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MULTICOMPONENT ITERATIVE METHOD FOR SOLVING TWO-DIMENSIONAL HEAT TRANSFER EQUATION ON MOVING GRIDS

S. SYTOVA

Institute for Nuclear Problems, Belarus State University
Bobruiskaya 11, Minsk, 220050, Republic of Belarus
E-mail: sytova@inp.minsk.by

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ABSTRACT

A multicomponent iterative method of domain decomposition on adaptive grids for solution of two-dimensional heat transfer equation is proposed. The adaptive grid is constructed in curvilinear space where Cartesian grid is non-stationary and depends on the solution behavior. In curvilinear space the initial two-dimensional heat transfer equation is converted to the system of nonlinear parabolic equations with mixed derivatives, a source and convective transfer.

1. INTRODUCTION

The heat transfer equation is one of the basic models of mathematical physics. It can be considered as a test problem for construction of new difference methods for solution of modern scientific problems. Such problems have many dimensions, complicated geometry, different types of nonlinearities.

Numerical and analytical investigations show that application of adaptive grids can increase appreciably the accuracy of numerical algorithms and reduce the grid dimension without the loss of solution accuracy. Adaptive grids allow us to eliminate oscillations, simulated viscosity. Such grids are very effective for solving multidimensional non-stationary problems with domains of strong variation of the solution. The theory of adaptive grids is developed now very intensively [10]-[15].

It was shown [1]-[3] that the multicomponent approach can be effectively used for the construction of numerical algorithms. The idea of combination of multicomponent alternating direction method and domain decomposition technique allow us to create a new class of numerical algorithms for different types of problems of mathematical physics, including elliptic and parabolic problems [5]-[9], [16], [17]. These algorithms are unconditionally stable as implicit schemes and are explicit in realization. They have the rate of convergence and computational costs similar to well-known explicit algorithms.

The main idea of this article is to combine adaptive grid technique with multicomponent domain decomposition method for solving a two-dimensional parabolic equation. The adaptive grid is constructed in curvilinear space where the Cartesian grid is non-stationary and moves to domains of strong variation of the solution. It is clear that the full mathematical model has to be supplemented by differential equations which describe the dynamic of Cartesian grid. So, in curvilinear space the full system to be solved is a system of nonlinear parabolic equations with mixed derivatives, a source and convective transfer. Proposed implicit finite difference schemes are nonlinear too. For their effective solution a domain decomposition method is used in the form suggested in [8]. For difference schemes of this method a multicomponent iterative algorithm of decomposition type is proposed. The domain is divided into minimal four-point sub-domains. Presented algorithms allow to obtain independent solution of initial problem in each of sub-domains. This can be important for working on multiprocessor computers. The number of equations which should be solved in such a sub-domain is equal to the number of pattern points. Detailed numerical studies confirmed efficiency of the proposed numerical methods.

2. STATEMENT OF THE PROBLEM

In an arbitrary domain G with a boundary Γ we consider a two-dimensional parabolic equation :

$$\frac{\partial \mathrm{u}}{\partial t} = \frac{\partial}{\partial \mathrm{x}}\left(k\frac{\partial \mathrm{u}}{\partial \mathrm{x}}\right) + \frac{\partial}{\partial \mathrm{y}}\left(k\frac{\partial \mathrm{u}}{\partial \mathrm{y}}\right) + \mathrm{g}, \quad k = k(\mathrm{x},\mathrm{y}) > 0, \qquad (2.1)$$

$$\mathrm{u}(\mathrm{x},\mathrm{y},0) = \mathrm{u}_0(\mathrm{x},\mathrm{y}), \quad (\mathrm{x},\mathrm{y}) \in G,$$

$$\mathrm{u}(\mathrm{x},\mathrm{y},t) = \mathrm{u}_1(\mathrm{x},\mathrm{y},t), \quad (\mathrm{x},\mathrm{y}) \in \Gamma, \quad t > 0. \qquad (2.2)$$

After a non-singular coordinates transformation of the general type:

$$z_1 = z_1(\mathrm{x},\mathrm{y},t), \quad z_2 = z_2(\mathrm{x},\mathrm{y},t), \quad \tau = t \qquad (2.3)$$

the domain G is converted to the rectangular domain Ω (see Fig.1).

Equation (2.1) takes the form:

$$\frac{\partial(\Psi \mathrm{u})}{\partial \tau} = L\mathrm{u} + I\mathrm{u} + \Psi \mathrm{g}, \qquad (2.4)$$

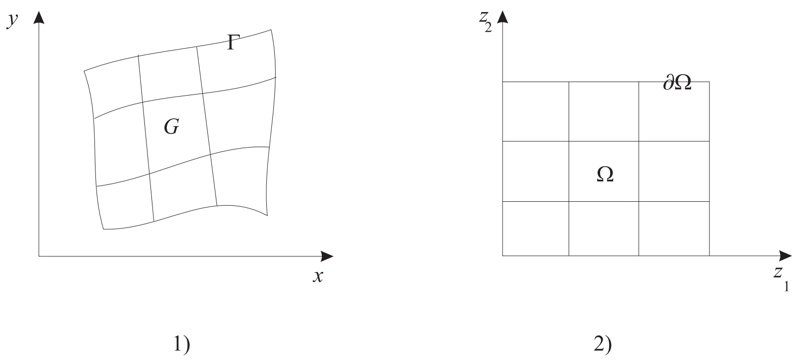

Figure 1. Domain G and domain Ω2) after transformation (2.3).

$$Lu = \sum_{\alpha,\beta=1}^{2} \frac{\partial}{\partial z_\alpha}\left(a_{\alpha\beta}\frac{\partial u}{\partial z_\beta}\right), \quad Iu = \sum_{\alpha=1}^{2} \frac{\partial}{\partial z_\alpha}(c_\alpha u),$$

$$a_{\alpha\alpha} = \frac{k}{\Psi}\left(\left(\frac{\partial x}{\partial z_\alpha}\right)^2 + \left(\frac{\partial y}{\partial z_\alpha}\right)^2\right), \quad \alpha = 1, 2,$$

$$a_{12} = a_{21} = -\frac{k}{\Psi}\left(\frac{\partial x}{\partial z_1}\frac{\partial x}{\partial z_2} + \frac{\partial y}{\partial z_1}\frac{\partial y}{\partial z_2}\right),$$

$$c_1 = \frac{\partial x}{\partial \tau}\frac{\partial y}{\partial z_2} - \frac{\partial y}{\partial \tau}\frac{\partial x}{\partial z_2}, \quad c_2 = \frac{\partial y}{\partial \tau}\frac{\partial x}{\partial z_1} - \frac{\partial x}{\partial \tau}\frac{\partial y}{\partial z_1},$$

$$\Psi = \frac{\partial x}{\partial z_1}\frac{\partial y}{\partial z_2} - \frac{\partial x}{\partial z_2}\frac{\partial y}{\partial z_1}, \tag{2.5}$$

$$\frac{\partial x}{\partial \tau} = D_1\frac{\partial^2 x}{\partial z_1^2} + D_2\frac{\partial^2 x}{\partial z_2^2} + C_1\Psi\frac{\partial}{\partial z_1}\left(\Psi^2\left(\left(\frac{\partial u}{\partial z_1}\right)^2 + \left(\frac{\partial u}{\partial z_2}\right)^2\right)^{1/2}\right), \tag{2.6}$$

$$\frac{\partial y}{\partial \tau} = D_1\frac{\partial^2 y}{\partial z_1^2} + D_2\frac{\partial^2 y}{\partial z_2^2} + C_2\Psi\frac{\partial}{\partial z_2}\left(\Psi^2\left(\left(\frac{\partial u}{\partial z_1}\right)^2 + \left(\frac{\partial u}{\partial z_2}\right)^2\right)^{1/2}\right). \tag{2.7}$$

Initial and boundary conditions (2.2) have the form:

$$\begin{aligned} u(z_1, z_2, 0) &= u_0(z_1, z_2), \quad (z_1, z_2) \in \Omega, \\ u(z_1, z_2, \tau) &= u_1(z_1, z_2, \tau), \quad (z_1, z_2) \in \partial\Omega, \quad \tau > 0. \end{aligned} \tag{2.8}$$

D_1, D_2, C_1, C_2 are positive constants of the order of $O(1)$.

Equations (2.6)-(2.7) describe the dynamics of rectangular Cartesian coordinates. They correspond to a grid with nodes that are collected in domain of strong variation of solution. Such form of adaptive grid was suggested in [11]. The type of (2.6) – (2.7) depends on concrete situation. One can take more simple equations that describe quasi-uniform grids [11]:

$$\frac{\partial \mathrm{x}}{\partial \tau} = D_1 \frac{\partial^2 \mathrm{x}}{\partial z_1^2} + D_2 \frac{\partial^2 \mathrm{x}}{\partial z_2^2}, \quad \frac{\partial \mathrm{y}}{\partial \tau} = D_1 \frac{\partial^2 \mathrm{y}}{\partial z_1^2} + D_2 \frac{\partial^2 \mathrm{y}}{\partial z_2^2}. \tag{2.9}$$

Equations (2.6) – (2.7) or (2.9) are equations of the transformation inverse for introduced above (2.3):

$$\mathrm{x} = \mathrm{x}(z_1, z_2, \tau), \quad \mathrm{y} = \mathrm{y}(z_1, z_2, \tau), \quad t = \tau.$$

Since we have supposed that transformation (2.3) is non-singular:

$$\frac{\partial z_1}{\partial \mathrm{x}} \frac{\partial z_2}{\partial \mathrm{y}} - \frac{\partial z_2}{\partial \mathrm{x}} \frac{\partial z_1}{\partial \mathrm{y}} > 0,$$

The Jacobian of the inverse transformation Ψ (2.5) will be positive too.

In this case the positive determinacy of the symmetric matrix $A = \{a_{\alpha\beta}\}$ takes place. And for operator L the strong ellipticity condition takes place:

$$b_1 \sum_{\alpha=1}^{2} \xi_\alpha^2 \le \sum_{\alpha,\beta=1}^{2} a_{\alpha\beta} \xi_\alpha \xi_\beta \le b_2 \sum_{\alpha=1}^{2} \xi_\alpha^2, \tag{2.10}$$

where $0 < b_1 < b_2$ are bounded constants and $(\xi_1, \xi_2)^T$ is an arbitrary nonzero real vector.

As a result of transformation (2.3) the initial equation (2.1) is converted to the system of nonlinear parabolic equations with mixed derivatives, a source and convective transfer that is determined by the coefficients c_1, c_2.

Let us transform the equations for adaptive grid (2.6) – (2.7) by differentiating over z_1 and z_2:

$$\frac{\partial \mathrm{x}}{\partial \tau} = D_1 \frac{\partial^2 \mathrm{x}}{\partial z_1^2} + D_2 \frac{\partial^2 \mathrm{x}}{\partial z_2^2} + \Phi_1 \mathrm{u}, \tag{2.11}$$

$$\frac{\partial \mathrm{y}}{\partial \tau} = D_1 \frac{\partial^2 \mathrm{y}}{\partial z_1^2} + D_2 \frac{\partial^2 \mathrm{y}}{\partial z_2^2} + \Phi_2 \mathrm{u}, \tag{2.12}$$

where

$$\Phi_1 \mathrm{u} = C_1 \Psi^2 \left(2\varphi_1 (\mathrm{f}(\mathrm{u}))^{1/2} + \Psi (\mathrm{f}(\mathrm{u}))^{-1/2} \left(\frac{\partial \mathrm{u}}{\partial z_1} \frac{\partial^2 \mathrm{u}}{\partial z_1^2} + \frac{\partial \mathrm{u}}{\partial z_2} \frac{\partial^2 \mathrm{u}}{\partial z_1 \partial z_2} \right) \right),$$

$$\Phi_2 \mathrm{u} = C_2 \Psi^2 \left(2\varphi_2 (\mathrm{f}(\mathrm{u}))^{1/2} + \Psi (\mathrm{f}(\mathrm{u}))^{-1/2} \left(\frac{\partial \mathrm{u}}{\partial z_1} \frac{\partial^2 \mathrm{u}}{\partial z_1 \partial z_2} + \frac{\partial u}{\partial z_2} \frac{\partial^2 \mathrm{u}}{\partial z_2^2} \right) \right),$$

$$\mathrm{f}(\mathrm{u}) = \left(\frac{\partial \mathrm{u}}{\partial z_1} \right)^2 + \left(\frac{\partial \mathrm{u}}{\partial z_2} \right)^2,$$

$$\varphi_1 = \frac{\partial \Psi}{\partial z_1} = \frac{\partial^2 \mathrm{x}}{\partial z_1^2} \frac{\partial \mathrm{y}}{\partial z_2} + \frac{\partial x}{\partial z_1} \frac{\partial^2 \mathrm{y}}{\partial z_1 \partial z_2} - \frac{\partial^2 x}{\partial z_1 \partial z_2} \frac{\partial \mathrm{y}}{\partial z_1} - \frac{\partial x}{\partial z_2} \frac{\partial^2 \mathrm{y}}{\partial z_1^2},$$

$$\varphi_2 = \frac{\partial \Psi}{\partial z_2} = \frac{\partial^2 \mathrm{x}}{\partial z_1 \partial \mathrm{x}_2} \frac{\partial \mathrm{y}}{\partial z_2} + \frac{\partial \mathrm{x}}{\partial z_1} \frac{\partial^2 \mathrm{y}}{\partial z_2^2} - \frac{\partial^2 x}{\partial z_2^2} \frac{\partial \mathrm{y}}{\partial z_1} - \frac{\partial x}{\partial z_2} \frac{\partial^2 \mathrm{y}}{\partial z_1 \partial z_2}.$$

3. FINITE DIFFERENCE SCHEMES

Let us write down finite difference schemes for nonlinear system (2.4) – (2.5), (2.11) – (2.12). Let us introduce uniform grids in the domain Ω:

$$\omega_h = \{(z_{i_1}, z_{i_2}), z_{i_1} = i_1 h_1,\, i_1 = \overline{0, N_1},\, N_1 = [L_1/h_1], z_{i_2} = i_2 h_2,$$

$$i_2 = \overline{0, N_2},\, N_2 = \lfloor L_2/h_2 \rfloor \},$$

and the time grid: $\omega_\tau = \{\tau_j = j h_\tau,\, j = \overline{0, N_\tau},\, N_\tau = \lfloor \mathrm{T}/h_\tau \rfloor \}$.

We use the notation from [14]. The finite difference schemes for adaptive grid equations are given in the form:

$$x_\tau = D_1 \widehat{x}_{\overline{z}_1 z_1} + D_2 \widehat{x}_{\overline{z}_2 z_2} + \Phi_1 u, \tag{3.1}$$

$$y_\tau = D_1 \widehat{y}_{\overline{z}_1 z_1} + D_2 \widehat{y}_{\overline{z}_2 z_2} + \Phi_2 u, \tag{3.2}$$

$$\Phi_1 u = C_1 \Psi^2 \left(2\phi_1 f^{1/2} + \Psi f^{-1/2} \left(u_{\overset{\circ}{z}_1} u_{\overline{z}_1 z_1} + 0.5 u_{\overset{\circ}{z}_2} (u_{\overline{z}_1 z_2} + u_{z_1 \overline{z}_2}) \right) \right),$$

$$\Phi_2 u = C_2 \Psi^2 \left(2\phi_2 f^{1/2} + \Psi f^{-1/2} \left(u_{\overset{\circ}{z}_2} u_{\overline{z}_2 z_2} + 0.5 u_{\overset{\circ}{z}_1} (u_{\overline{z}_1 z_2} + u_{z_1 \overline{z}_2}) \right) \right),$$

$$\phi_1 = x_{\overline{z}_1 z_1} y_{\overset{\circ}{z}_2} + 0.5 x_{\overset{\circ}{z}_1} (y_{\overline{z}_1 z_2} + y_{z_1 \overline{z}_2}) - 0.5 (x_{\overline{z}_1 z_2} + x_{z_1 \overline{z}_2}) y_{\overset{\circ}{z}_1} - x_{\overset{\circ}{z}_2} y_{\overline{z}_1 z_1},$$

$$\phi_2 = 0.5 (x_{\overline{z}_1 z_2} + x_{z_1 \overline{z}_2}) y_{\overset{\circ}{z}_2} + x_{\overset{\circ}{z}_1} y_{\overline{z}_2 z_2} - x_{\overline{z}_2 z_2} y_{\overset{\circ}{z}_1} - 0.5 x_{\overset{\circ}{z}_2} (y_{\overline{z}_1 z_2} + y_{z_1 \overline{z}_2}),$$

$$f = (u_{\overset{\circ}{z}_1})^2 + (u_{\overset{\circ}{z}_2})^2.$$

The following system of difference equations can be written for (2.4) – (2.5):

$$(\Psi u)_\tau = A \widehat{u} + \widehat{\Psi} \widehat{g}, \tag{3.3}$$

$$Au = \frac{1}{2} \sum_{\alpha,\beta=1}^{2} \Big(\big(a_{\alpha\beta} u_{\overline{z}_\beta}\big)_{z_\alpha} + \big(a_{\alpha\beta} u_{z_\beta}\big)_{\overline{z}_\alpha} \Big) + \sum_{\alpha=1}^{2} (c_\alpha u)_{\mathring{z}_\alpha}, \tag{3.4}$$

$$a_{\alpha\alpha} = \frac{k}{\Psi} \Big((x_{\mathring{z}_\beta})^2 + (y_{\mathring{z}_\beta})^2 \Big), \quad \alpha, \beta = 1, 2, \quad \beta \neq \alpha,$$

$$a_{12} = a_{21} = -\frac{k}{\Psi} \Big(x_{\mathring{z}_1} x_{\mathring{z}_2} + y_{\mathring{z}_1} y_{\mathring{z}_2} \Big),$$

$$c_1 = x_\tau y_{\mathring{z}_2} - y_\tau x_{\mathring{z}_2}, \quad c_2 = y_\tau x_{\mathring{z}_1} - x_\tau y_{\mathring{z}_1},$$

$$\Psi = x_{\mathring{z}_1} y_{\mathring{z}_2} - x_{\mathring{z}_2} y_{\mathring{z}_1}. \tag{3.5}$$

Schemes (3.1) – (3.5) approximate the initial problem (2.4) – (2.5), (2.11) – (2.12) with the order $O(h_\tau + h_1^2 + h_2^2)$. Positivity of the operators in equations (2.4)-(2.5), (2.11)-(2.12) guarantees the stability and the convergence of difference schemes (3.1)–(3.5). This can be proved by the energy inequalities method [14].

4. DOMAIN DECOMPOSITION METHOD

In order to solve nonlinear system (3.1)-(3.5) we use a domain decomposition method in the form [8].

Let us break up the domain Ω into minimal sub-domains $\omega_{i_1 i_2} = \{(z_{i_1}, z_{i_2}), (z_{i_1}, z_{i_2+1}), (z_{i_1+1}, z_{i_2+1}), (z_{i_1+1}, z_{i_2})\}$ (see Fig. 2). In each of four points of a sub-domain we define four values of grid functions:
$x_{i_1 i_2} = \Big(x^{(0)}_{i_1 i_2}, \ldots, x^{(3)}_{i_1 i_2}\Big)$ $y_{i_1 i_2} = \Big(y^{(0)}_{i_1 i_2}, \ldots, y^{(3)}_{i_1 i_2}\Big)$, $u_{i_1 i_2} = \Big(u^{(0)}_{i_1 i_2}, \ldots, u^{(3)}_{i_1 i_2}\Big)$.

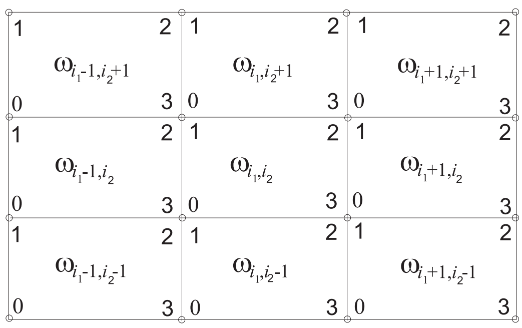

Figure 2. Fragment of the grid ω_h.

Let us consider the equation for the space coordinate x (3.1) :

$$x_\tau = A_x \widehat{x} + \Phi_1 u, \tag{4.1}$$

where operator A_x has the form:

$$A_x x = D_1 \widehat{x}_{\overline{z}_1 z_1} + D_2 \widehat{x}_{\overline{z}_2 z_2}.$$

In each grid point (z_{i_1}, z_{i_2}) it is represented by the sum of four operators:

$$A_x x = \sum_{\alpha=0}^{3} A_x^{(\alpha)} x. \tag{4.2}$$

Operators $A_x^{(\alpha)}$, $\alpha = \overline{0,3}$, have the form:

$$A_x^{(0)} x_{i_1,i_2} = 0.5 \sum_{\alpha=1}^{2} \Upsilon_\alpha^+ x_{i_1,i_2}^{(0)}, \quad A_x^{(1)} x_{i_1,i_2} = 0.5(\Upsilon_1^+ x_{i_1,i_2}^{(1)} + \Upsilon_2^- x_{i_1,i_2}^{(1)}),$$

$$A_x^{(2)} x_{i_1,i_2} = 0.5 \sum_{\alpha=1}^{2} \Upsilon_\alpha^- x_{i_1,i_2}^{(2)}, \quad A_x^{(3)} x_{i_1,i_2} = 0.5(\Upsilon_1^- x_{i_1,i_2}^{(3)} + \Upsilon_2^+ x_{i_1,i_2}^{(3)}), \tag{4.3}$$

$$\Upsilon_1^+ x = h_1^{-1} D_1 x_{z_1}, \quad \Upsilon_2^+ x = h_2^{-1} D_2 x_{z_2},$$

$$\Upsilon_1^- x = -h_1^{-1} D_1 x_{\overline{z}_1}, \quad \Upsilon_2^- x = -h_2^{-1} D_2 x_{\overline{z}_2}.$$

We repeat the same operations for the equation (3.2):

$$y_\tau = A_y \widehat{y} + \Phi_2 u.$$

Since the operator $A_y = A_x$, so it has the form (4.2)–(4.3).

Let us introduce the following designations:

$$\widetilde{v}_{i_1,i_2} = (\widetilde{v}_{i_1,i_2}^{(0)}, \ldots, \widetilde{v}_{i_1,i_2}^{(3)}),$$

$$\widetilde{v}_{i_1,i_2}^{(0)} = 0.25(v_{i_1,i_2}^{(0)} + v_{i_1,i_2-1}^{(1)} + v_{i_1-1,i_2-1}^{(2)} + v_{i_1-1,i_2}^{(3)}),$$

$$\widetilde{v}_{i_1,i_2}^{(1)} = 0.25(v_{i_1,i_2}^{(1)} + v_{i_1-1,i_2}^{(2)} + v_{i_1-1,i_2+1}^{(3)} + v_{i_1,i_2+1}^{(0)}),$$

$$\widetilde{v}_{i_1,i_2}^{(2)} = 0.25(v_{i_1,i_2}^{(2)} + v_{i_1,i_2+1}^{(3)} + v_{i_1+1,i_2+1}^{(0)} + v_{i_1+1,i_2}^{(1)}),$$

$$\widetilde{v}_{i_1,i_2}^{(3)} = 0.25(v_{i_1,i_2}^{(3)} + v_{i_1+1,i_2}^{(0)} + v_{i_1+1,i_2-1}^{(1)} + v_{i_1,i_2-1}^{(2)}).$$

Let us write down the difference scheme of domain decomposition for equation (4.1) in the domain $\omega_{i_1 i_2}$:

$$(\widehat{x}_{i_1,i_2} - \widetilde{x}_{i_1,i_2})/h_\tau + A_{x\,i_1,i_2}(\widehat{x} - x) + 0.25 B_{x\,i_1,i_2}(x) = \Phi_1 \, \widetilde{u}_{i_1,i_2}, \tag{4.4}$$

where

$$A_{x\,i_1,i_2}(x) = (-A_x^{(0)} x_{i_1,i_2}, \ldots, -A_x^{(3)} x_{i_1,i_2}),$$

$$B_{x\,i_1,i_2}(x) = (-B_{i_1,i_2}^{(0)}(x), \ldots, -B_{i_1,i_2}^{(3)}(x)),$$

$$B_{i_1,i_2}^{(0)}(x) = A_x^{(0)} x_{i_1,i_2}^{(0)} + A_x^{(1)} x_{i_1,i_2-1}^{(1)} + A_x^{(2)} x_{i_1-1,i_2-1}^{(2)} + A_x^{(3)} x_{i_1-1,i_2}^{(3)},$$

$$B_{i_1,i_2}^{(1)}(x) = A_x^{(1)} x_{i_1,i_2}^{(1)} + A_x^{(2)} x_{i_1-1,i_2}^{(2)} + A_x^{(3)} x_{i_1-1,i_2+1}^{(3)} + A_x^{(0)} x_{i_1,i_2+1}^{(0)},$$

$$B_{i_1,i_2}^{(2)}(x) = A_x^{(2)} x_{i_1,i_2}^{(2)} + A_x^{(3)} x_{i_1,i_2+1}^{(3)} + A_x^{(0)} x_{i_1+1,i_2+1}^{(0)} + A_x^{(1)} x_{i_1+1,i_2}^{(1)},$$

$$B_{i_1,i_2}^{(3)}(x) = A_x^{(3)} x_{i_1,i_2}^{(3)} + A_x^{(0)} x_{i_1+1,i_2}^{(0)} + A_x^{(1)} x_{i_1+1,i_2-1}^{(1)} + A_x^{(2)} x_{i_1,i_2-1}^{(2)}.$$

$$\Phi_1\, \tilde{u}_{1,i_2} = (\Phi_1\, \tilde{u}_{i_1,i_2}^{(0)}, \ldots, \Phi_1\, \tilde{u}_{i_1,i_2}^{(3)}) = (\Phi_1^{(0)}, \ldots, \Phi_1^{(3)}).$$

Corresponding iterative algorithm of domain decomposition looks like this:

$$(\overset{s+1}{x}_{i_1,i_2} - \tilde{x}_{i_1,i_2})/h_\tau + A_{x\,i_1,i_2}(\overset{s+1}{x} - \overset{s}{x}) + 0.25 B_{x\,i_1,i_2}(\overset{s}{x}) = \Phi_1\, \tilde{u}_{i_1,i_2}, \quad (4.5)$$

where $\overset{0}{x}_{i_1,i_2} = \tilde{x}_{i_1,i_2}$.

Analogous difference scheme of domain decomposition and iterative process can be written for the difference equation (3.2).

Let us consider the equation (3.3). Operator A can be represented in an analogous to (4.2) way. Operators $A^{(\alpha)}$, $\alpha = \overline{0,3}$ have the following form:

$$A^{(0)} u_{i_1,i_2} = M^{(0)} u_{i_1,i_2} + \Pi^{(0)} u_{i_1,i_2}, \quad A^{(1)} u_{i_1,i_2-1} = M^{(1)} u_{i_1,i_2-1} + \Pi^{(1)} u_{i_1,i_2-1},$$

$$A^{(2)} u_{i_1-1,i_2-1} = M^{(2)} u_{i_1-1,i_2-1} + \Pi^{(2)} u_{i_1-1,i_2-1},$$

$$A^{(3)} u_{i_1-1,i_2} = M^{(3)} u_{i_1-1,i_2} + \Pi^{(3)} u_{i_1-1,i_2}, \quad (4.6)$$

$$M^{(0)} u = 0.5 \sum_{\alpha=1}^{2} \Lambda_\alpha^+ u, \quad M^{(1)} u = 0.5(\Lambda_1^+ u + \Lambda_2^- u),$$

$$M^{(2)} u = 0.5 \sum_{\alpha=1}^{2} \Lambda_\alpha^- u, \quad M^{(3)} u = 0.5(\Lambda_1^- u + \Lambda_2^+ u),$$

$$\Lambda_\alpha^+ u = h_\alpha^{-1} a_{\alpha\alpha} u_{z_\alpha} - h_\alpha^{-1} c_\alpha^{(+1_\alpha)} u^{(+1_\alpha)},$$

$$\Lambda_\alpha^- u = -h_\alpha^{-1} a_{\alpha\alpha}^{(-1_\alpha)} u_{\bar{z}_\alpha} + h_\alpha^{-1} c_\alpha^{(-1_\alpha)} u^{(-1_\alpha)}, \quad \alpha = 1, 2,$$

$$\Pi^{(0)} u = 0.5(h_1^{-1} a_{21} u_{z_2} + h_2^{-1} a_{12} u_{z_1}),$$

$$\Pi^{(1)}u = 0.5(-h_2^{-1}a_{21}^{(-1_2)}u_{z_1}^{(-1_2)} + h_1^{-1}a_{12}^{(+1_1)}u_{\overline{z}_2}^{(+1_1)}),$$

$$\Pi^{(2)}u = -0.5(h_2^{-1}a_{21}u_{\overline{z}_1} + h_1^{-1}a_{12}u_{\overline{z}_2}),$$

$$\Pi^{(3)}u = 0.5(h_2^{-1}a_{12}^{(+1_2)}u_{\overline{z}_1}^{(+1_2)} - h_1^{-1}a_{21}^{(-1_1)}u_{z_2}^{(-1_1)}).$$

The difference scheme of domain decomposition for equation (3.3) in the domain ω_{i_1,i_2} can be written as:

$$\left((\widehat{\Psi}\widehat{u})_{i_1,i_2} - (\widetilde{\Psi}\widetilde{u})_{i_1,i_2}\right)/h_\tau + A_{i_1,i_2}(\widehat{u} - u) + 0.25R_{i_1,i_2}(u) = \widehat{\Psi}\widehat{g}, \qquad (4.7)$$

where

$$A_{i_1,i_2}(u) = (-A^{(0)}u_{i_1,i_2}, \ldots, -A^{(3)}u_{i_1,i_2}),$$

$$R_{i_1,i_2}(u) = (-R_{i_1,i_2}^{(0)}(u), \ldots, -R_{i_1,i_2}^{(3)}(u)),$$

$$R_{i_1,i_2}^{(0)}(u) = A^{(0)}u_{i_1,i_2}^{(0)} + A^{(1)}u_{i_1,i_2-1}^{(1)} + A^{(2)}u_{i_1-1,i_2-1}^{(2)} + A^{(3)}u_{i_1-1,i_2}^{(3)},$$

$$R_{i_1,i_2}^{(1)}(u) = A^{(1)}u_{i_1,i_2}^{(1)} + A^{(2)}u_{i_1-1,i_2}^{(2)} + A^{(3)}u_{i_1-1,i_2+1}^{(3)} + A^{(0)}u_{i_1,i_2+1}^{(0)},$$

$$R_{i_1,i_2}^{(2)}(u) = A^{(2)}u_{i_1,i_2}^{(2)} + A^{(3)}u_{i_1,i_2+1}^{(3)} + A^{(0)}u_{i_1+1,i_2+1}^{(0)} + A^{(1)}u_{i_1+1,i_2}^{(1)},$$

$$R_{i_1,i_2}^{(3)}(u) = A^{(3)}u_{i_1,i_2}^{(3)} + A^{(0)}u_{i_1+1,i_2}^{(0)} + A^{(1)}u_{i_1+1,i_2-1}^{(1)} + A^{(2)}u_{i_1,i_2-1}^{(2)}.$$

The iterative algorithm for equation (3.3) can be formulated as:

$$\left((\widehat{\Psi}\overset{s+1}{u})_{i_1,i_2} - (\widetilde{\Psi}\widetilde{u})_{i_1,i_2}\right)/h_\tau + A_{i_1,i_2}(\overset{s+1}{u} - \overset{s}{u}) + 0.25R_{i_1,i_2}(\overset{s}{u}) = \widehat{\Psi}\widehat{g}, \quad (4.8)$$

where $\overset{0}{u}_{i_1,i_2} = \widetilde{u}_{i_1,i_2}$.

The stability relative to initial data and also the convergence of the difference problem solution (4.4), (4.7) and the iterative algorithms (4.5), (4.8) were investigated in linearized case. We proved that difference schemes (4.4) and (4.7) were unconditionally stable relative to the initial data and converged. Iterative methods (4.5) and (4.8) converge too. It seems to be impossible to examine the full nonlinear system. But since all our operators are positive definite, we can suppose that the stability and the convergence take place. This was confirmed by numerical experiments. We considered a problem of two-dimensional flow of incompressible viscous fluid as a test problem. A next article will be devoted to the discussion of results of numerical experiments carried out.

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DVIMATĖS ŠILUMOS LAIDUMO LYGTIES SPRENDIMAS JUDANČIUOSE TINKLUOSE DAUGIAKOMPONENTINIU ITERACINIU METODU

S. SYTOVA

Darbe nagrinėjamas daugiakomponentinis iteracinis metodas, kuriuo sprendžiamas dvimatis šilumos laidumo uždavinys. Naudojamas adaptyvus diskretusis tinklas ir skaičiavimo sritis skaidoma į atskiras nepriklausomas dalis. Adaptyvusis tinklas konstruojamas kreivinėse koordinatėse ir jis priklauso nuo uždavinio sprendinio. Po transformacijos gaunama netiesinių skirtumų lygčių sistema tenkinanti paraboliškumo sąlygą. Lygtyje atsiranda nariai su mišriomis išvestinėmis, konvekcijos nariai ir šilumos šaltiniai. Gautoji lygčių sistema sprendžiama panaudojant srities skaidymo ir daugiakomponentinių iteracinių metodų kombinaciją.