laser

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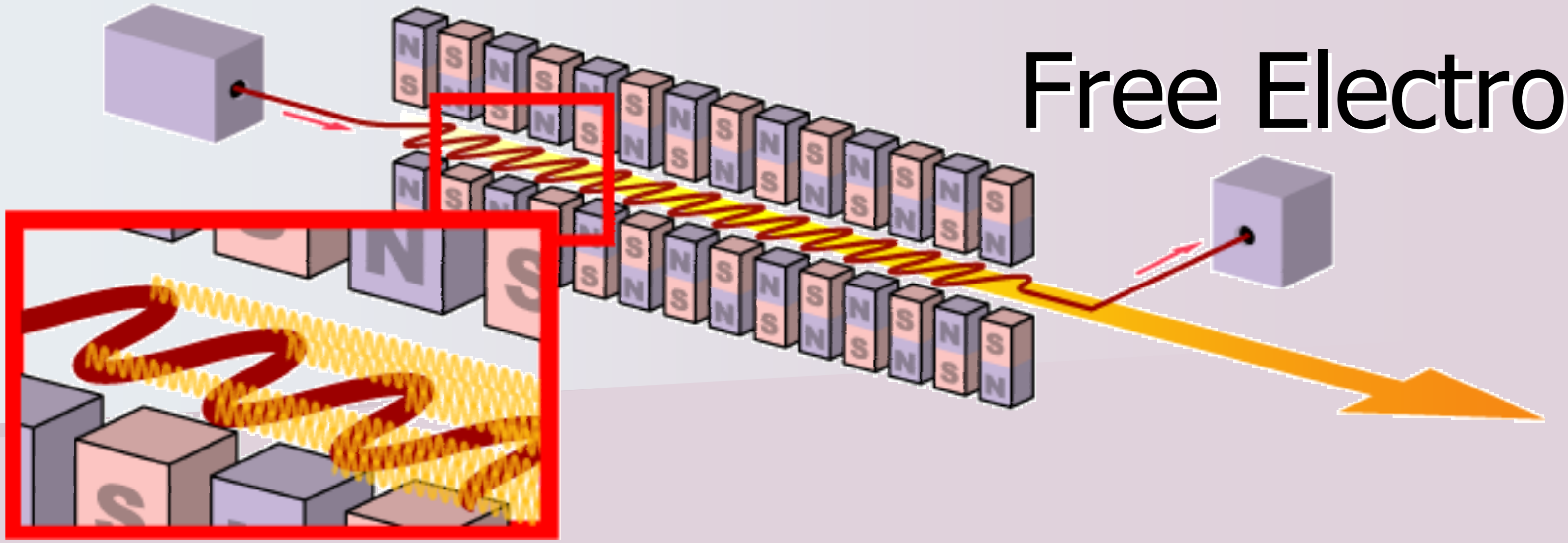
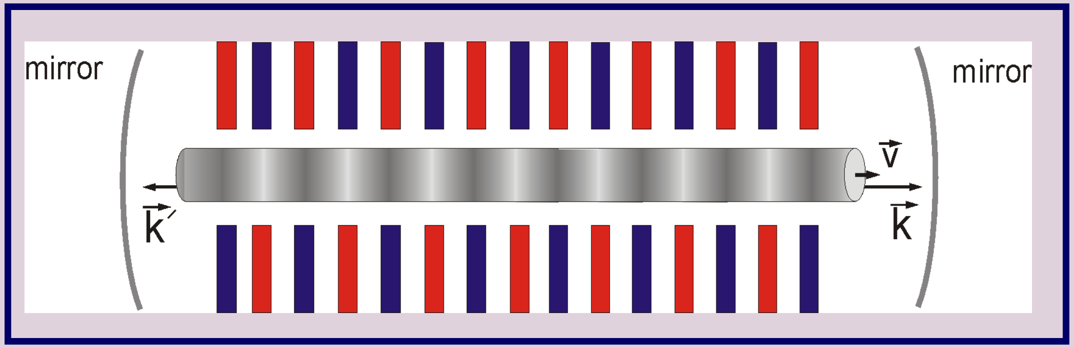
Free Electron Laser



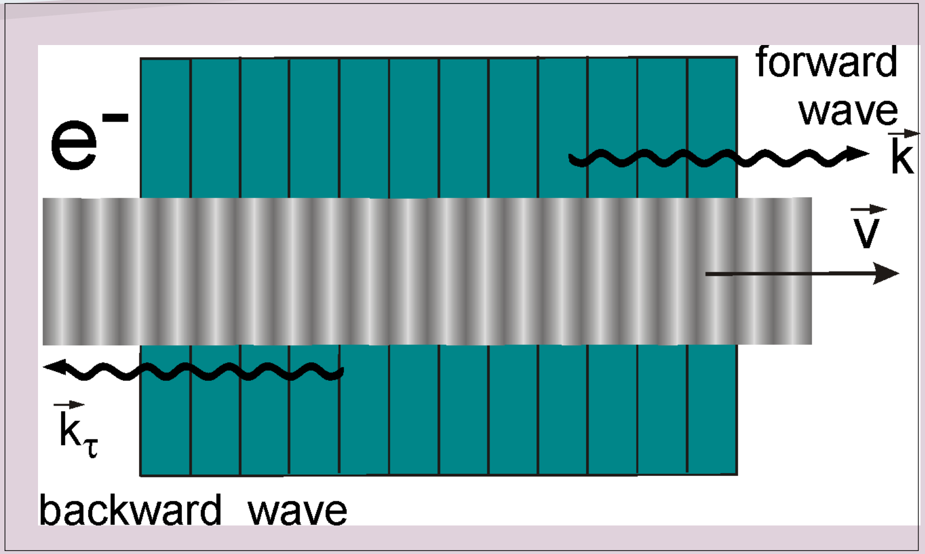
FEL lasing is aroused by different types of spontaneous radiation: undulator radiation, Smith-Purcell or Cherenkov radiation and so on. Using positive feedback in FELs reduces the working length and provides oscillation regime of generation. This feedback is usually one-dimensional and can be formed either by two parallel mirrors or by one-dimensional diffraction grating, in which incident and diffracted waves move along the electron beam.

Theoretical investigations show that it is one of the effective schemes with n-wave volume distributed feedback (VDFB)

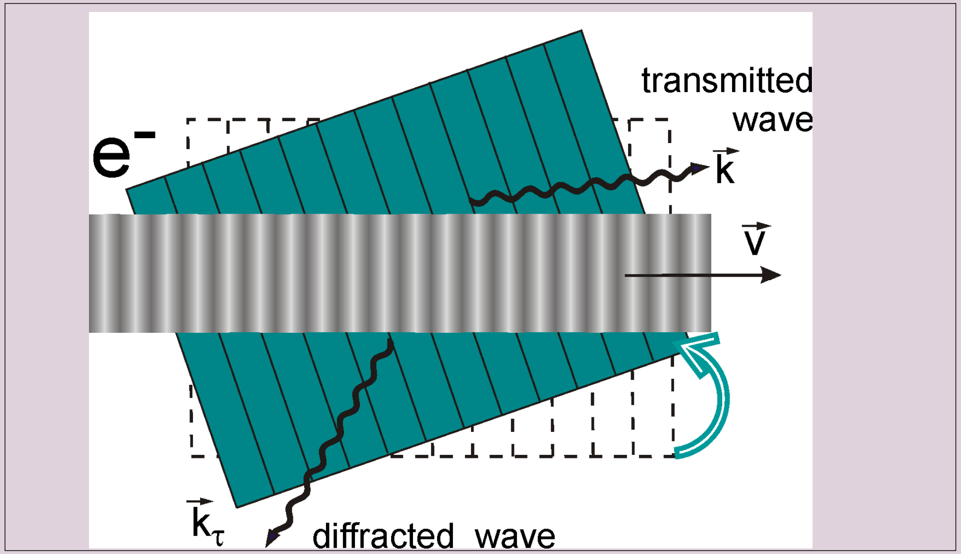


What is volume distributed feedback ?

one-dimensional distributed feedback



two-dimensional distributed feedback



Volume (non-one-dimensional) multi-wave distributed feedback is the distinctive feature of Volume Free Electron Laser (VFEL)

Benefits provided by the volume distributed feedback

The new law of instability for an electron beam passing through a spatially-periodic medium provides the increment of instability in degeneration points proportional to $\rho^{1/(3+s)}$, here **s** is the number of surplus waves appearing due to diffraction. ***This increment differs from the conventional increment for single-wave system (TWTA and FEL), which is proportional to*** $\rho^{1/3}$.

(**V.G.Baryshevsky, I.D.Feranchuk**, *Phys.Lett.* 102A (1984) 141)

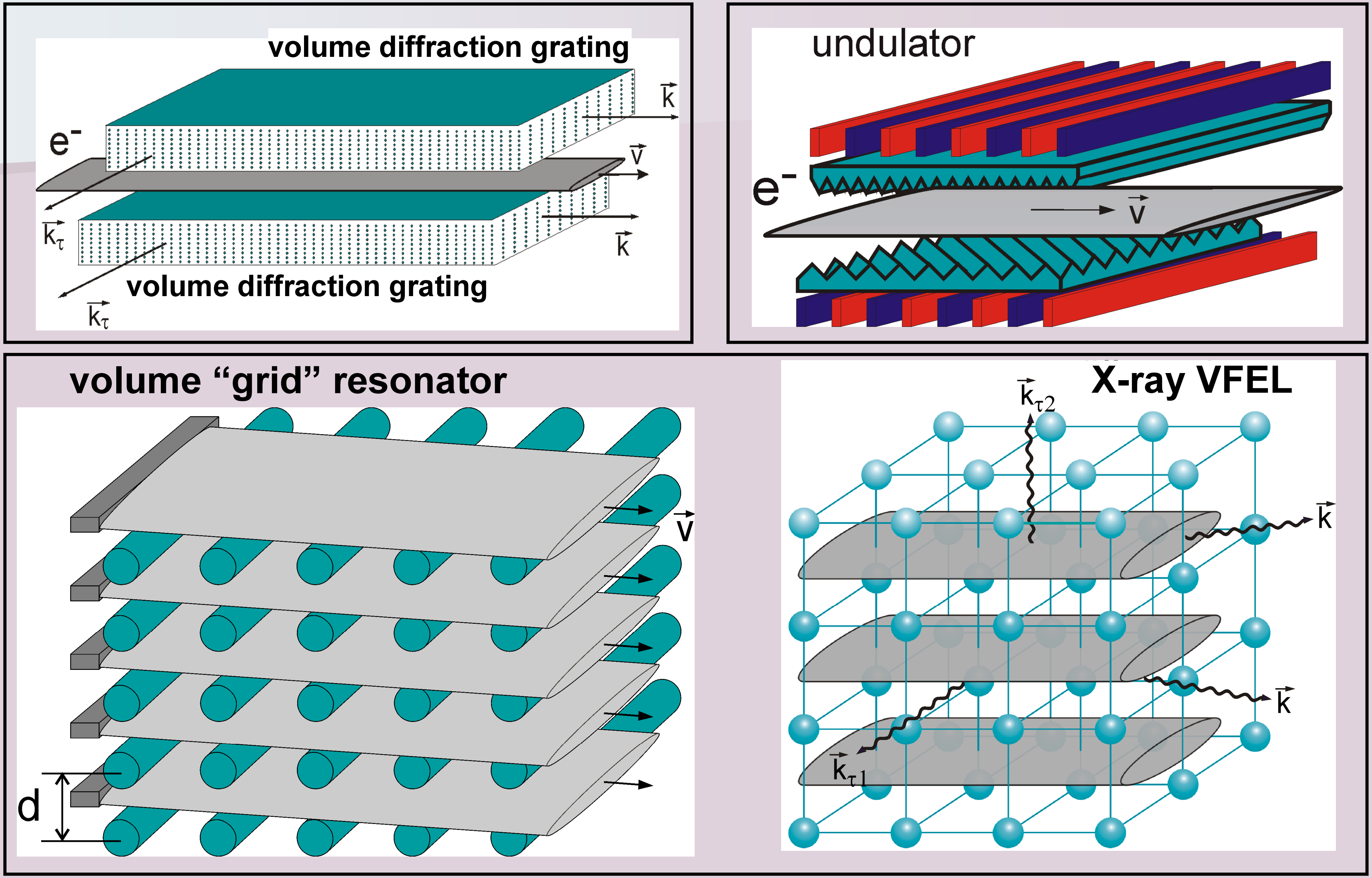
This new law provides for noticeable reduction of electron beam current density necessary for achievement the generation threshold. In X-ray range this generation threshold can be reached for the induced parametric X-ray radiation in crystals, i.e. to create X-ray laser

(V.G.Baryshevsky, K.G.Batrakov, I.Ya. Dubovskaya, *J.Phys* D24 (1991) 1250)

This law is universal and valid for all wavelength ranges regardless the spontaneous radiation mechanism

$$j_{start} \sim 1 \Big/ \left[(kL)^3 (k\chi_\tau L)^{2s} \right]$$

What is Volume Free Electron Laser ? *



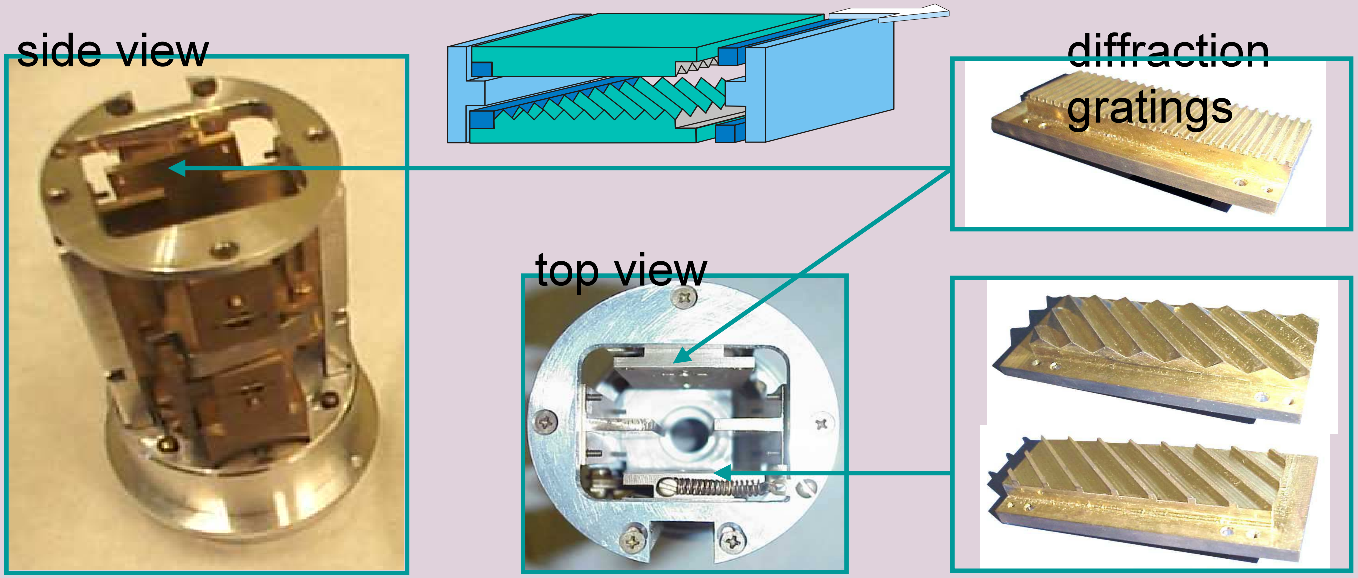
* Eurasian Patent no. 004665

Use of volume distributed feedback makes available:

- ✓ frequency tuning at fixed energy of electron beam in significantly wider range than conventional systems can provide
- ✓ more effective interaction of electron beam and electromagnetic wave allows significant reduction of threshold current of electron beam and, as a result, miniaturization of generator
- ✓ reduction of limits for available output power by the use of wide electron beams and diffraction gratings of large volumes
- ✓ simultaneous generation at several frequencies

VFEL experimental setup (2001)

2001 - First lasing of volume free electron laser in mm-wavelength range.
Demonstration of validity of VFEL principles.
Demonstration of possibility for frequency tuning at constant electron energy
(**V.G.Baryshevsky et al.** ***NIM 483 A (2002) 21)***



The interaction of the exciting diffraction grating with the electron beam arouses Smith-Purcell radiation. The second resonant grating provides distributed feedback of generated radiation with electron beam by Bragg dynamical diffraction.

New VFEL generator (2004) :

First observation of generation in the backward wave oscillator with a "grid" diffraction grating and lasing of the volume FEL with a "grid" volume resonator*

Main features:

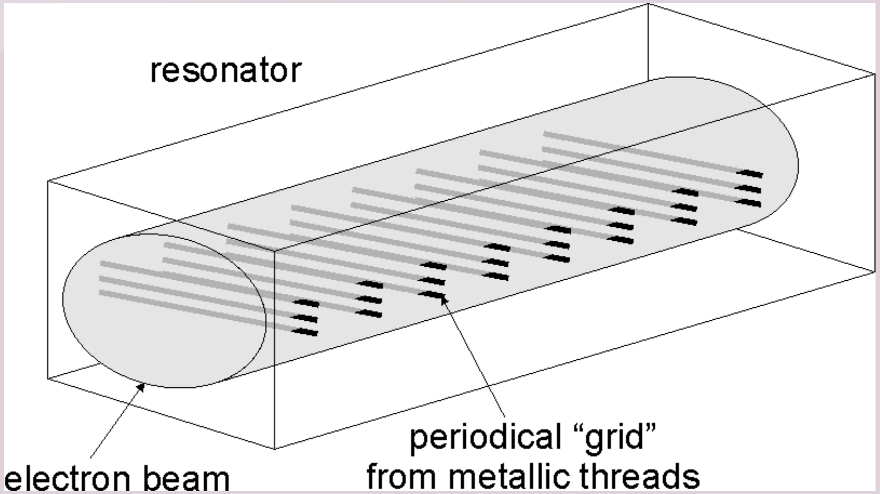
- volume gratings (volume “grid”)
- electron beam of large cross-section
- electron beam energy 180-250 keV
- possibility of gratings rotation
- operation frequency 10 GHz
- tungsten threads with diameter 100 μm

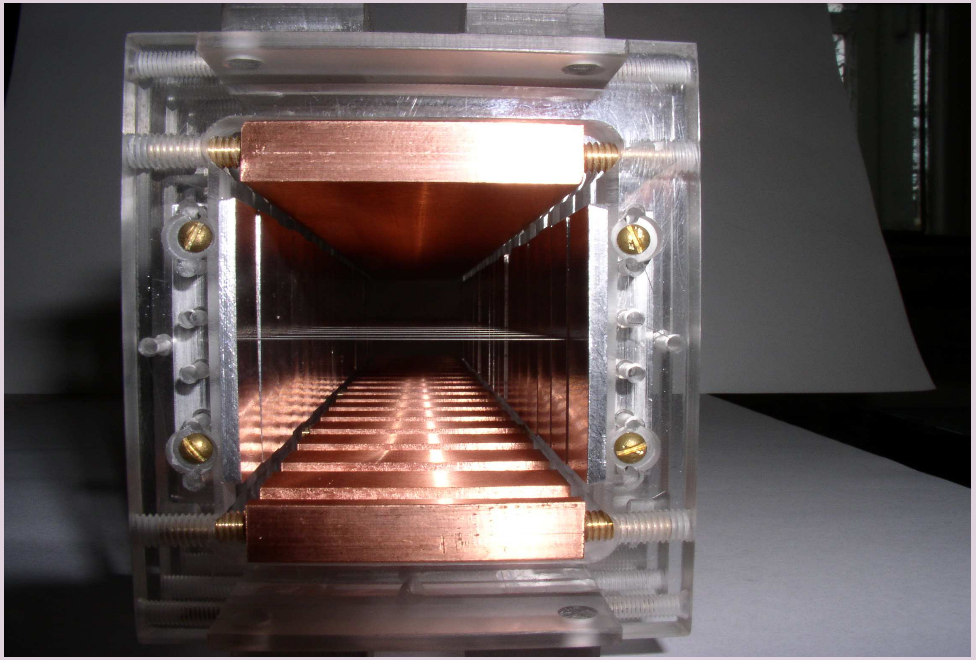
* V. Baryshevsky et al., Nucl. Instr. Meth. B 252 (2006) 86

V.G.Baryshevsky et al., Proceedings of FEL06, Germany (2006), p.331

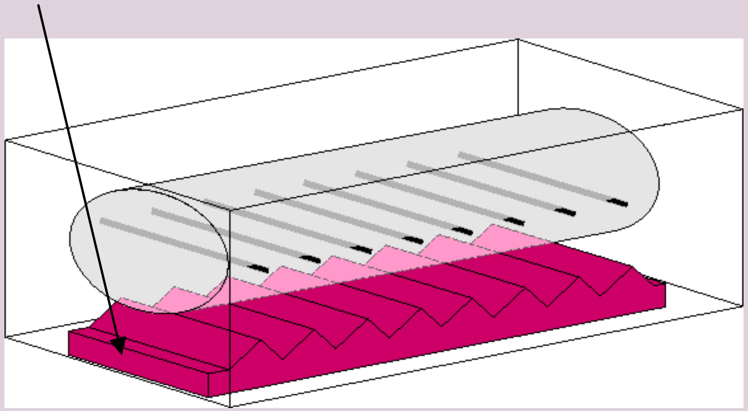
Electrodynamical properties of a "grid" volume resonator *

Electrodynamical properties of a volume resonator that is formed by a periodic structure built from the metallic threads inside a rectangular waveguide depend on diffraction conditions.





Resonant grating provides VDFB of generated radiation with electron beam



*** V. Baryshevsky, A. Gurinovich, Proceedings of FEL06, Germany (2006), p.335**

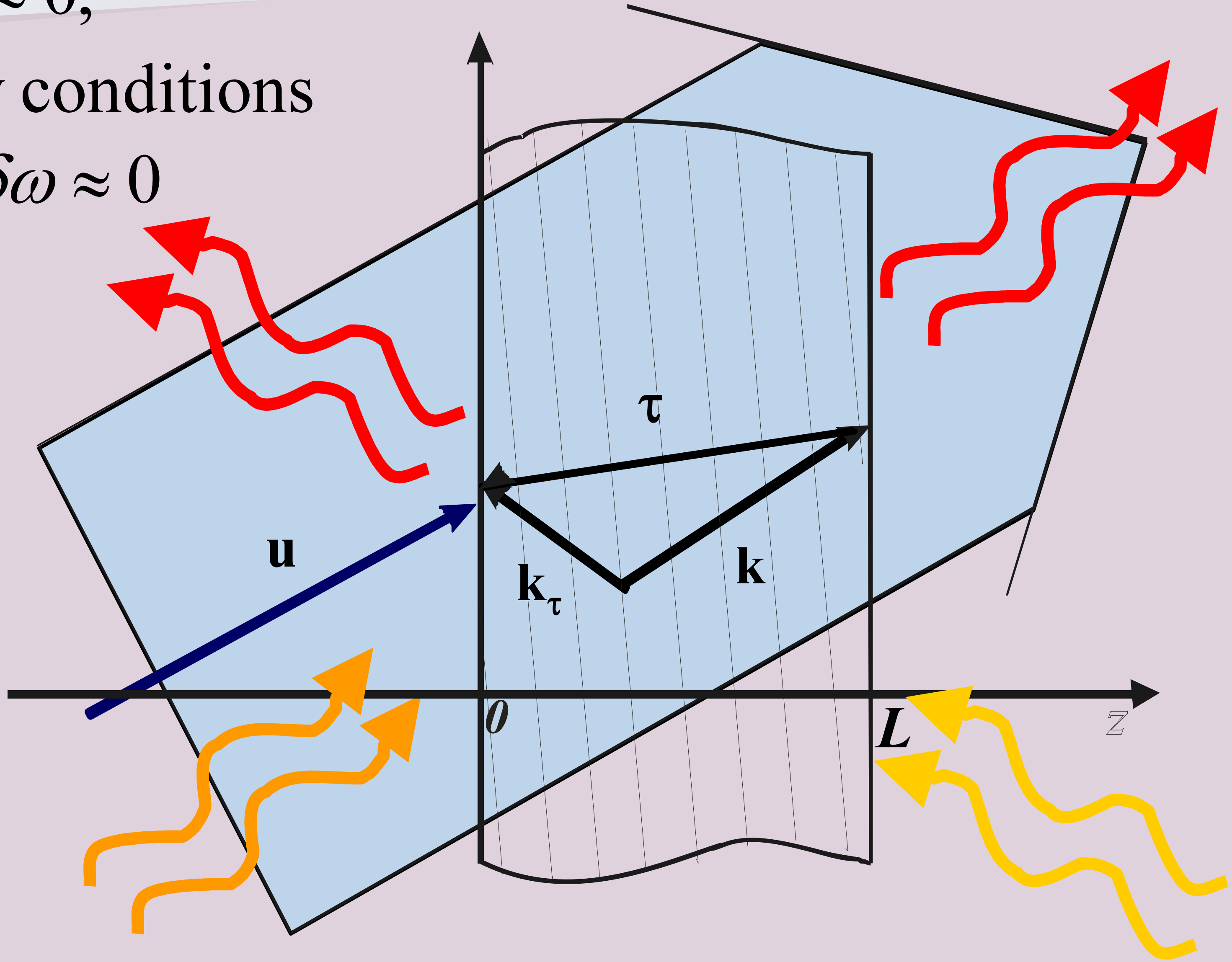
VFEL in Bragg geometry

Bragg conditions

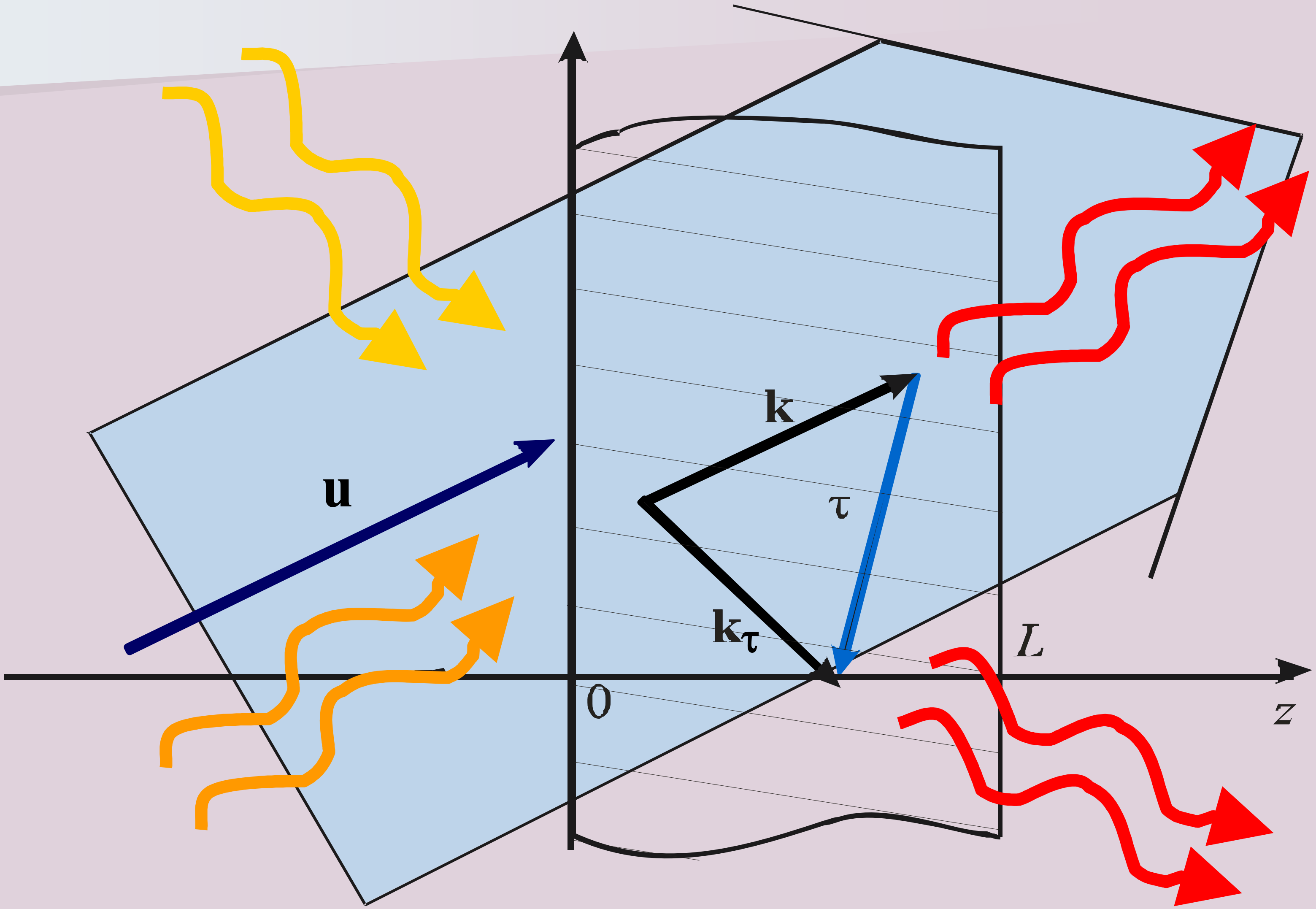
$2\mathbf{k\tau} + \boldsymbol{\tau}^2 \approx 0,$

Cherenkov conditions

$\left|\omega - \mathbf{ku}\right| = \delta\omega \approx 0$

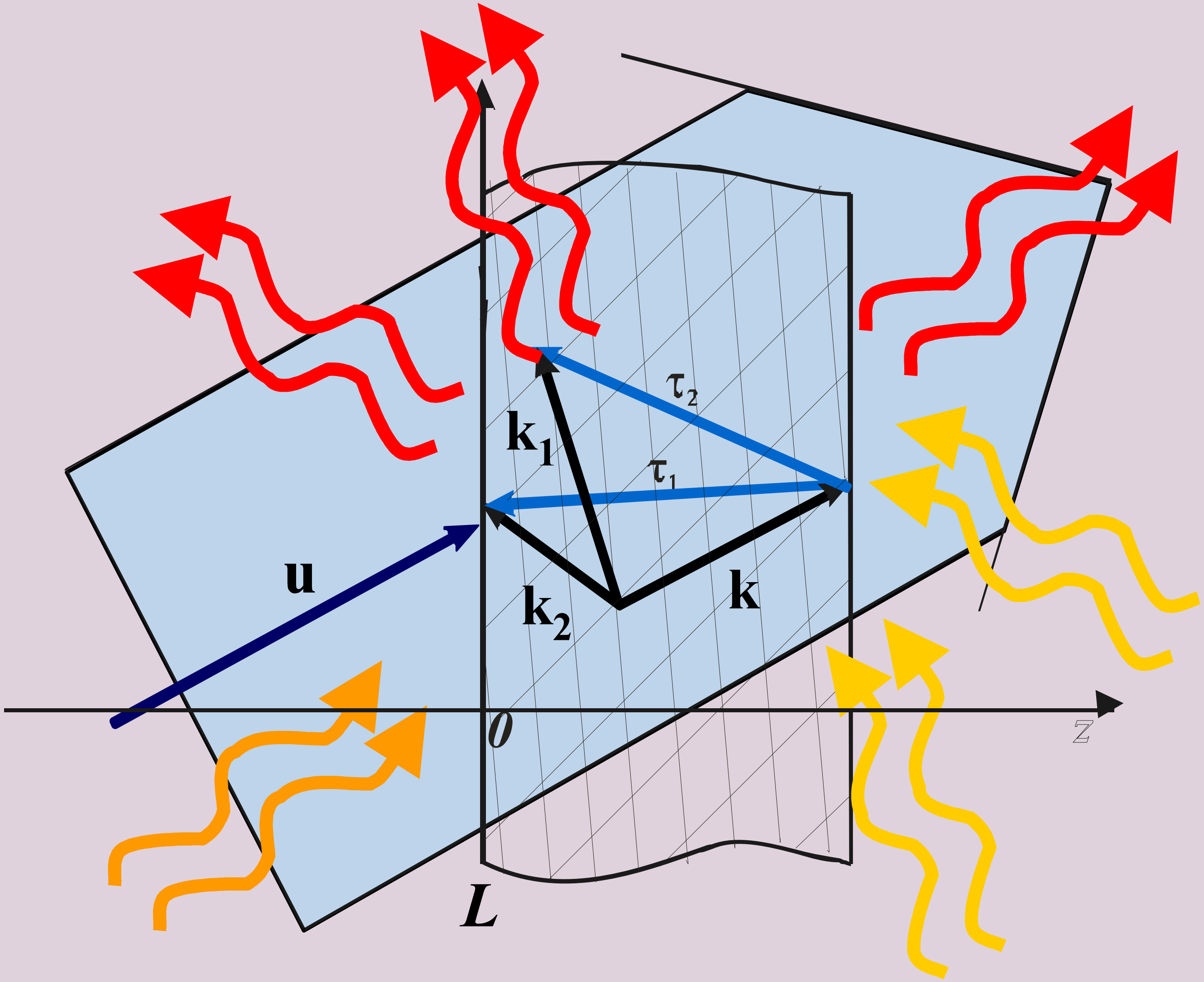


Laue geometry



Three-wave VFEL, Bragg-Bragg geometry

Additional parameters give possibility to adjust generation to more optimal region. Besides three-wave distributed feedback can be realized in region of three root degeneration. In this case all three modes are in synchronism with electron beam and interaction occurs more intensively.



Equations for electron beam

$$\frac{d^2\theta}{dt^2} = \frac{e}{m\gamma^3}(\mathbf{e}_\sigma \mathbf{n})\,\mathrm{Re}\left\{E\exp\left(i(\mathbf{k}_\perp \mathbf{r}_\perp + k_z z - \omega t)\right)\right\},$$

$\theta(t, z, p)$ – electron phase in a wave

$$\frac{d\theta(t,0,p)}{dz} = k - \omega / u, \quad \theta(t,0,p) = p,$$

$$t > 0, \quad z \in [0, L], \quad p \in [-2\pi, 2\pi]$$

Main equations

$$\Delta\mathbf{E} - \nabla(\nabla\mathbf{E}) - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t} = \frac{\partial\mathbf{j}_b}{\partial t},$$

$$\mathbf{E} = \mathbf{e}\ \left(E_0 e^{i(\mathbf{kr}-\omega t)} + E_1 e^{i(\mathbf{k}_\tau\mathbf{r}-\omega t)}\right),$$

$$\mathbf{j}_b = \mathbf{e}\ j e^{i(\mathbf{kr}-\omega t)}$$

In the common $n-$wave case:

$$\mathbf{E} = \mathbf{e}\sum_{i=0}^{n-1} E_i e^{i(\mathbf{k}_i\mathbf{r}-\omega t)}$$

System for two-wave VFEL:

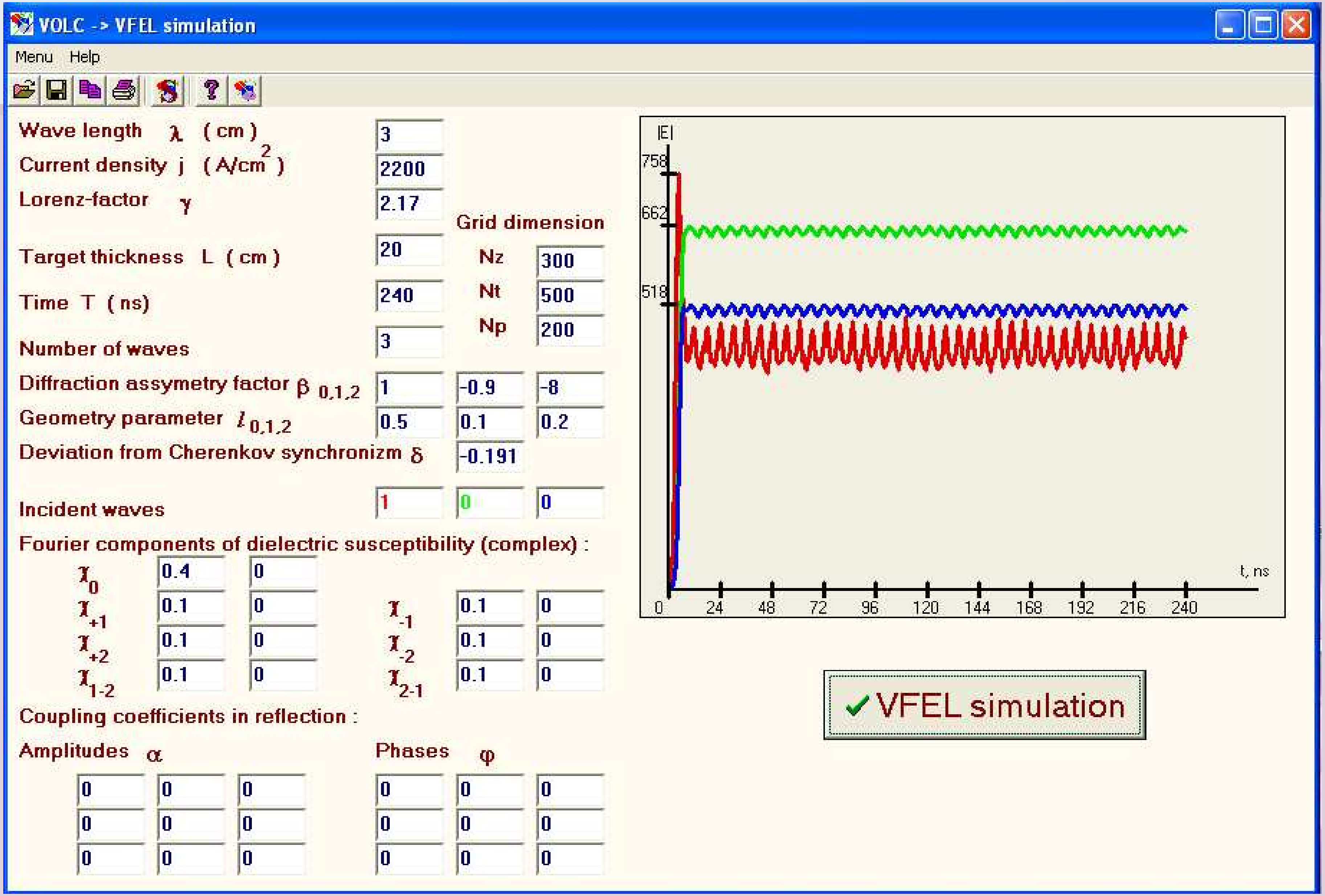
$$\frac{\partial E_0}{\partial t} + \gamma_0 c \frac{\partial E_0}{\partial z} + 0.5 i\omega l E_0 - 0.5 i\omega \chi_1 E_1 =$$

$$= 2\pi j \Phi \int_0^{2\pi} \frac{2\pi - p}{8\pi^2} \left(e^{-i\theta(t,z,p)} + e^{-i\theta(t,z,-p)} \right) dp,$$

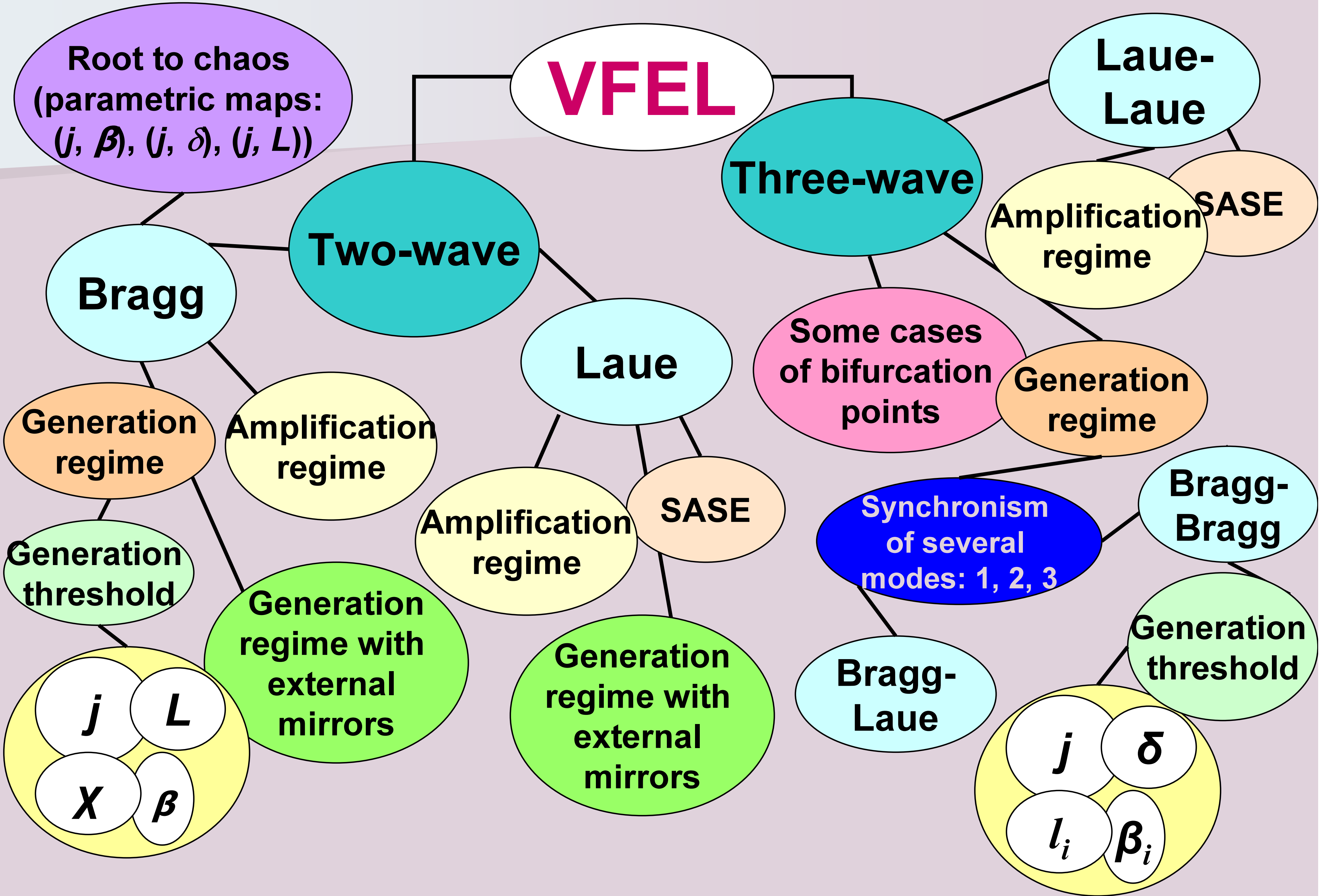
$$\frac{\partial E_1}{\partial t} + \gamma_1 c \frac{\partial E_1}{\partial z} - 0.5 i\omega \chi_{-1} E_0 + 0.5 i\omega l_1 E_1 = 0$$

The integral form of current is obtained by averaging over two initial phases: entrance time of electron in interaction zone and transverse coordinate of entrance point in interaction zone. In the mean field approximation double integration over two initial phases can be reduced to single integration.

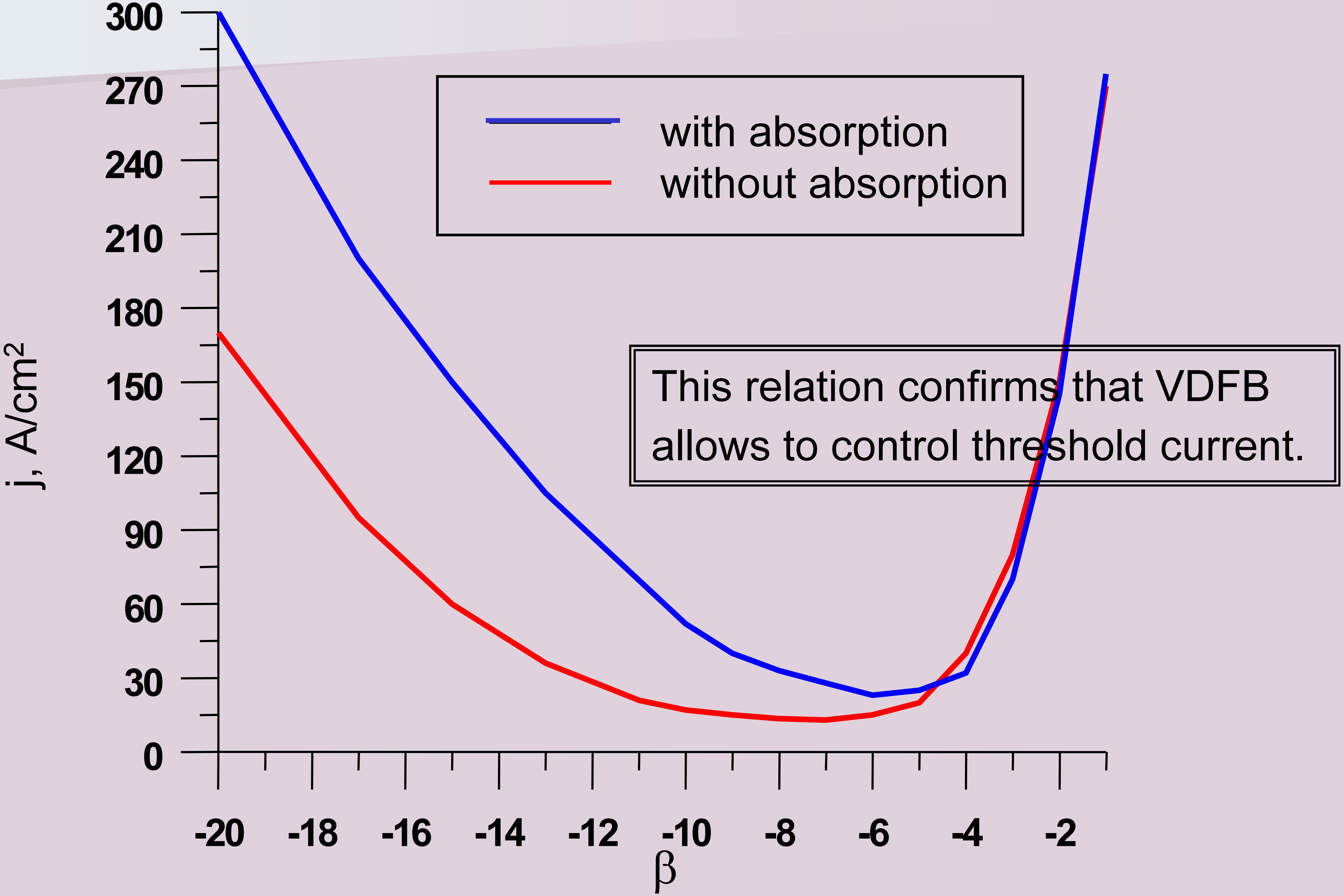
Code VOLC for VFEL simulation



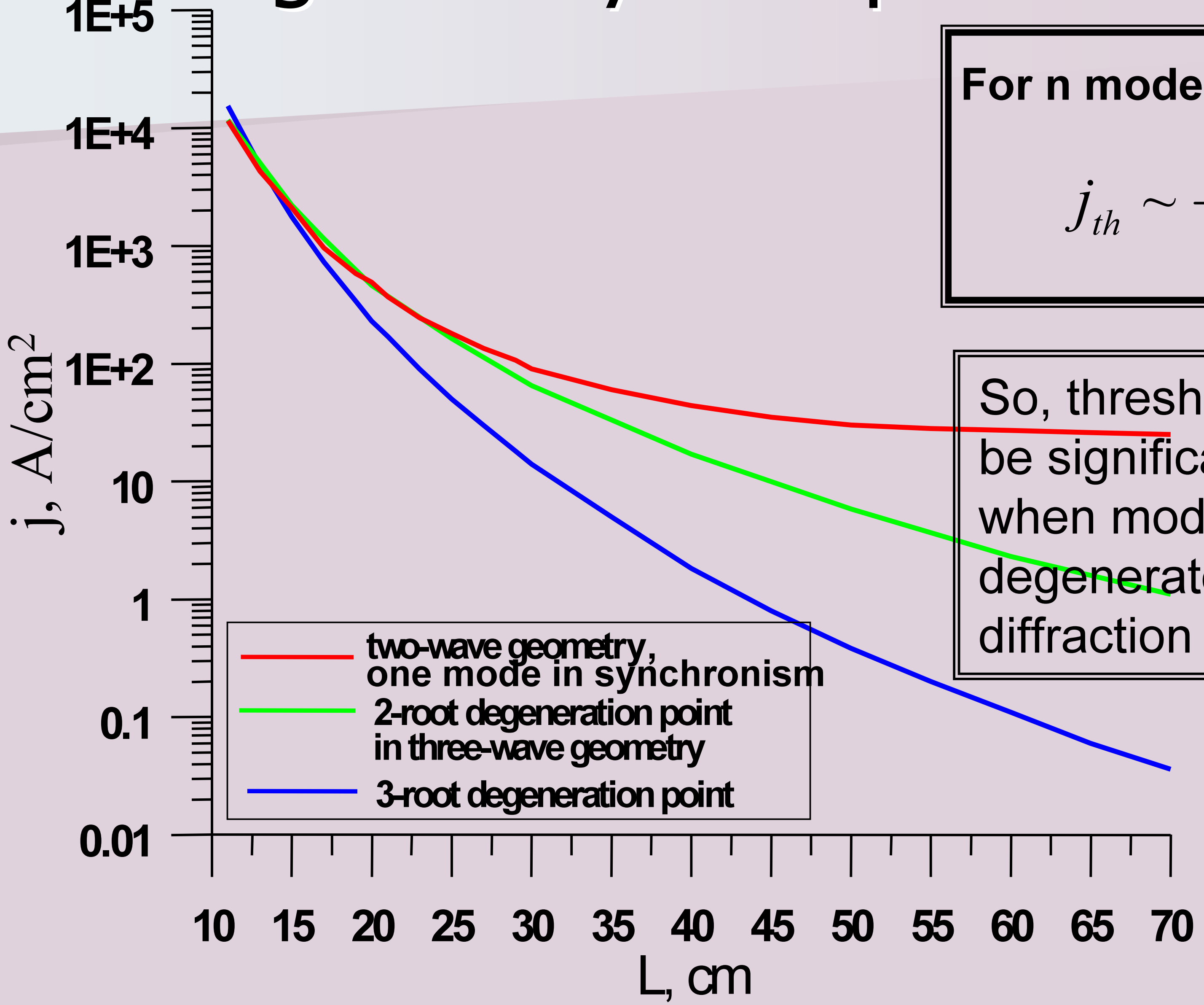
Results of numerical simulation (2002-2007):



Dependence of the threshold current on asymmetry factor β of VDFB



Current threshold for two- and three-wave geometry in dependence on *L*



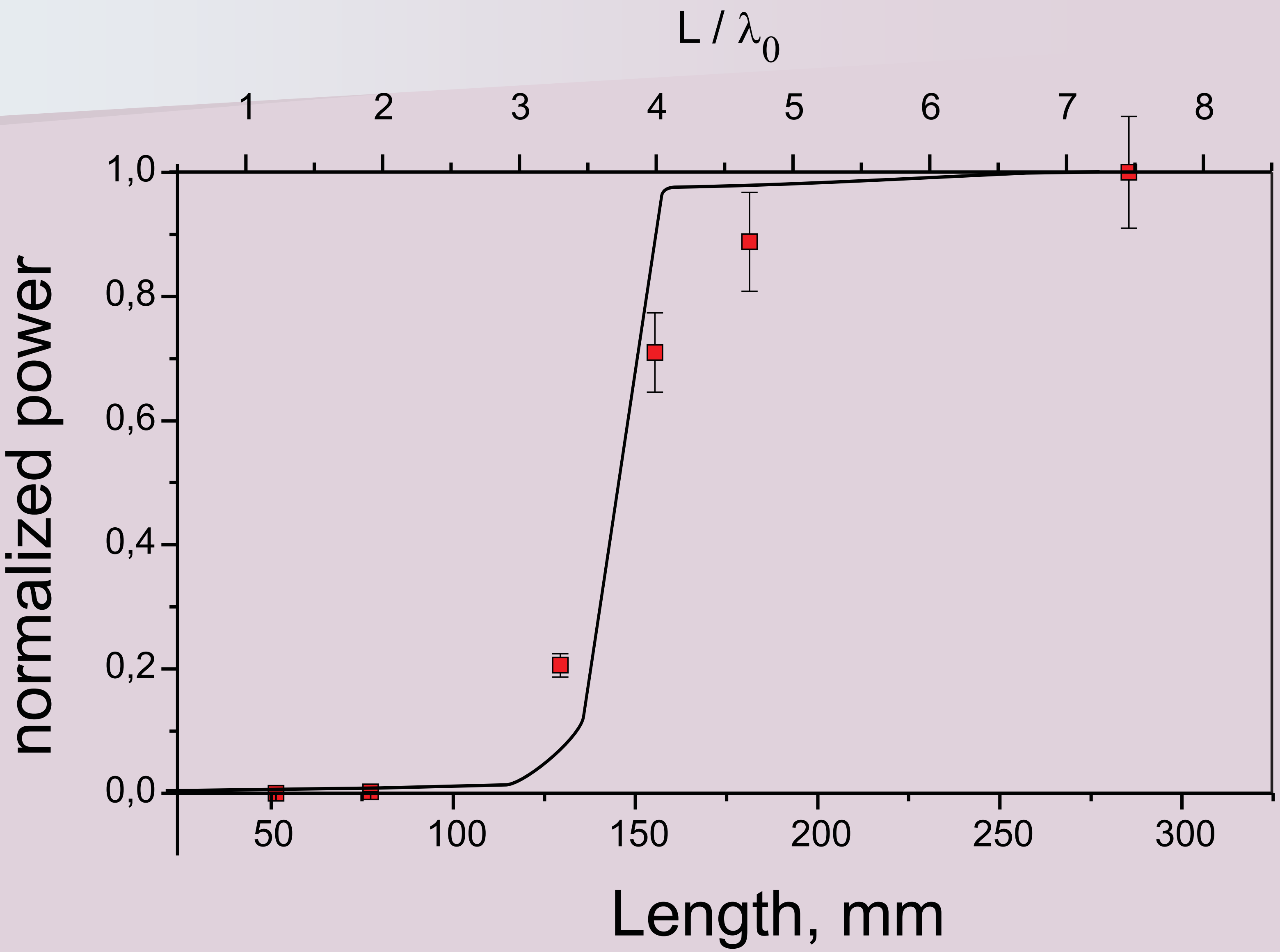
For n modes in synchronism*:

$$j_{th} \sim \frac{1}{(kL)^{3+2(n-1)}}$$

So, threshold current can be significantly decreased when modes are degenerated in multiwave diffraction geometry

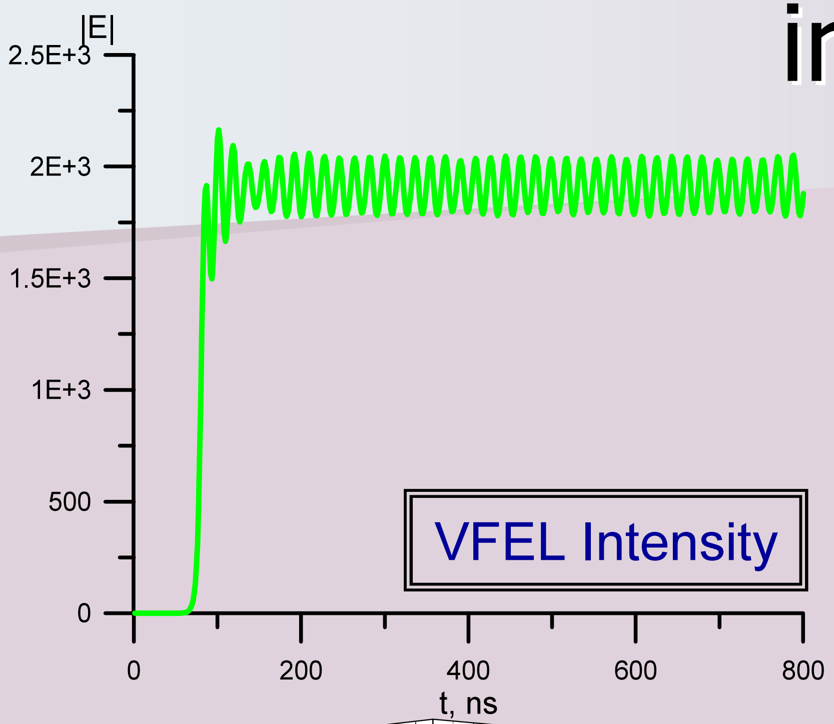
***V.Baryshevsky, K.Batrakov, I.Dubovskaya, NIM 358A (1995) 493**

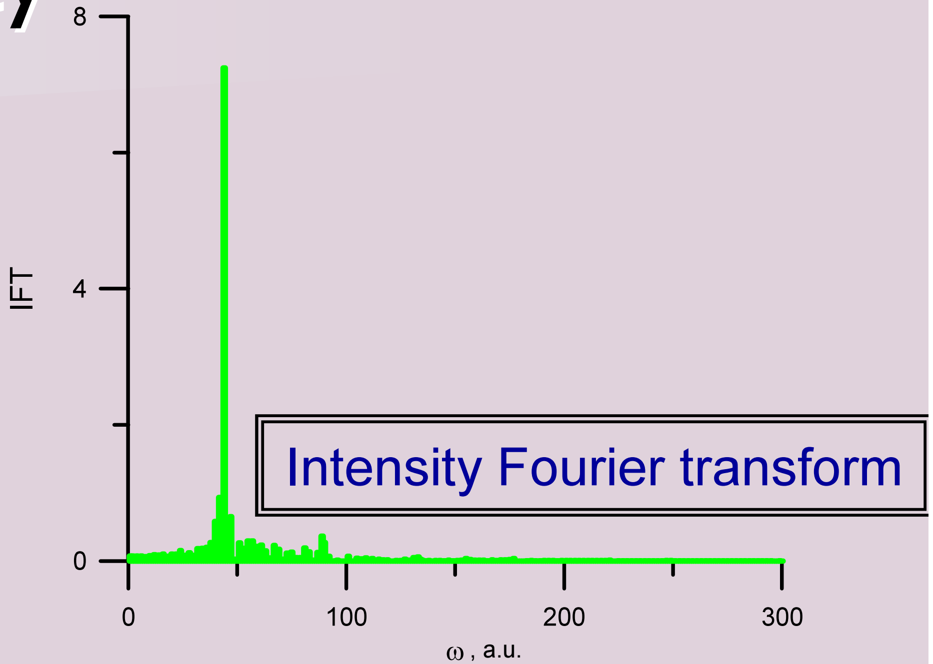
Dependence of electromagnetic radiation on *L* for experimental setup*

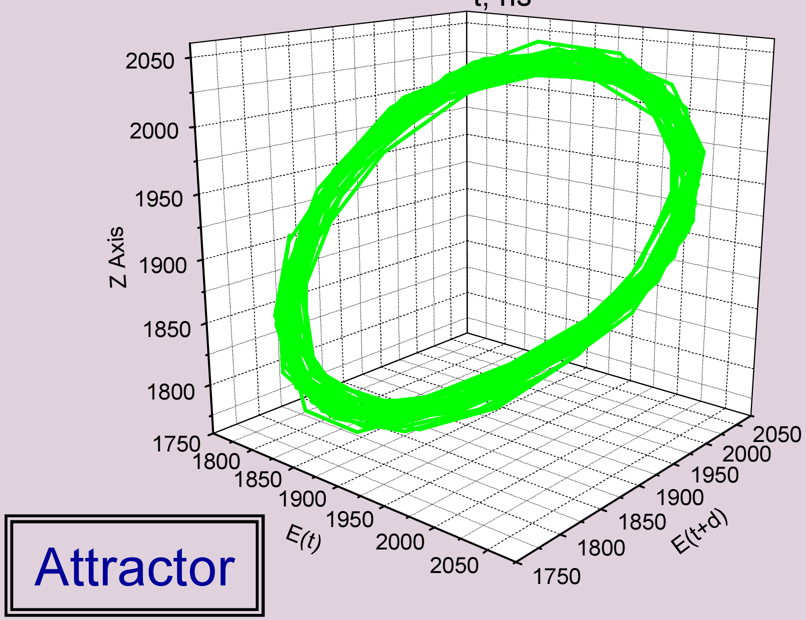


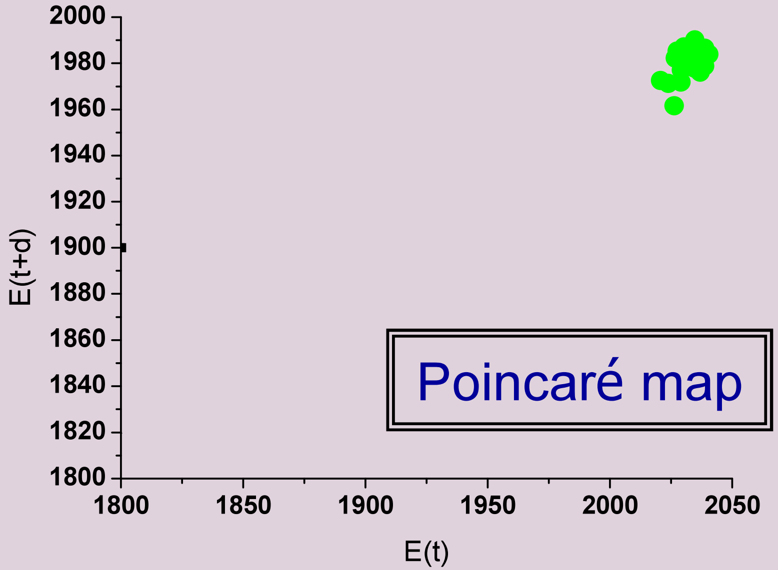
***V.G.Baryshevsky, N.A.Belous, A.Gurinovich et al., Proceedings of FEL06, Germany (2006), p.331**

Example of periodic regimes of VFEL intensity

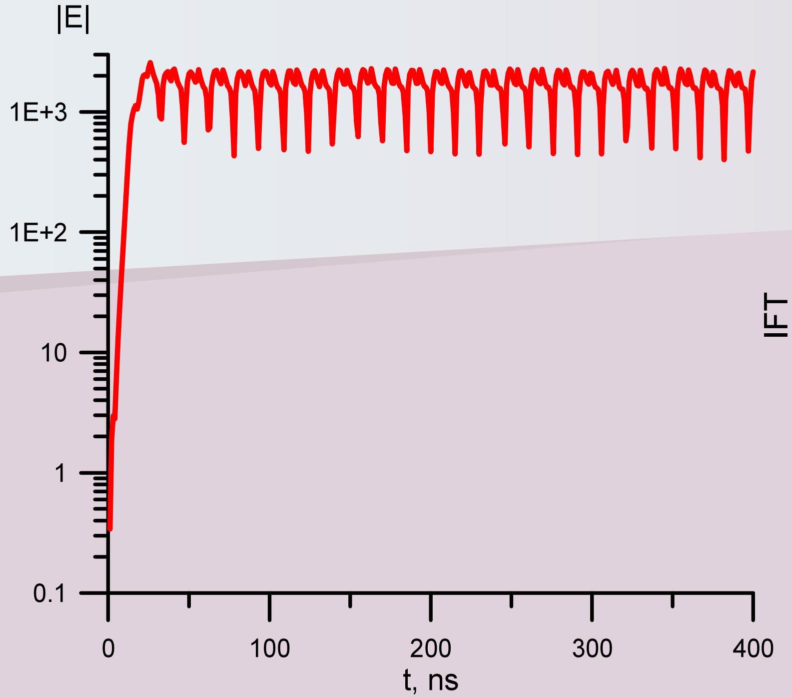


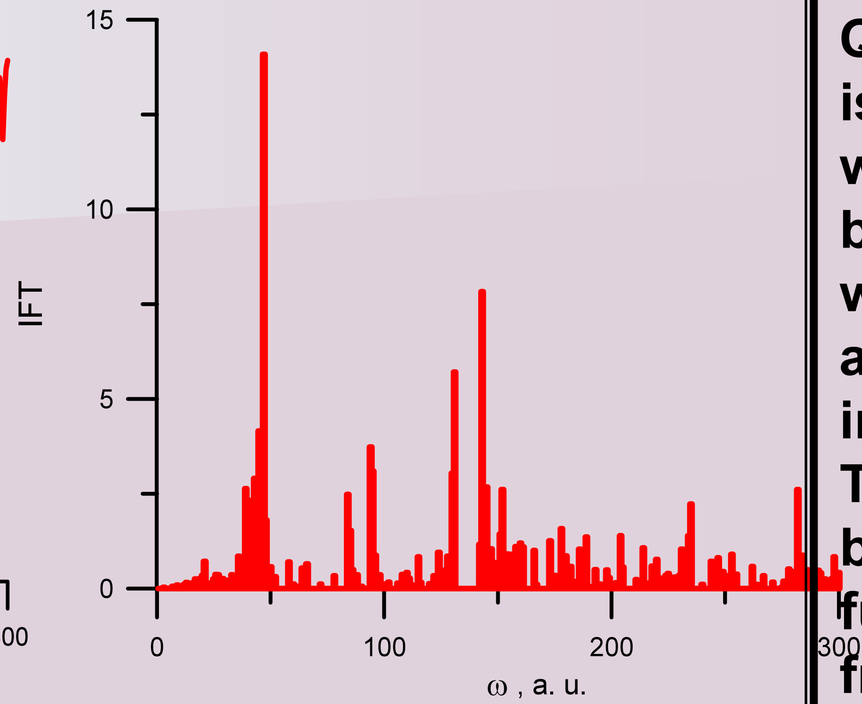


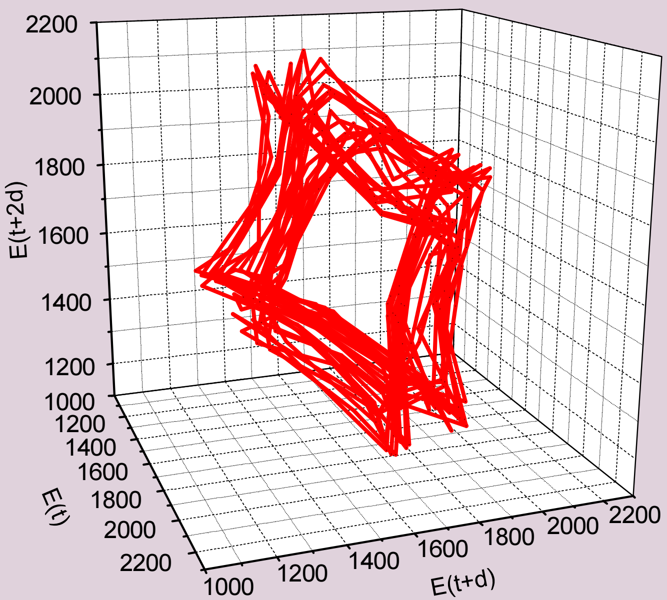


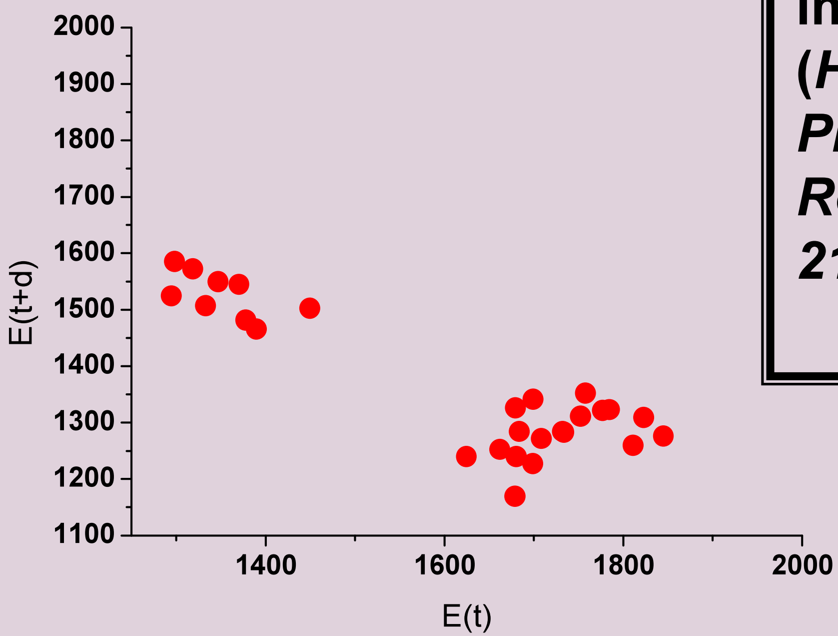


Quasiperiodic oscillations



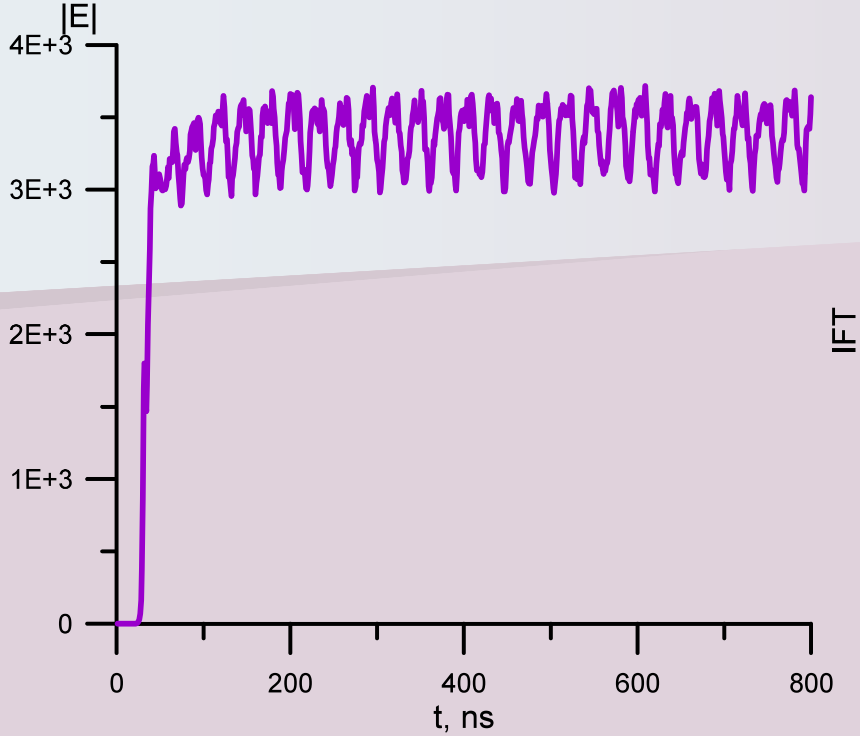


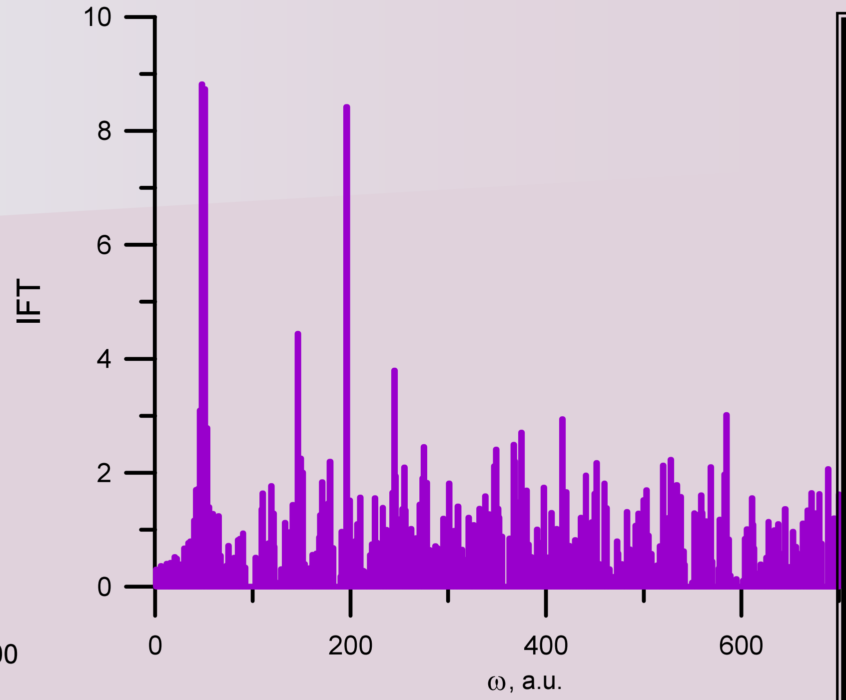




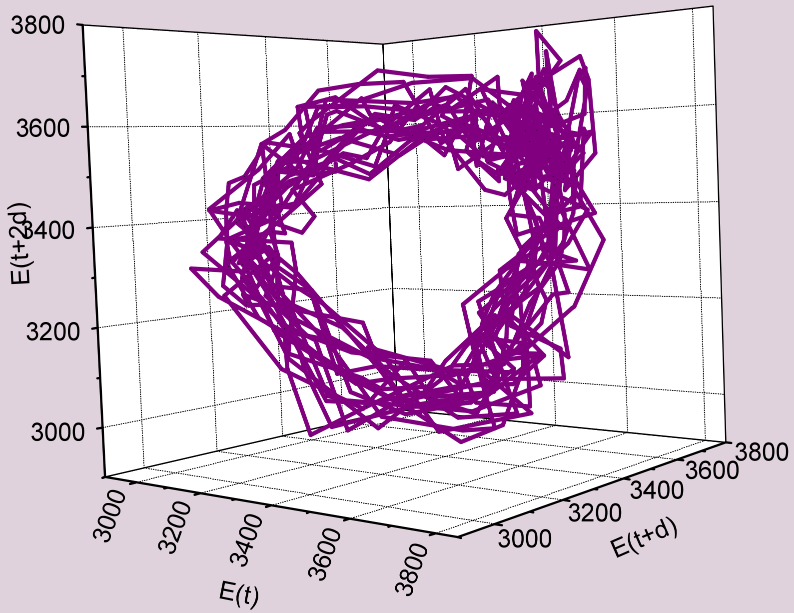
Quasiperiodicity is associated with the Hopf bifurcations which introduces a new frequency into the system. The ratios between the fundamental frequencies are incommensurate (*Hahn, Lee, Phys. Rev.E(1993),48, 2162*)

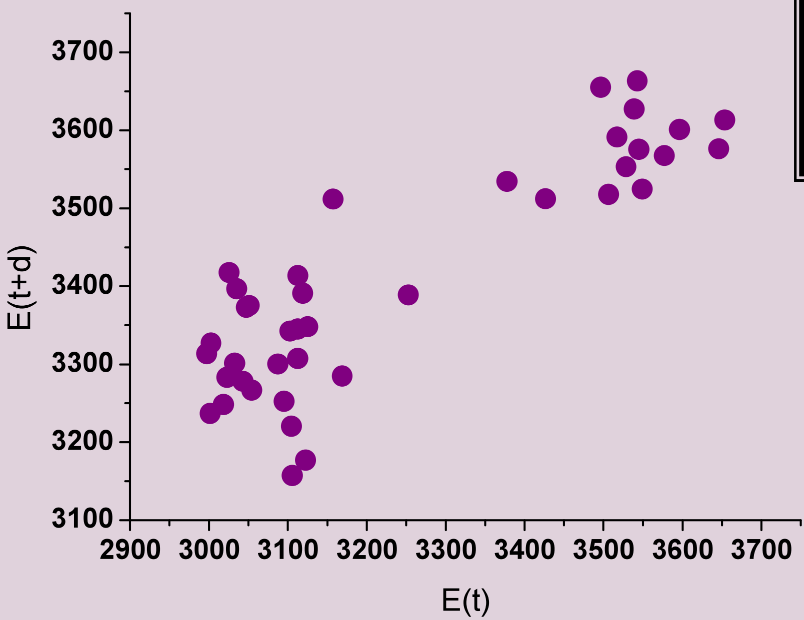
“Weak” chaotic regime



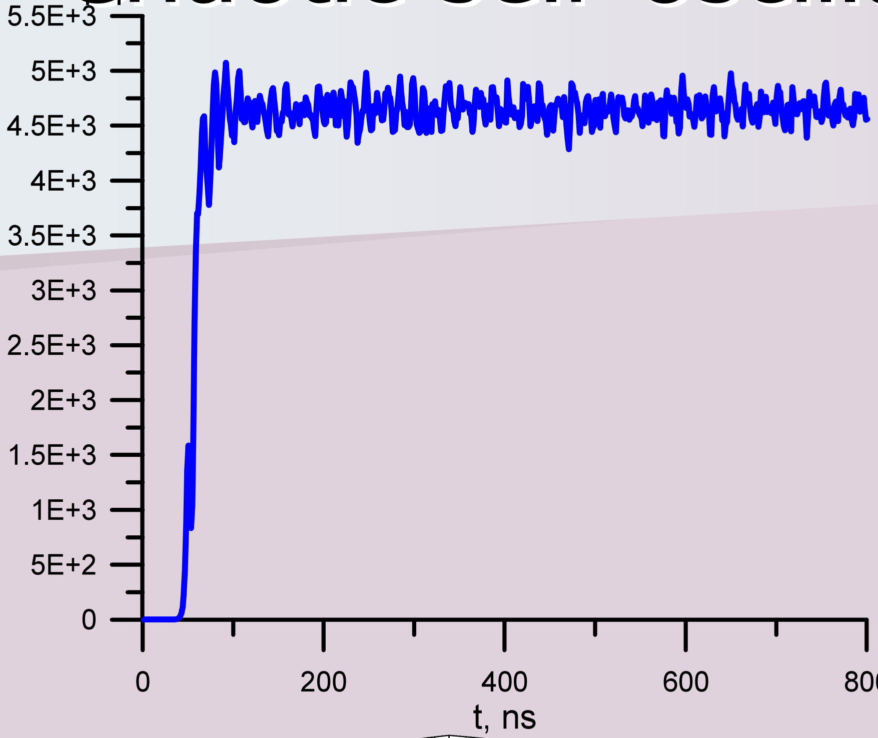


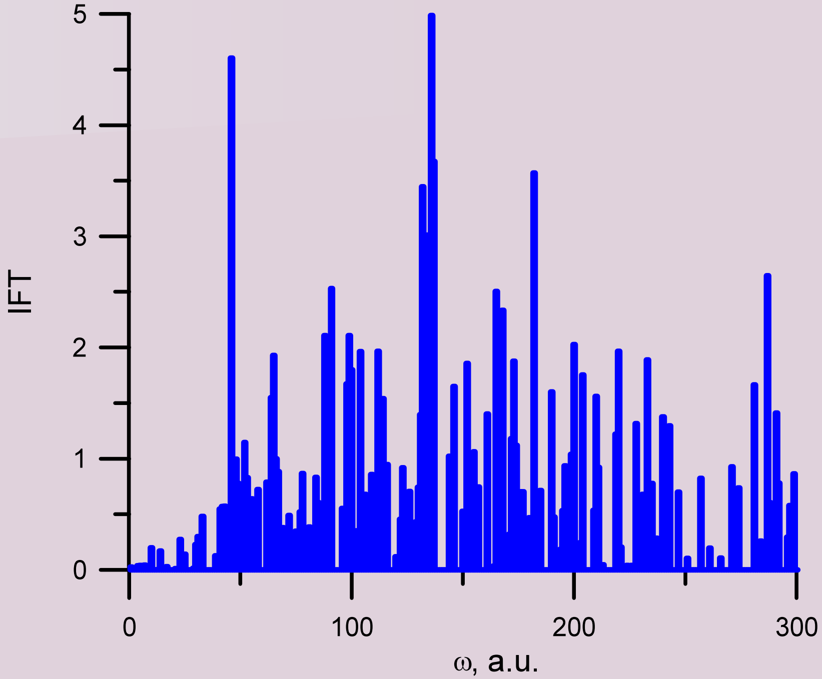
Dependence of amplitude in time seems as approximate repetition of equitype spikes close in dimensions per approximately equal time space.

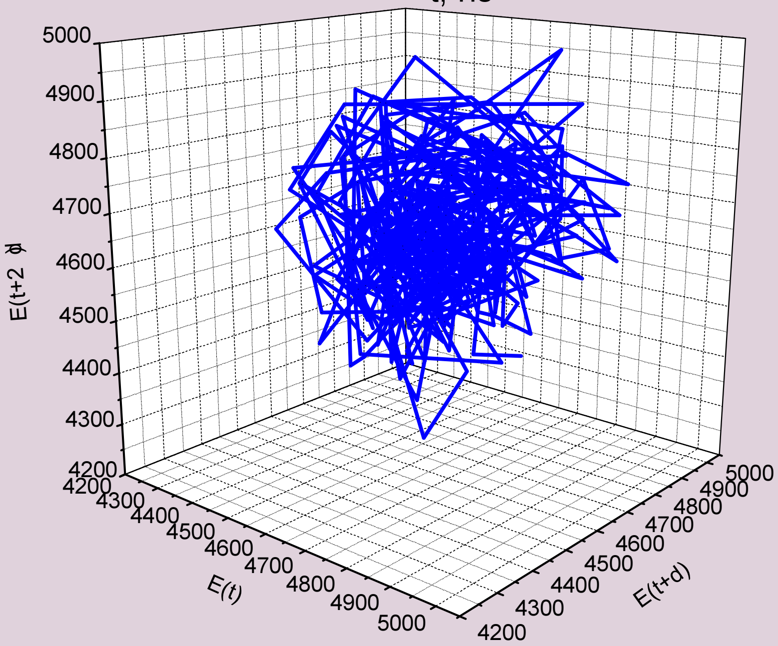


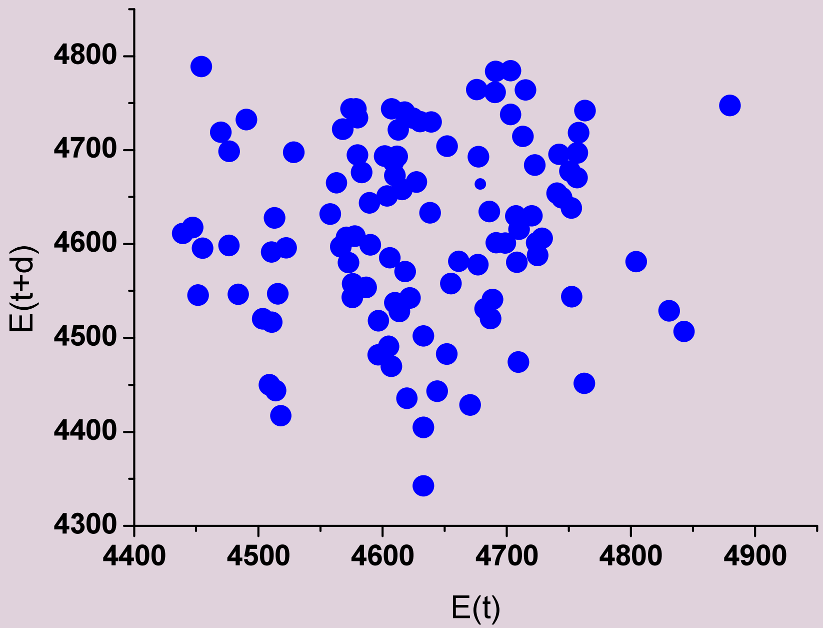


Chaotic self-oscillations (hyperchaos)



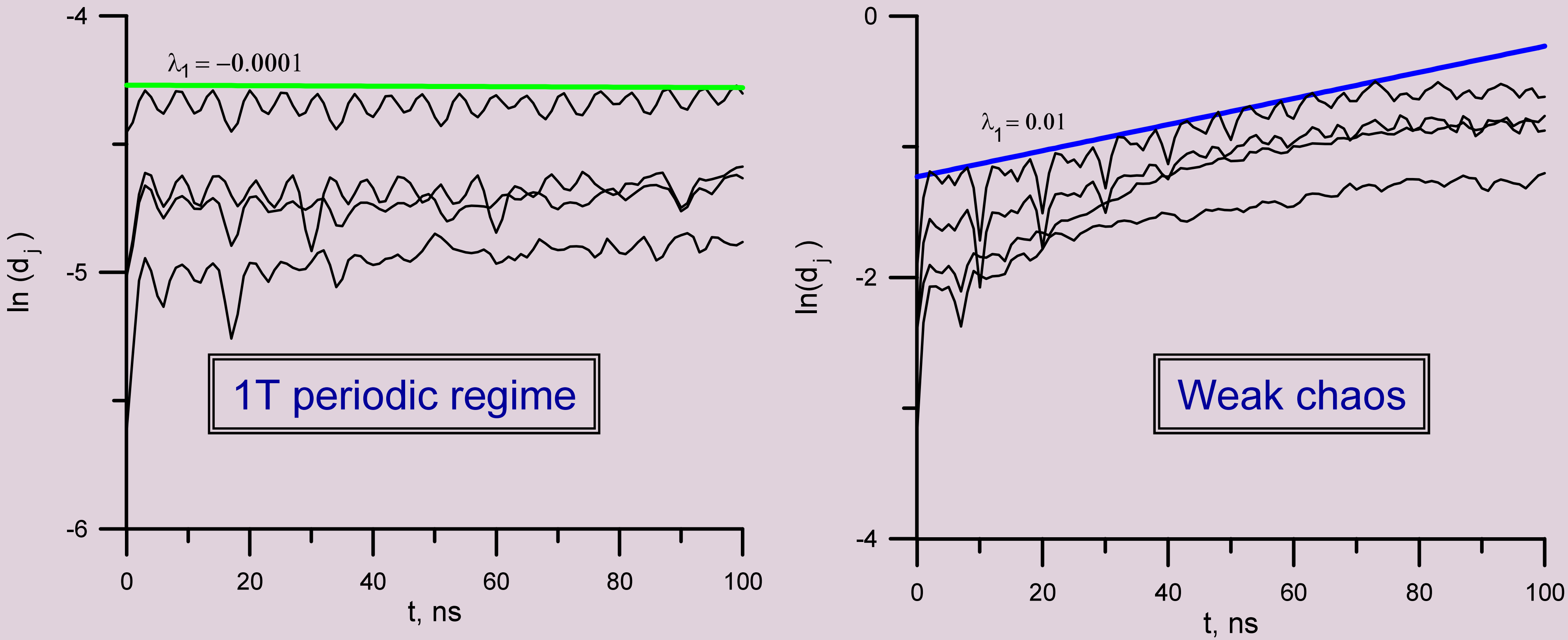






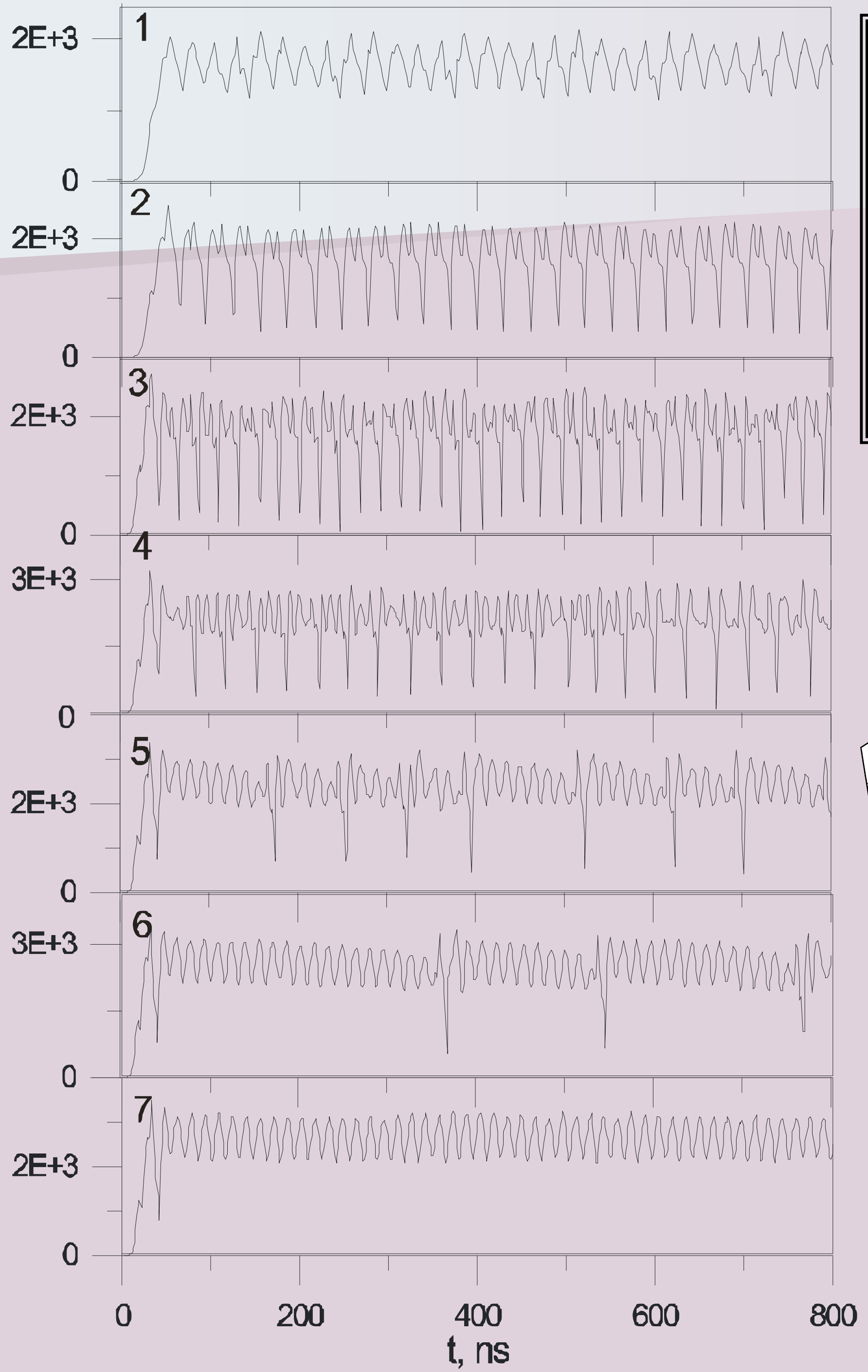
The largest Lyapunov exponent reconstructed with Rosenstein approach*

The largest Lyapunov exponent is a measurement of the stability of the underlying dynamics of time series. It specifies the mean velocity of divergence of neighboring points.



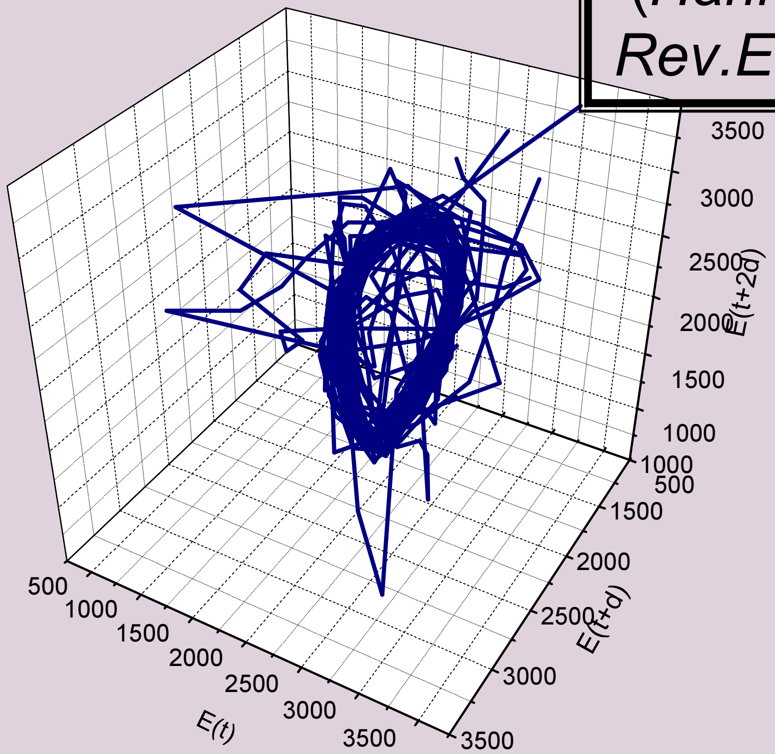
***M.T.Rosenstein et al. Physica D65 (1993), 117-134**

Quasiperiodicity and intermittency

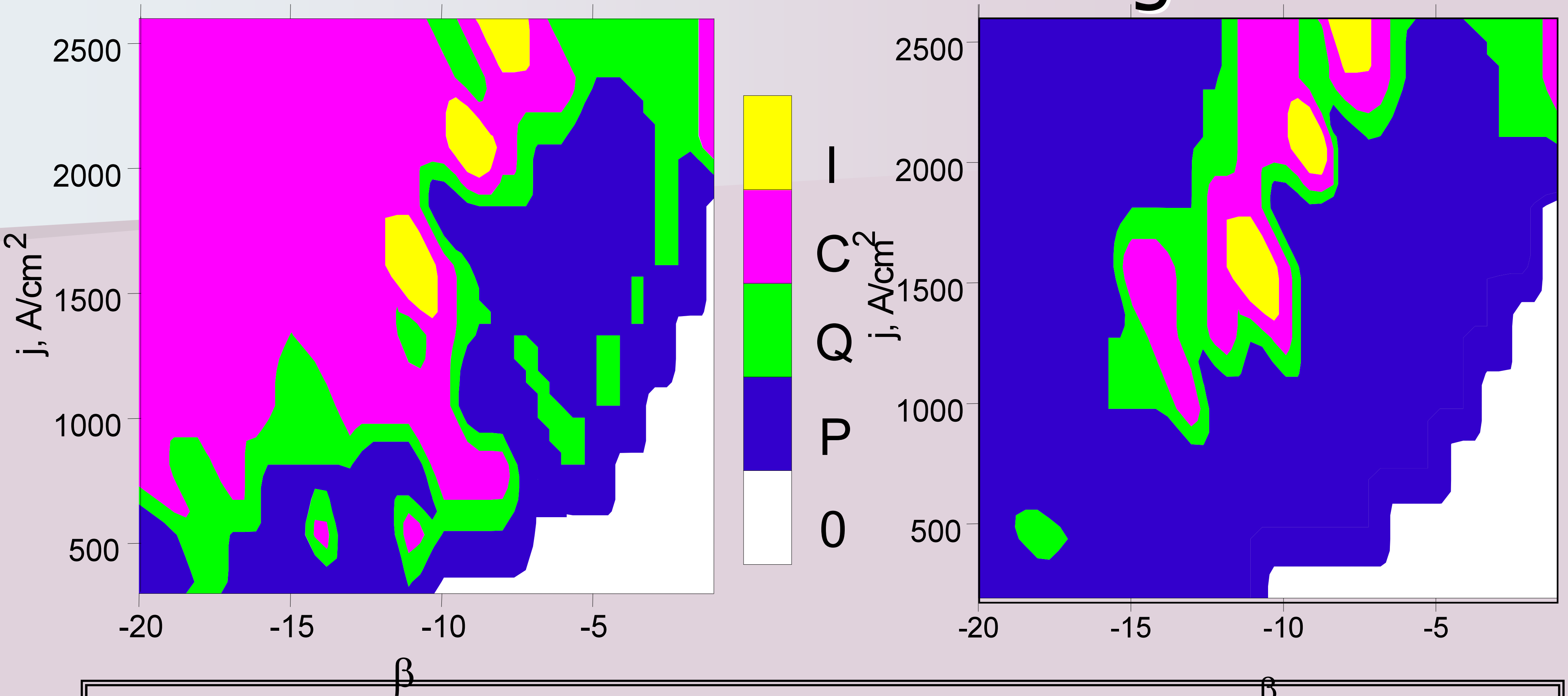


1) **1750 A/ см²**
2) **1950 A/ см²**
3) **2150 A/ см²**
4) **2220 A/ см²**
5) **2300 A/ см²**
6) **2340 A/ см²**
7) **2350 A/ см²**

Intermittency is closely related to saddle-node bifurcations. This means the collision between stable and unstable points, that then disappears.
(*Hahn, Lee, Phys. Rev.E(1993),48, 2162*)



Root to chaotic lasing



0 depicts a domain under beam current threshold. P – periodic regimes, QP –quasiperiodicity, I – intermittency, C – weak chaos.

Larger number of principle frequencies for transmitted wave can be explained the fact that in VFEL simultaneous generation at several frequencies is available. Here electrons emit radiation namely in the direction of transmitted wave.

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