On Numerical Methods for One Problem of Mixed Type

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Test problem:

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial z} + b\frac{\partial u}{\partial x} = 0, \qquad (1)$$

$$u(0, x, t) = u_0\ ,\ t > 0,$$
$$u(z, x, 0) = 0,\ 0 \le z \le L. \qquad (2)$$

From Maxwell's equations:

$$\Delta\mathbf{E} - \nabla(\nabla\mathbf{E}) - \frac{1}{c^2}\frac{\partial^2\mathbf{E}}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial\mathbf{j}_\mathrm{b}}{\partial t}$$

$\mathbf{E} = Ee^{i(\mathbf{kr}-\omega t)}$ – the electric field strength,

$\mathbf{j}_\mathrm{b} = je^{i(\mathbf{kr}-\omega t)}$ – the electron beam current density,

$\mathbf{k} = (k_x, 0, k_z)$ – the wave vector with the frequency ω.

$$-\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} + \frac{1}{c^2}\frac{\partial^2 E}{\partial z^2} + \frac{2i\omega}{c^2}\frac{\partial E}{\partial t} + 2ik_z\frac{\partial E}{\partial z} + 2ik_x\frac{\partial E}{\partial x} = \frac{4\pi}{c^2}\frac{\partial j}{\partial t} - \frac{4\pi i\omega}{c^2}j.$$

$$\frac{\partial E}{\partial z} + \frac{k_z c^2}{\omega}\frac{\partial E}{\partial z} + \frac{k_x c^2}{\omega}\frac{\partial E}{\partial x} = F(j) \qquad (3)$$

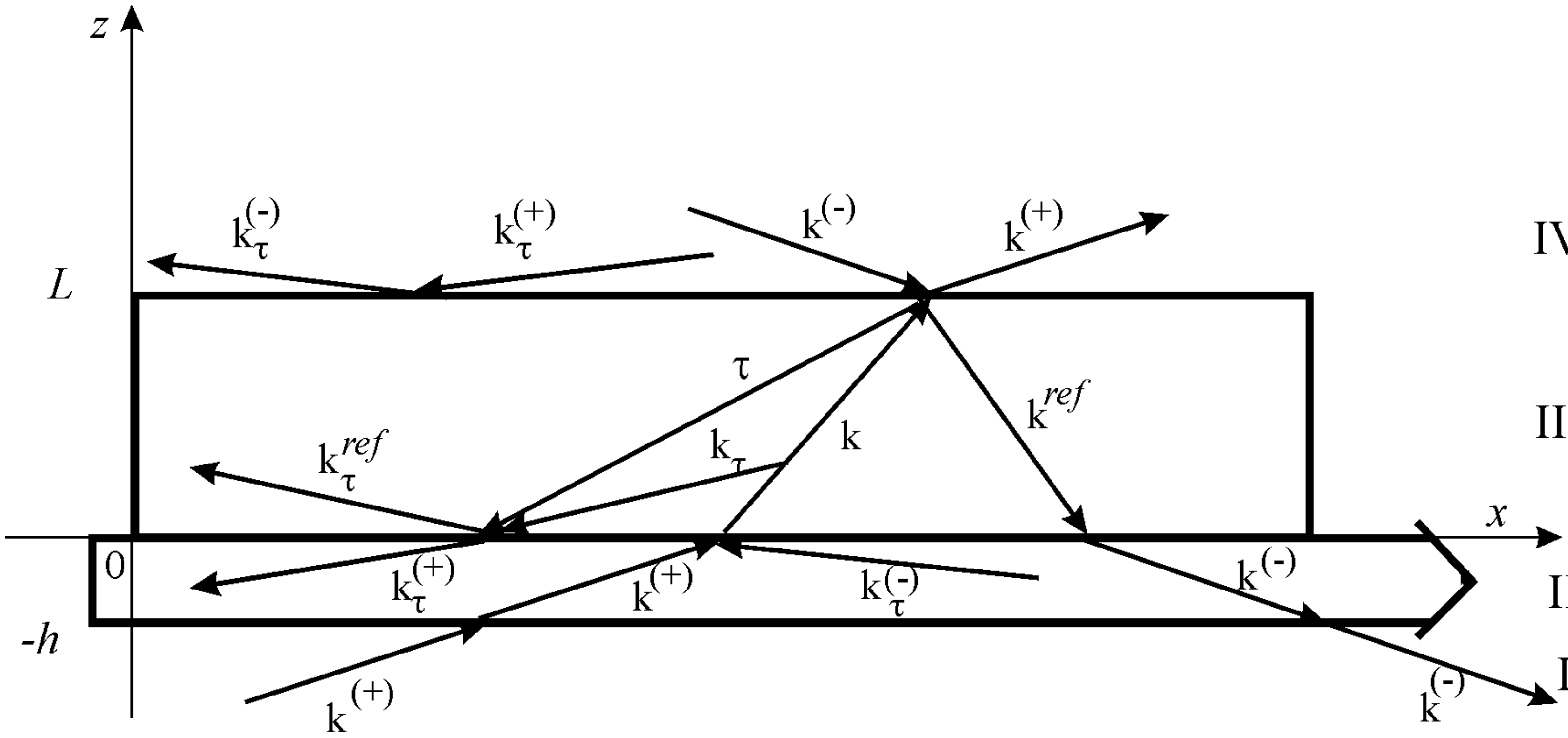


Fig. 1. Scheme of surface quasi-Cherenkov FEL

$$k_z = k_z' + ik_z''$$

$$\mathbf{E} = \mathrm{E}\, e^{(ik_z' z + k_x x - \omega t)}, \quad \mathrm{E} = E\, e^{-k_z'' z}, \quad k_z'' > 0.$$

In the vacuum, inside the particle beam:

$$\frac{\partial E}{\partial z}+\frac{k_z c^2}{\omega}\frac{\partial E}{\partial z}+\frac{k_x c^2}{\omega}\frac{\partial E}{\partial x}=F(j) \qquad (3)$$

Boundary conditions with respect to z
for waves with amplitudes E_1, E_2, E_3 :

$$A_l^- \frac{\partial E_l}{\partial t}+B_l^- \frac{\partial E_l}{\partial x}+C_l^- E_l+D_l E_3+$$

$$+A_k^+ \frac{\partial E_k}{\partial t}+B_k^+ \frac{\partial E_k}{\partial x}+C_k^+ E_k = f_k(x,t,E_k^{(0)}, j), \qquad (4)$$

$$l=2,1, \quad k=1,2$$

Elliptic problem:

$$\frac{\partial u}{\partial t} + a\frac{\partial u}{\partial z} = 0 \qquad (5)$$

$$u(0, x, t) = u_0\ ,\ t > 0,$$

$$u(z, x, 0) = 0,\ 0 \le z \le L.$$

First-order implicit scheme:

$$\frac{y_k^{j+1} - y_k^j}{\tau} + a\frac{y_k^{j+1} - y_{k-1}^{j+1}}{h} = 0 \qquad (6)$$

$$a = ia'',\ a'' > 0,\ i = \sqrt{-1}: \qquad \frac{\tau |a|}{h} \geq 2$$

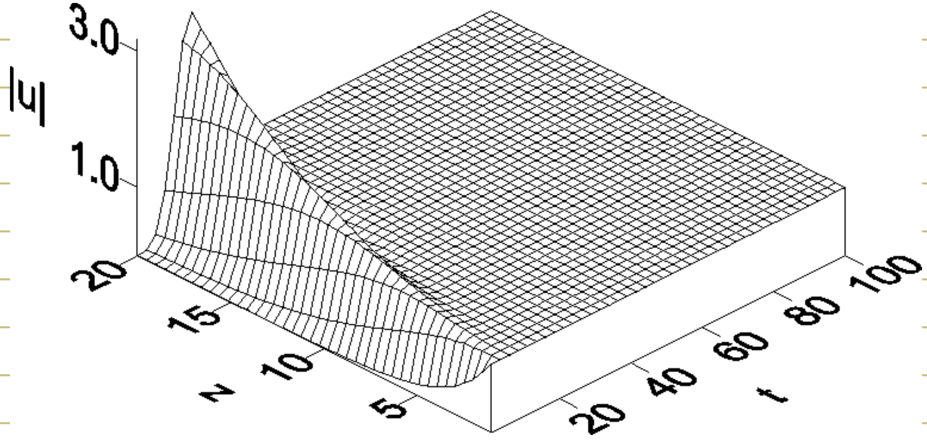
$$a = a' + ia'',\ a', a'' > 0: \qquad \frac{\tau |a|^2}{h} \geq 2a''$$

Weighted scheme:

$$\frac{y_k^{j+1} - y_k^j}{\tau} + \sigma a \frac{y_k^{j+1} - y_{k-1}^{j+1}}{h} + (1-\sigma) a \frac{y_k^j - y_{k-1}^j}{h} = 0$$

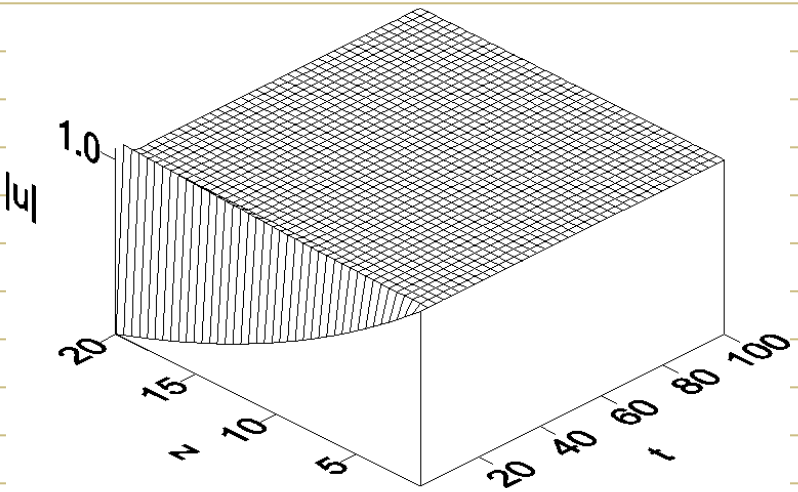
$$\frac{\tau |a|}{h} \geq \frac{2}{2\sigma - 1}, \qquad 0.5 < \sigma < 1$$

Fig.2. Numerical solution of (5) by scheme (6)



$$\frac{\tau |a|^2}{h} = 2a',$$

$$a = 1 + i$$



$$\frac{\tau |a|^2}{h} > 2a',$$

$$a = 10 + i \cdot 10$$

Numerical method for modelling of surface quasi-Cherenkov FEL:

$$\mathrm{E}_t{}^1 + \mathrm{A}_1\hat{\mathrm{E}}_z{}^1 + \mathrm{A}_2\mathrm{E}_x{}^2 = \mathrm{F}(\hat{j}),$$
$$\mathrm{E}_t{}^2 + \mathrm{A}_1\hat{\mathrm{E}}_z{}^1 + \mathrm{A}_2\hat{\mathrm{E}}_x{}^2 = \mathrm{F}(\hat{j}). \quad (8)$$

$$A_l^- E_{lt}^1 + B_l^- E_{lx}^2 + C_l^- \hat{E}_l^1 + D_l \hat{E}_3^1 + A_k^+ E_{kt}^1 + B_k^+ E_{kx}^2 + C_k^+ \hat{E}_k^1 = f(\hat{E}_k^{(0)}, \hat{j}),$$
$$l = 1, 2, k = 2, 1;$$
$$A_l^- E_{lt}^2 + B_l^- \hat{E}_{lx}^2 + C_l^- \hat{E}_l^1 + D_l \hat{E}_3^1 + A_k^+ E_{kt}^1 + B_k^+ E_{kx}^2 + C_k^+ \hat{E}_k^1 = f(\hat{E}_k^{(0)}, \hat{j}),$$
$$l = 1, k = 2,$$
$$A_l^- E_{lt}^1 + B_l^- E_{lx}^2 + C_l^- \hat{E}_l^1 + D_l \hat{E}_3^1 + A_k^+ E_{kt}^2 + B_k^+ \hat{E}_{kx}^2 + C_k^+ \hat{E}_k^1 = f(\hat{E}_k^{(0)}, \hat{j}),$$
$$l = 2, k = 1.$$

(9)

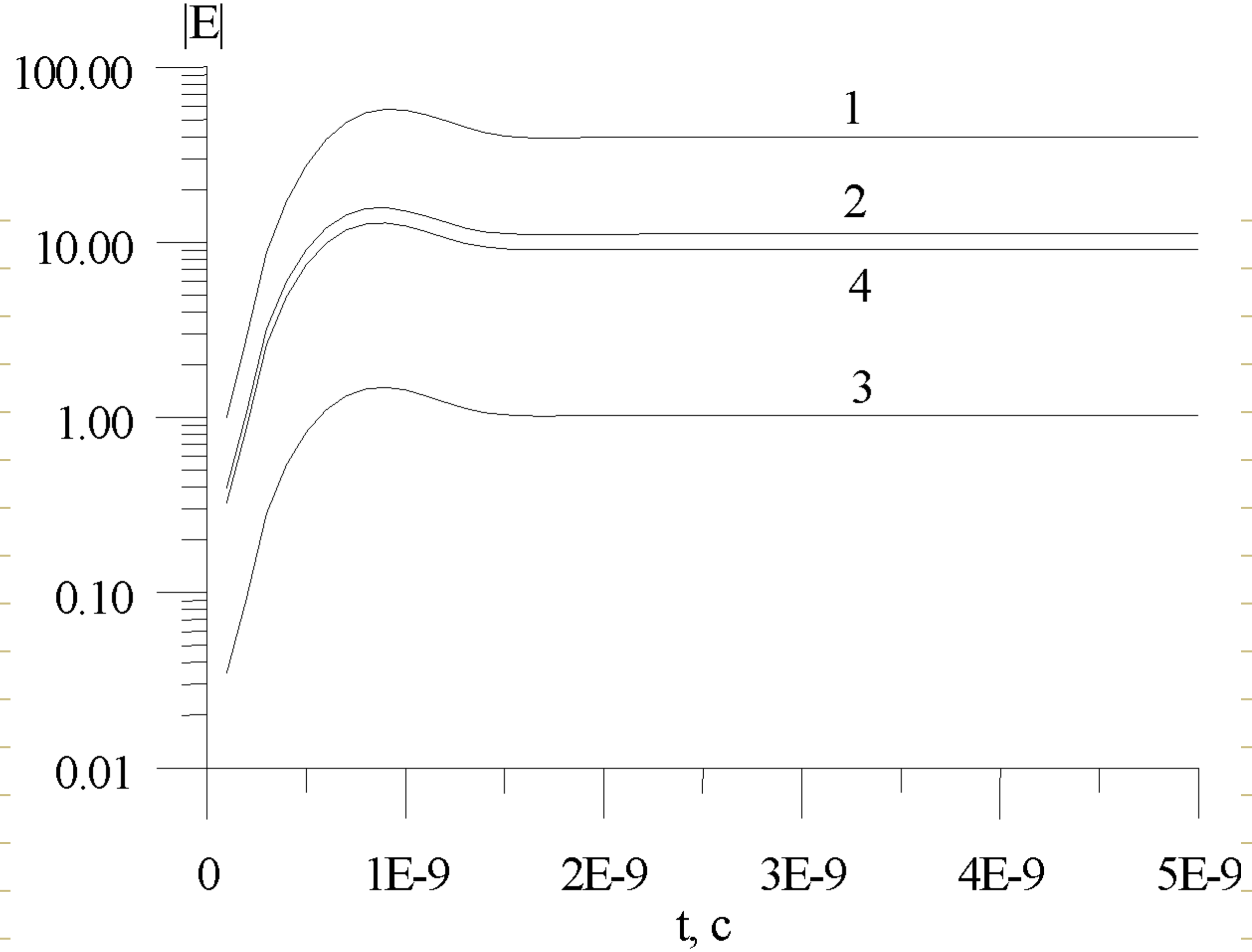


Fig.3.Amplification of electromagnetic fields as a function of time in visible surface quasi-Cherenkov FEL

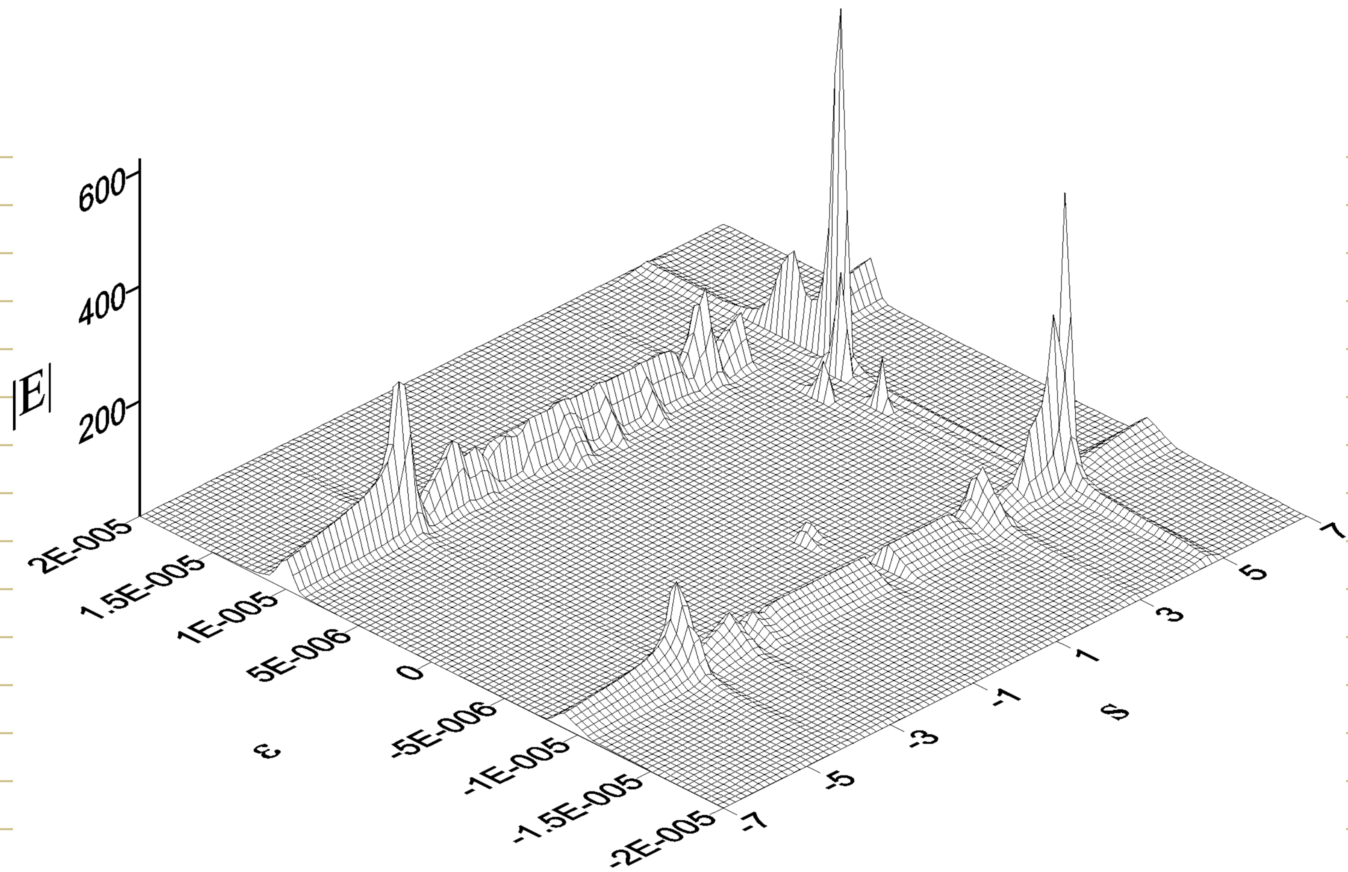


Fig.4. Attempt of optimisation of the amplification process in visible surface quasi-Cherenkov FEL

References

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