Mathematical modelling of multiwave Volume Free Electron Laser (VFEL)

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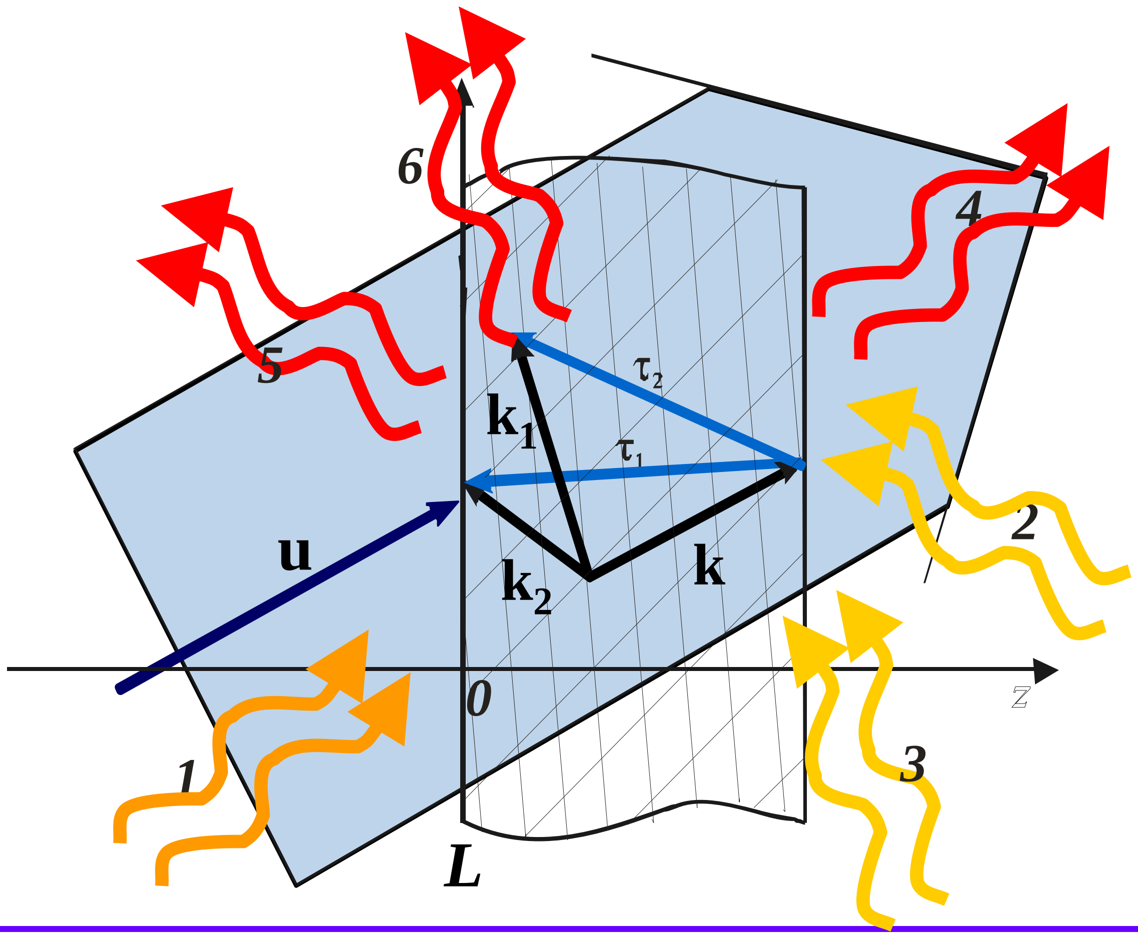
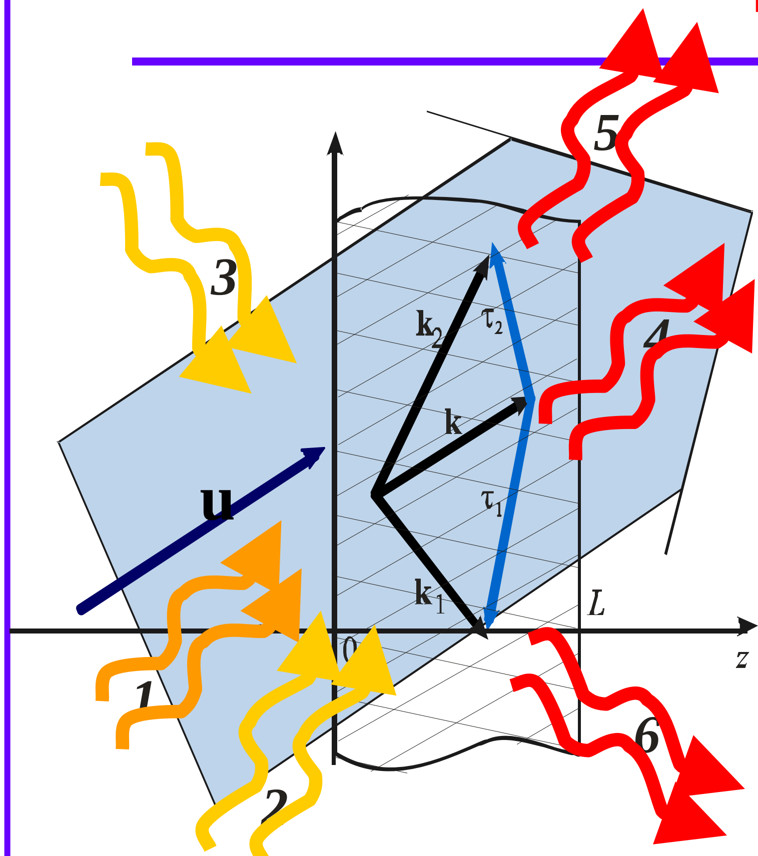
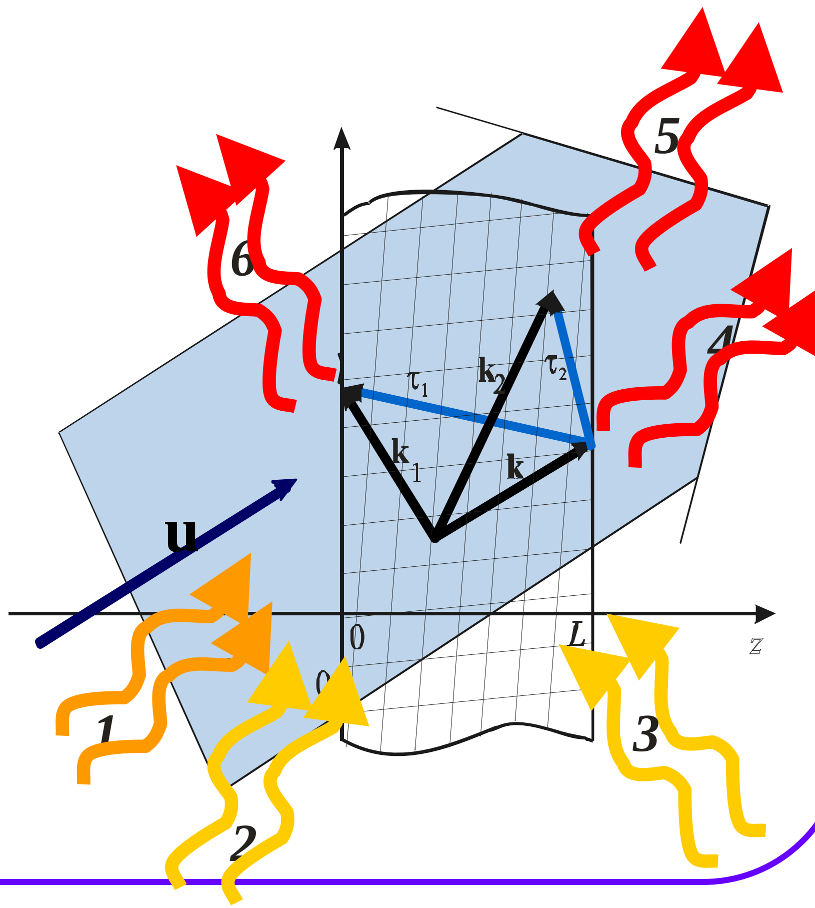
Three-wave VFEL in Bragg-Bragg geometry



Laue-Laue geometry

Bragg-Laue geometry





- If **one mode** is in synchronism, the threshold current ***j*** :

$$j \sim \frac{1}{(kL)^3}$$

- If **two modes are** in synchronism, the threshold current ***j*** :

$$j \sim \frac{1}{(kL)^5}$$

- If **n modes are** in synchronism, the threshold current ***j*** :

$$j \sim \frac{1}{(kL)^{3+2(n-1)}}$$

We assume *kL* ⑦1

System for three-wave VFEL

$$\frac{\partial E_0}{\partial t} + a_1 \frac{\partial E_0}{\partial z} + b_{11} E_0 + b_{12} E_1 + b_{13} E_2 = \Phi I,$$

$$\frac{\partial E_1}{\partial t} + a_2 \frac{\partial E_1}{\partial z} + b_{21} E_0 + b_{22} E_1 + b_{23} E_2 = 0,$$

$$\frac{\partial E_2}{\partial t} + a_3 \frac{\partial E_2}{\partial z} + b_{31} E_0 + b_{32} E_1 + b_{33} E_2 = 0,$$

$$\frac{d^2\theta(t,z,p)}{dz^2} = \Psi\left(k - \frac{d\theta}{dz}\right)^3 \mathrm{Re}\left(E_0\left(t - z/u, z\right)\right) e^{i\theta(t,z,p)},$$

$$I = \int_0^{2\pi} \frac{2\pi - p}{8\pi^2}\left(e^{-i\theta(t,z,-p)} + e^{-i\theta(t,z,-p)}\right) dp$$

Initial and boundary conditions:

$$E_0\Big|_{z=0} = E_0^0, \quad E_1\Big|_{z=L_1} = E_1^0,$$

$$E_2\Big|_{z=L_2} = E_2^0, \quad E_j\Big|_{t=0} = 0,$$

$$\frac{d\theta(t,0,p)}{dz} = k - \omega/u, \quad \theta(t,0,p) = p,$$

$$t > 0, \quad z \in [0,L], \quad p \in [-2\pi, 2\pi]$$

System for n-wave VFEL

$$\frac{\partial \mathbf{E}}{\partial t} + \mathbf{A}\frac{\partial \mathbf{E}}{\partial z} + \mathbf{B}\mathbf{E} = \mathbf{F}(I),$$

$$\mathbf{E} = (E_j)^T,\ j = 1,\dots,n$$

Numerical algorithm

$$\hat{\theta}^{j}_{\bar{z}z} = \Psi\left|\left(k - \hat{\theta}^{j}_{\underset{z}{o}}\right)\right|^{3} \mathrm{Re}\left(\widetilde{E}_0 \exp\left(i\theta^{j}\right)\right),$$

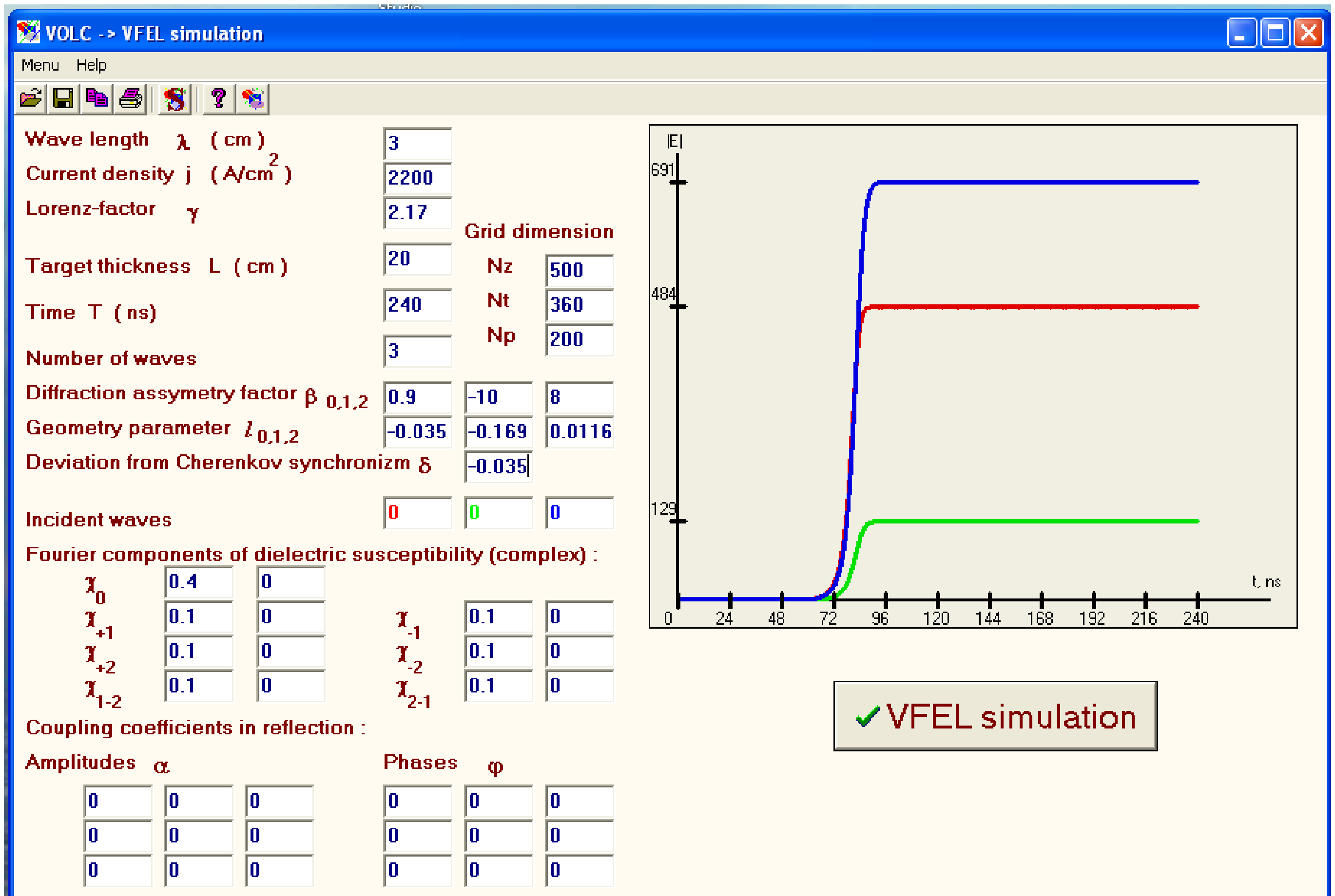
$$E_{0t} + a_1 \hat{E}_{0\bar{z}} + b_{11}\hat{E}_0 + b_{12}\hat{E}_1 + b_{13}\hat{E}_2 =$$

$$= \Phi \sum_{j=0}^{N} c_j \left(\exp(-i\hat{\theta}^{j}) + \exp(-i\hat{\theta}^{-j})\right),$$

$$E_{1t} + a_2 \hat{E}_{1\tilde{z}} + b_{21}\hat{E}_0 + b_{22}\hat{E}_1 + b_{23}\hat{E}_2 = 0,$$

$$E_{2t} + a_3 \hat{E}_{2\tilde{z}} + b_{31}\hat{E}_0 + b_{32}\hat{E}_1 + b_{33}\hat{E}_2 = 0$$

Code VOLC (Volume Code) for VFEL simulation



Dispersion equation:

$$l_0 l_1 l_2 - l_0 r_{12} - l_1 r_2 - l_2 r_1 - \chi_1 \chi_{-2} \chi_{2-1} - \chi_2 \chi_{-1} \chi_{1-2} = 0$$

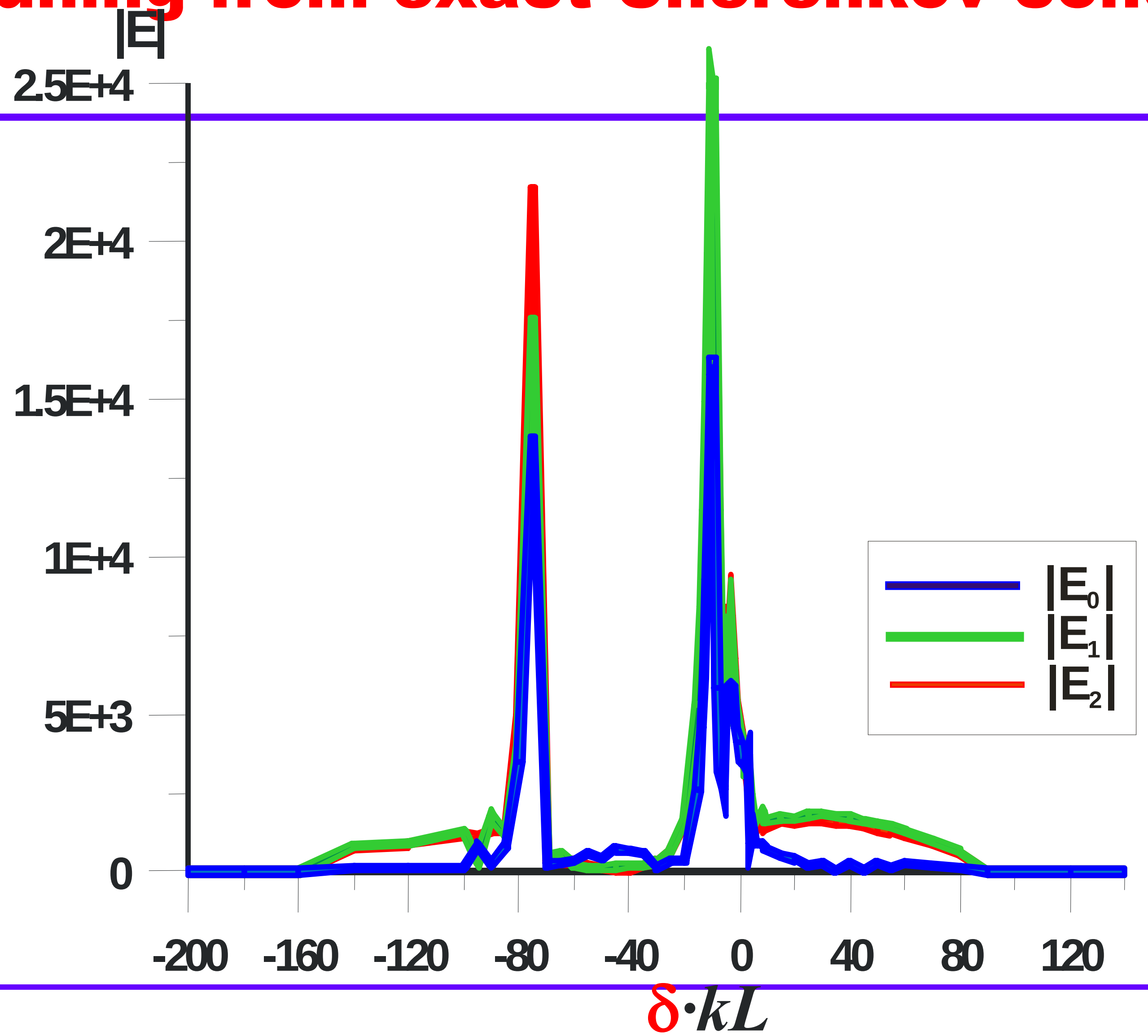
Two-root degeneration case:

$$\beta_1 \beta_2 l_1 l_2 + (\beta_1 l_1 + \beta_2 l_2) l_0 - \beta_1 \beta_2 r_{12} - \beta_1 r_1 - \beta_2 r_2 = 0$$

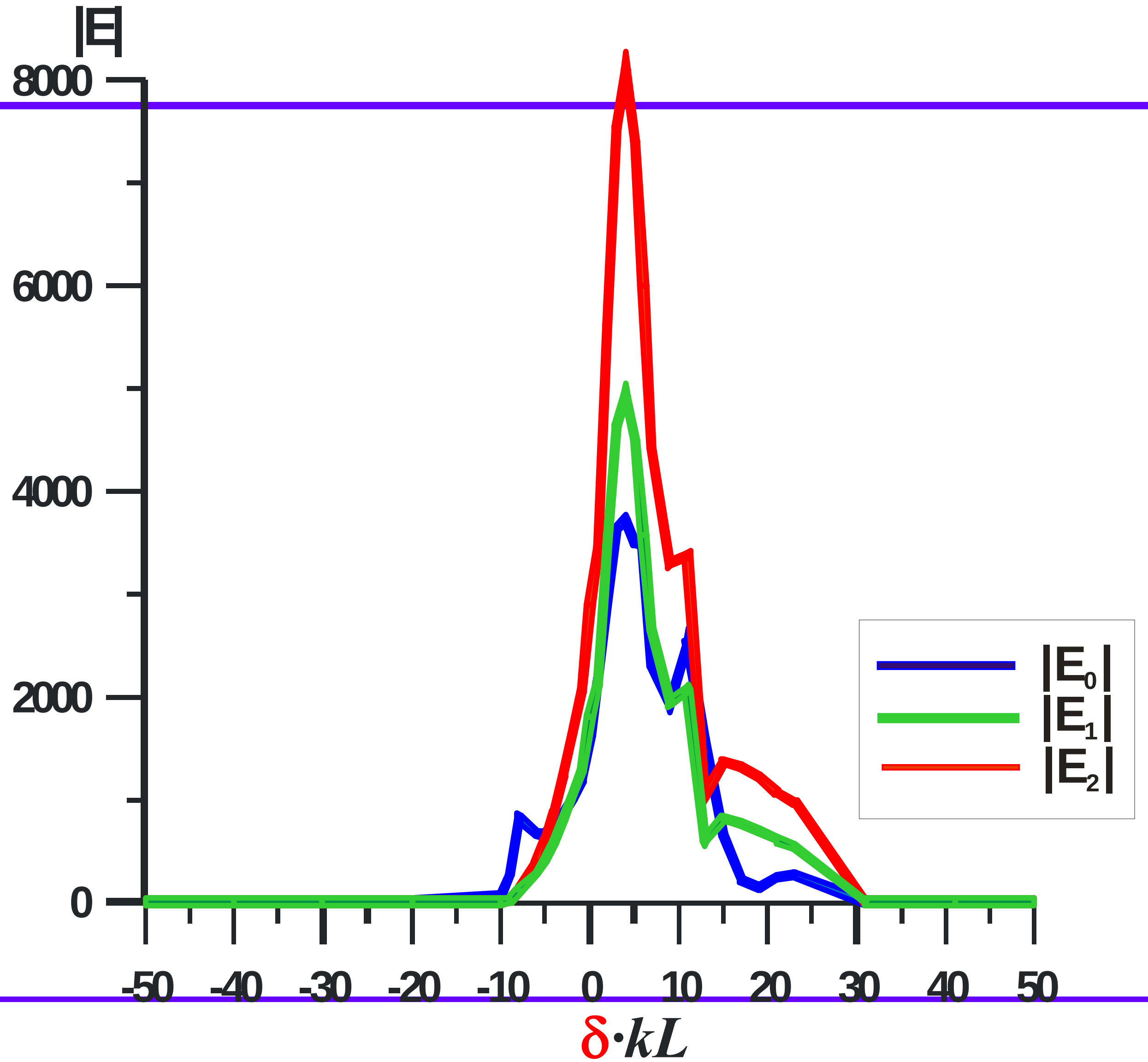
Three-root degeneration case:

$$\beta_1 l_1 + \beta_2 l_2 + l_0 = 0$$

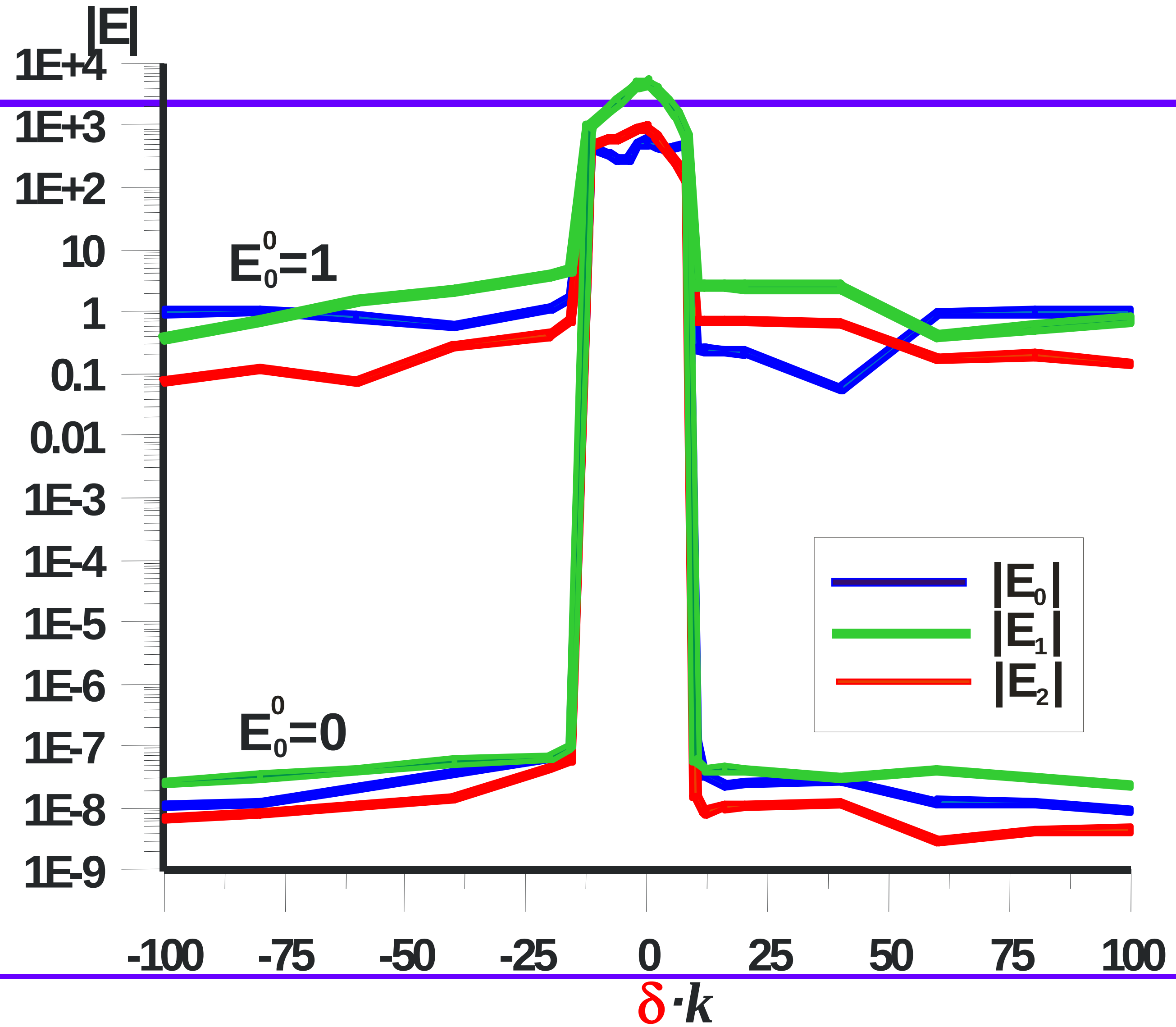
One-mode synchronism, dependence on detuning from exact Cherenkov condition δ



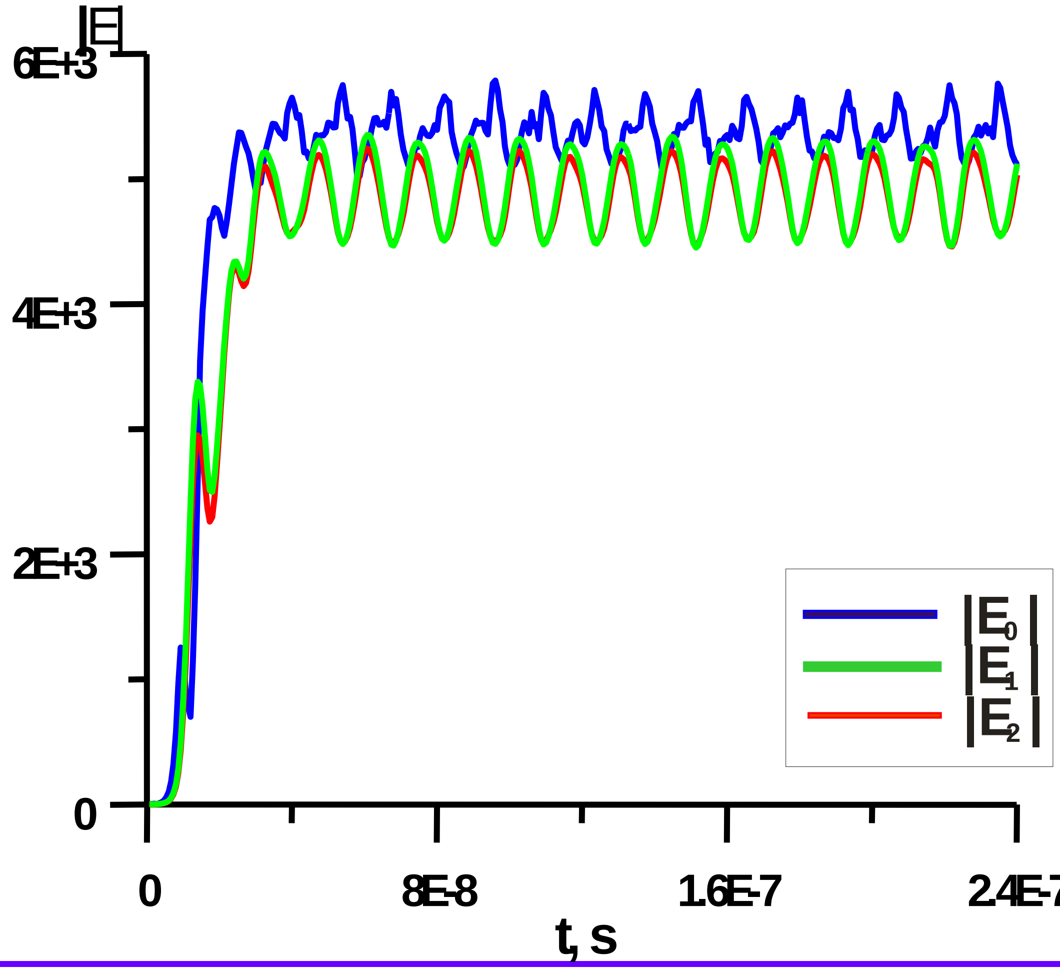
Two-root degeneration case



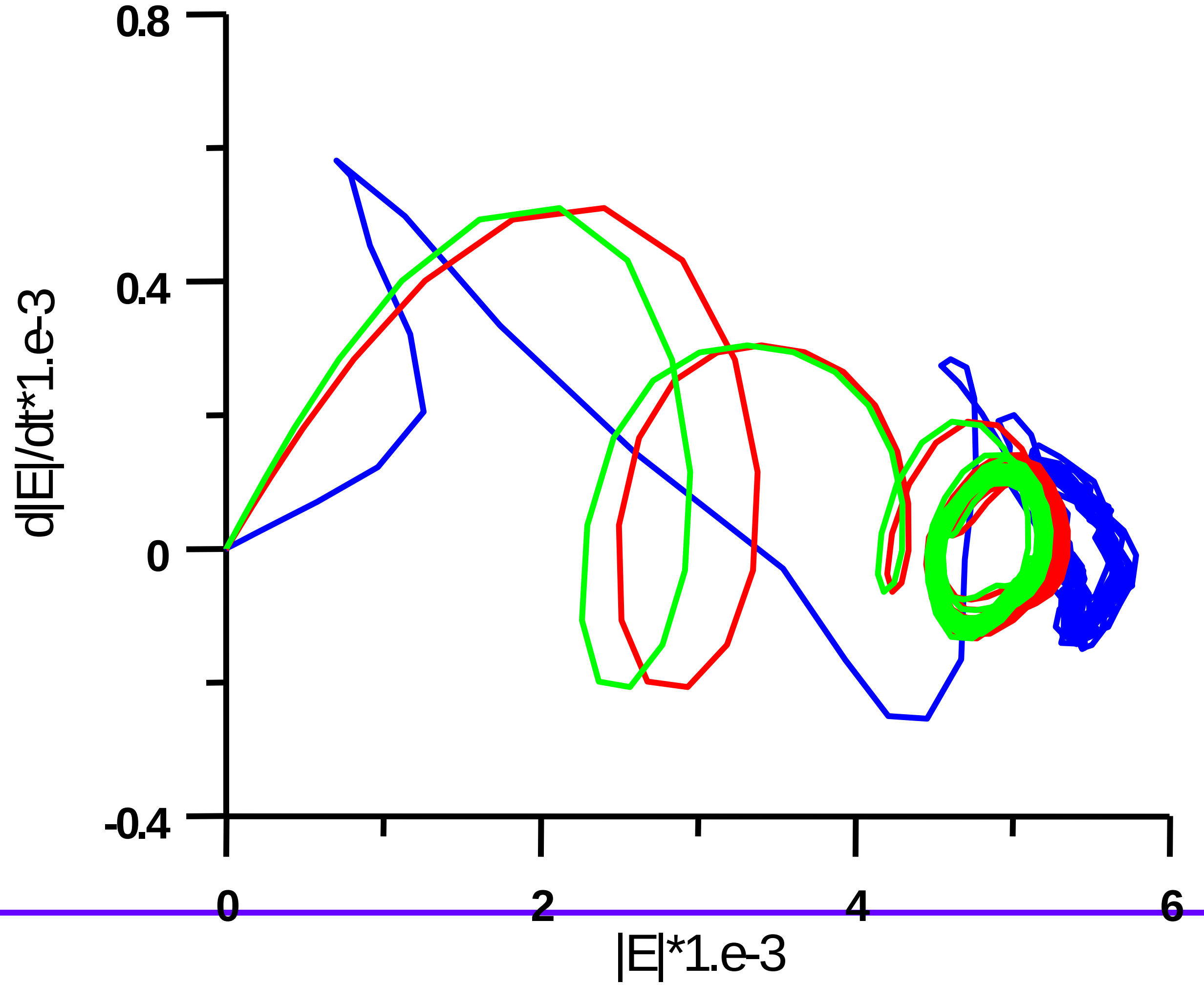
Three-root degeneration case



Periodic regime of VFEL intensity



Phase space portrait



References (2005)

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- *Batrakov K., Sytova S.* Nonlinear analysis of quasi-Cherenkov electron beam instability in VFEL (Volume Free Electron Laser). ***Nonlinear Phenomena in Complex Systems*, 8** : 1(2005) 42–48
- *Batrakov K., Sytova S.* Dynamics of electron beam instabilities under conditions of multiwave distributed feedback. *Submitted to **Nonlinear Phenomena in Complex Systems***