Volume Free Electron Laser (VFEL) as a Dynamical System

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At the present time Volume Free Electron Lasers (VFEL) with volume distributed feedback (VDFB) are developed actively. It was shown that VFEL is a dynamical system that has different bifurcation points depending on various VDBF and other VFEL parameters. Nonlinear phenomena originating in VFEL are investigated by methods of mathematical modelling using computer code VOLC. Parametric map for beam current and detuning parameter from exact Cherenkov condition presents complicated root to chaos in two-wave VFEL in Bragg geometry and oscillator generation regime. This root is carried out via period-doubling cascade and quasiperiodicity.

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1. Introduction

First lasing of Volume Free Electron Lasers (VFEL) in mm wavelength range was obtained recently [1]. So-called volume multi-wave distributed feedback where electromagnetic waves and electron beam spread angularly one to other is the distinctive feature of VFEL. The principles and theoretical foundations of VFEL operation based on mechanism of multi-wave VDFB were proposed in [2]. There it was shown that the increment of instability for an electron beam passing through a spatially-periodic target in degeneration points essentially increased in comparison with single-wave system. This means the noticeable reduction of electron beam current density necessary for achievement the generation threshold for all wavelength ranges regardless the spontaneous radiation mechanism. Prototypes of VFEL based on induced radiation in three-dimensional periodical structures were investigated in [2] and [3]. In VFEL operation the linear stage investigated in [2] - [4] quickly changes into the nonlinear one where most of the electron beam energy is transformed into electromagnetic radiation. A detailed numerical analysis of this stage is necessary for experiment design, optimal geometry determination and result processing. VFEL experimental investigations [1], [5] validate theoretical foundations.

Mathematical model and numerical methods for VFEL modelling were proposed [6], [7]. They are implemented in computer code VOLC [8] with dimensionality 2D (one spatial coordinate and one phase space coordinate) plus time. Different VFEL geometries were investigated [6]–[10].

Chaotic dynamics means the tendency of a wide range of systems to transition between different states with deterministic periodic and nonperiodic behavior. Under modern concept of deterministic chaos [11] dynamical system under changing of external control parameter gives series of bifurcations leading to complication of self-oscillations right up to stochastic oscillations with continuous spectrum. Examples of such nonlinear systems are nonlinear optical devices, lasers, particle accelerators, free electron lasers (FEL) etc.

In electronic generators and amplifiers such

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as VFEL, FEL, BWT (backward wave tube), TWB (travelling wave tube) self-oscillations are due to interaction of electron beam and electromagnetic field under distributed feedback. Investigation of chaos in such devices is of great interest in modern physics [12]–[17].

Usually a system of equation describing chaotic behavior in electronic devices has the following generalized form [16]:

$$\frac{\partial F}{\partial \tau} - \frac{\partial F}{\partial \zeta} = \frac{1}{\pi}\int_0^{2\pi} \exp(-i\theta) d\theta_0,$$

$$\frac{\partial^2 \theta}{\partial \zeta^2} = -\mathrm{Re}\left(F \exp(i\theta)\right), \qquad (1)$$

$$\theta(\zeta = 0) = \theta_0, \quad \frac{\partial \theta}{\partial \zeta}(\zeta = 0) = 0, \quad F(\zeta = L) = 0.$$

Equations (1) are versatile in the sense that they remain the same within some normalization for a wide range of electronic devices (FEL, BWT, TWB etc).

Three main routes to chaos for nonlinear systems under control parameter changing are the following [11]: period doubling, quasiperiodicity and intermittency. In [12] it was shown that in FEL there exists many other routes to chaos in a complicatedly combined way of these routes.

Under simulation different chaotic regimes originating in VFEL were examined [8]–[10]. It was shown [7] that changing of control parameters j and l leads to transition between different periodical regimes. It is very interesting to investigate route to chaos in VFEL more profoundly.

2. VFEL schemes and mathematical formulation

As was shown [18] VFEL resonator of the experimental installation [1] and volume resonator (so-called "grid" volume resonator) of the installation [5] can be reduced to the following simple scheme of VFEL (see Fig.1) by recounting of dielectric susceptibility of the target. Here an electron beam with electron velocity **u** passes through spatially periodic target. When Bragg conditions are fulfilled two strong waves can be excited in the target. If simultaneously electrons of the beam are under Cherenkov condition, they emit electromagnetic radiation in directions depending on diffraction regime. Case without incident electromagnetic waves corresponds to oscillator generation regime. In previous work [6]-[10] other schemes of VFEL such as two-wave Laue geometry, three-wave Bragg-Bragg, Bragg-Laue and Laue-Laue geometry were considered.

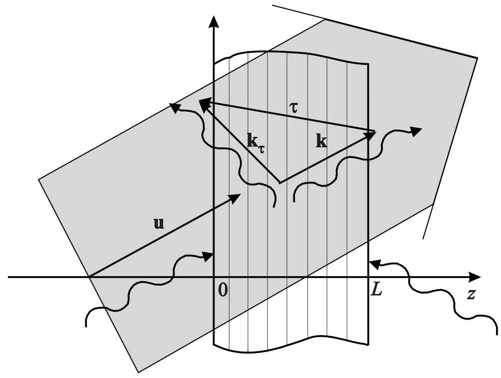

FIG. 1. Scheme of two-wave VFEL in Bragg geometry.

The system of equations for all cases of VFEL is obtained from Maxwell equations in the slowly-varying envelope approximation using the field representation in the form $\mathbf{E} = \mathbf{e}_i E_i \exp\{i\mathbf{k}_{\tau_i}\mathbf{r} - \omega t\}$, $i = 0, ..., n-1$. In this paper two-wave VDFB will be considered. The system for n-wave VDFB can be written by evident generalization.

$$\frac{\partial E}{\partial t} + \gamma_0 c \frac{\partial E}{\partial z} + 0.5 i\omega l E - 0.5 i\omega \chi_\tau E_\tau =$$

$$2\pi j \Phi \int_0^{2\pi} \frac{2\pi - p}{8\pi^2} \left(\exp(-i\Theta(t,z,p) + \exp(-i\Theta(t,z,-p))\right) dp, \tag{2}$$

$$\frac{\partial E_\tau}{\partial t} + \gamma_1 c \frac{\partial E_\tau}{\partial z} + 0.5 i\omega \chi_{-\tau} E - 0.5 i\omega l_1 E = 0.$$

Here $l_i = \left(k_{\tau_i}^2 c^2 - \omega^2 \varepsilon_0\right)/\omega^2$, $i = 0, 1$, $l = l_0 + \delta$. δ is detuning from exact Cherenkov condition. γ_0, γ_1 are VDFB cosines. $\Phi = \sqrt{l_0 + \chi_0 - 1/(\beta\gamma)^2}$. $\chi_{\pm\tau}$ are Fourier components of the dielectric susceptibility of the target.

System (2) must be supplemented with proper initial and boundary conditions as well as equations for the phase dynamics:

$$\frac{d^2\Theta(t,z,p)}{dz^2} = \frac{e\Phi}{m\gamma^3\omega^2} \left(k_z - \frac{d\Theta(t,z,p)}{dz}\right)^3 \mathrm{Re}\left(E \exp(i\Theta(t,z,p)\right), \tag{3}$$

$$\frac{d\Theta(t,0,p)}{dz} = k_z - \omega/u, \quad \Theta(t,0,p) = p.$$

The integral form of beam current in the right hand side of (3) is obtained by averaging over the following initial phases of electrons in the beam: entrance time of electron in interaction zone and transverse coordinate of entrance point in interaction zone. Method of averaging over initial phases of electrons is well-known [19] and widely used in simulation of BWT, TWB, FEL and other electronic devices. Equations (2)–(3) are more complicated than usually used (1) but they allow to simulate electron beam dynamics more precisely.

3. Numerical results

Let us consider results of mathematical modelling of chaotic regimes in VFEL. Analytical investigation of chaos in the system (2)–(3) seems to be impossible because of its strong nonlinearity. There exists a wide range of external control parameters such as beam current, target length, VDFB cosines, target absorption, diffraction asymmetry factors, tuning parameter, system parameters l_i etc. Electron beam moving though spatially-periodic target in VFEL leads to a diversity of features of generation dynamics that is due to nonlocal nature of interaction between electron beam and electromagnetic field under VDFB. Since VFEL is a nonlinear distributed dynamical system it is characterized by some type of generation regimes such as stationary state, periodicity and chaos.

Let us restrict ourselves here to investigate the chaotic behavior in two-dimensional system for Bragg geometry in oscillator generation regime with respect to two parameters: current density j and tuning parameter δ. Firstly let us consider examples of different regimes obtained

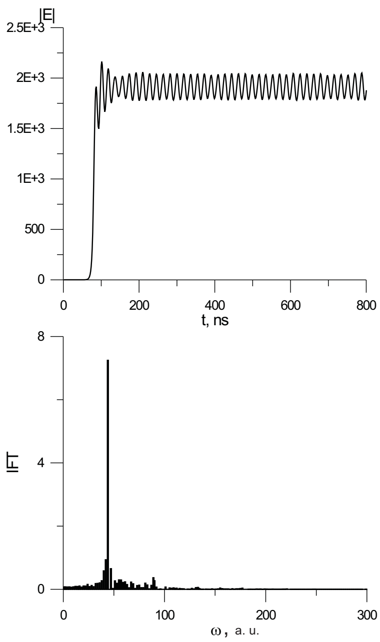

FIG. 2. Periodic regime (a) for transmitted wave and (b) its intensity Fourier transform

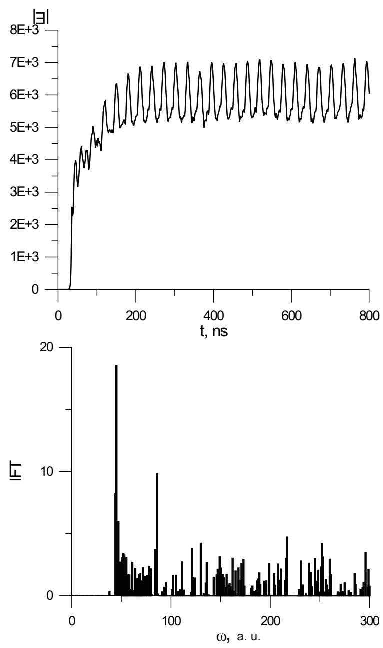

FIG. 4. "Weak" chaotic regime (a) for transmitted wave and (b) its intensity Fourier transform

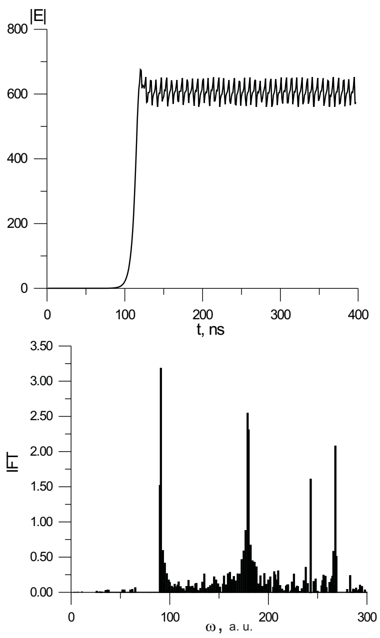

FIG. 3. Quasiperiodic regime (a) for transmitted wave and (b) its intensity Fourier transform

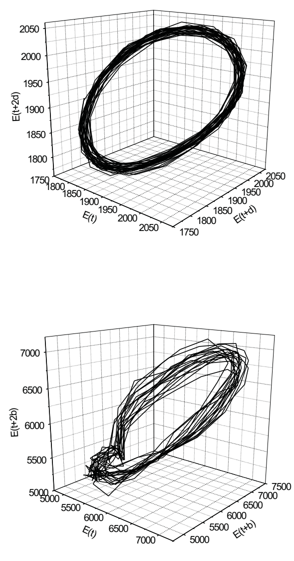

FIG. 5. Attractor (a) for periodic and (b) "weak" chaotic regime

and depicted in Fig.2–Fig.7. Some input parameters are the following: λ =3 cm, L = 20 cm, $j = 500 \div 3200\text{A/cm}^2$, $\beta = -10$, $\delta kL = -20 \div 20$, $l_0 = 1.0$, $\chi_0 = 0.1$.

In Fig.2 the typical 1T periodic regime is shown together with its intensity Fourier trans-

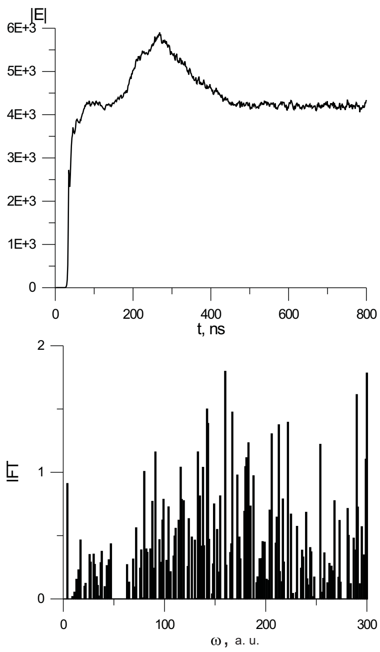

FIG. 6. Chaotic self-oscillations (a) for transmitted wave and (b) its intensity Fourier transform

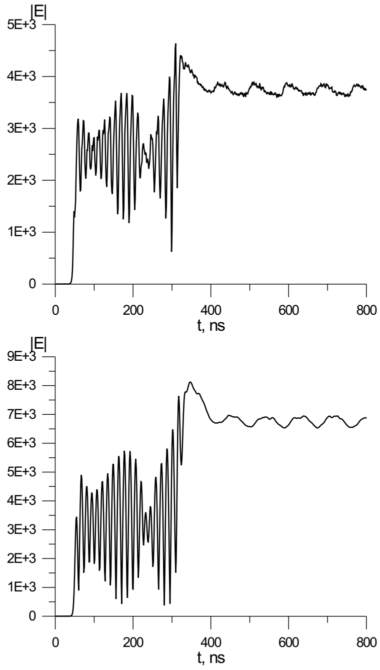

FIG. 7. Transition between two modes in VFEL (a) for transmitted and (b) diffracted wave

form. In some domains of the parametric map (j, l) this period becomes doubled and quadru-

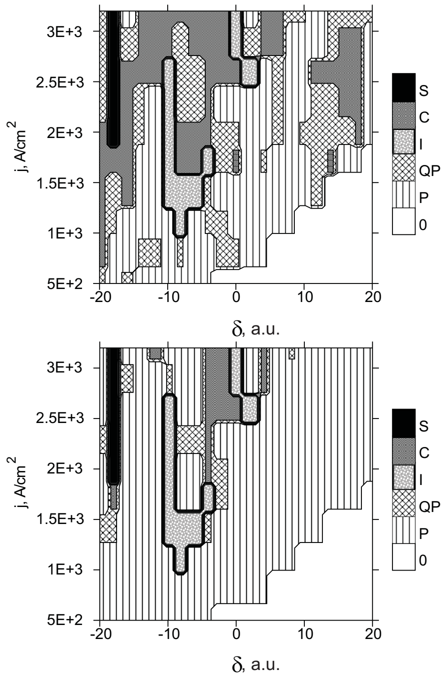

FIG. 8. Root to chaos (a) for transmitted and (b) diffracted wave. 0 means that the beam current is under threshold current. P – periodic regimes 1T, 2T etc. QP — quasiperiodicity. I — transition between two mode. C — weak chaos. S — stochastic self-oscillations.

pled.

Other typical regime is quasiperiodical oscillations with incommensurate frequencies originating in Hopf bifurcation. It is demonstrated in Fig.3. In some domains of the map (j, l) this regime goes to so-called "weak" chaos [16] where dependance of amplitude in time seems as approximate repetition of equitype spikes close in dimensions per approximately equal time space. This is illustrated in Fig.4. Corresponding to Fig.2 and Fig.4 attractors can be seen in Fig.5. "Weak" chaos and quasiperiodicity goes in one domain of the parametric map to stochastic self-oscillations presented in Fig.6.

In multi-wave VFEL as in dynamical distributed system some modes for each electromagnetic wave are excited. Moreover VFEL theory allows to adjust generation to more optimal region with root degeneration where all modes are in synchronism with electron beam and interaction occurs more intensively [4]. In simulation carried out the single mode generation was considered and it was founded some domains where oscillation at starting mode was unstable and the second mode were excited. This is induced by the nonlinear mode competition mechanism. After mode transition periodic, quasiperiodic or chaotic regimes are established. Example of such scenario of VFEL operation is proposed in Fig.7. Analogous regime was obtained [14] for FEM (free electron maser) oscillator with a Bragg resonator.

So, in conclusion a root to chaos in the form of the parametric map (j, δ) where δ is normalized with respect to kL is presented in Fig.8. It is well seen that in the middle of different regimes of bifurcations there are a lot of window of periodicity both for transmitted and diffracted waves. Increase of beam current density does not lead to chaos automatically. Larger number of main frequencies for transmitted wave (compare Fig.8 (a) and (b)) can be explained by the fact that in VFEL it exists the possibility to obtain generation of electron beam at several frequencies [5]. In simulation Cherenkov conditions are fulfilled namely for transmitted wave. Unfortunately, because of enormous computational work parametric map is rough and it is not possible to precise more accurately limits of transition between regimes "period doubling – chaos" and "quiasiperiodicity – chaos".

4. Conclusion

An instrument to model real VFEL experiments was developed. Mathematical model with computer code VOLC allows to obtain all main VFEL physical dependencies and to investigate the nonlinear stage of its operation. As more than ten control parameters are in the system it is very complicated to investigate the full picture of possible chaotic behavior in the system. But parametric map (j,δ) of root to chaos presents intricate root to chaos via period-doubling cascade and quasiperiodicity.

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