Mathematical modelling of multiwave Volume Free Electron Laser (VFEL)

K. Batrakov, S. Sytova Institute for Nuclear Problems, Belarusian State University

Three-wave VFEL in Bragg-Bragg geometry



Laue-Laue geometry Bragg-Laue geometry



- If one mode is in synchronism, the threshold current j:
- If two modes are in synchronism, the threshold current j:

 If **n modes are** in synchronism, the threshold current **j**:



We assume

*kL©*1

System for three-wave VFEL

J 0

$$\frac{\partial E_{0}}{\partial t} + a_{1} \frac{\partial E_{0}}{\partial z} + b_{11}E_{0} + b_{12}E_{1} + b_{13}E_{2} = \Phi I,$$

$$\frac{\partial E_{1}}{\partial t} + a_{2} \frac{\partial E_{1}}{\partial z} + b_{21}E_{0} + b_{22}E_{1} + b_{23}E_{2} = 0,$$

$$\frac{\partial E_{2}}{\partial t} + a_{3} \frac{\partial E_{2}}{\partial z} + b_{31}E_{0} + b_{32}E_{1} + b_{33}E_{2} = 0,$$

$$\frac{d^{2}\theta(t, z, p)}{dz^{2}} = \Psi \left(k - \frac{d\theta}{dz}\right)^{3} \operatorname{Re}\left(E_{0}(t - z/u, z)\right)e^{i\theta(t, z, p)},$$

$$I = \int_{0}^{2\pi} \frac{2\pi - p}{8\pi^{2}}\left(e^{-i\theta(t, z, -p)} + e^{-i\theta(t, z, -p)}\right)dp$$

C

 μp

Initial and boundary conditions:

$$\begin{split} E_0 \big|_{z=0} &= E_0^0, \quad E_1 \big|_{z=L_1} = E_1^0, \\ E_2 \big|_{z=L_2} &= E_2^0, \quad E_j \big|_{t=0} = 0, \end{split}$$

$$\frac{d\theta(t,0,p)}{dz} = k - \omega/u, \quad \theta(t,0,p) = p,$$

$$t > 0, \quad z \in [0,L], \quad p \in [-2\pi, 2\pi]$$

System for n-wave VFEL

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$$\frac{\partial \mathbf{E}}{\partial t} + \mathbf{A} \frac{\partial \mathbf{E}}{\partial z} + \mathbf{B} \mathbf{E} = \mathbf{F}(I)$$
$$\mathbf{E} = (E_j)^T, \ j = 1, ..., n$$

Numerical algorithm

$$\begin{aligned} \hat{\theta}_{\bar{z}z}^{j} &= \Psi \left(k - \hat{\theta}_{o_{\bar{z}}}^{j} \right)^{3} \operatorname{Re} \left(\widetilde{E}_{0} \exp \left(i \theta^{j} \right) \right), \\ E_{0t} &+ a_{1} \hat{E}_{0\bar{z}} + b_{11} \hat{E}_{0} + b_{12} \hat{E}_{1} + b_{13} \hat{E}_{2} = \\ &= \Phi \sum_{j=0}^{N} c_{j} \left(\exp(-i \hat{\theta}^{j}) + \exp(-i \hat{\theta}^{-j}) \right), \end{aligned}$$

$$\begin{split} E_{1t} + a_2 \hat{E}_{1\tilde{z}} + b_{21} \hat{E}_0 + b_{22} \hat{E}_1 + b_{23} \hat{E}_2 &= 0, \\ E_{2t} + a_3 \hat{E}_{2\tilde{z}} + b_{31} \hat{E}_0 + b_{32} \hat{E}_1 + b_{33} \hat{E}_2 &= 0 \end{split}$$

Code VOLC (Volume Code) for VFEL simulation

SVOLC -> VFEL simulation		
Menu Help		
6 6 6 5 ? %		
Wave length χ (cm) Current density j (A/cm ²) Lorenz-factor γ Target thickness L (cm) Time T (ns) Number of waves	3 Image: Second system 2200 Second system 2.17 Grid dimension 20 Nz 500 484 3 Np 200 N	
Geometry parameter $l_{0,1,2}$ Deviation from Cherenkov synchror	$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	
Fourier components of dielectric su χ_0 0.4 0 χ_{+1} 0.1 0 χ_{+2} 0.1 0 χ_{1-2} 0.1 0	sceptibility (complex): X ₋₁ 0.1 0 X ₋₂ 0.1 0 X ₂ 0.1 0 X ₂ 0.1 0 X ₂ 0.1 0	t, ns
Coupling coefficients in reflection :	VFEL simulation	
Amplitudes Ct 0 0 0 0 0 0 0 0 0 0 0 0	Phases φ 0 0 0 0 0 0 0 0 0 0 0 0	

Dispersion equation:

$$l_0 l_1 l_2 - l_0 r_{12} - l_1 r_2 - l_2 r_1 - \chi_1 \chi_{-2} \chi_{2-1} - \chi_2 \chi_{-1} \chi_{1-2} = 0$$

Two-root degeneration case:

$$\beta_1 \beta_2 l_1 l_2 + (\beta_1 l_1 + \beta_2 l_2) l_0 - \beta_1 \beta_2 r_{12} - \beta_1 r_1 - \beta_2 r_2 = 0$$

Three-root degeneration case: $\beta_1 l_1 + \beta_2 l_2 + l_0 = 0$





Three-root degeneration case



Periodic regime of VFEL intensity



Phase space portrait



References (2005)

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