Physics of Particles and Nuclei, Vol. 36, No. 2, 2005, pp. 233–245. Translated from Fizika Elementarnykh Chastits i Atomnogo Yadra, Vol. 36, No. 2, 2005, pp. 436–459.

The Formation of ^{8}Be Nuclei and Their Role in the Fragmentation of Light Nuclei

F. G. Lepekhin

Petersburg Institute of Nuclear Physics, Russian Academy of Sciences, Gatchina, Leningrad oblast, 188350 Russia
http://hepd.pnpi.spb.ru/ofve/nni/nni.htm

Abstract—In the processes related to nuclear clustering in light nuclei, a special role belongs to the ^{8}Be nucleus. This nucleus is the transition state through which a considerable fraction of α particles, the fragments of all relativistic nuclei, is formed. The emphasis of this review is on the experimental features involved in identifying ^{8}Be nuclei in the fragmentation of relativistic ^{10}B nuclei, chosen due to the fact that they are the lightest nuclei in which this cluster can be formed. The formation probabilities of ^{8}Be nuclei are calculated for the fragmentation of relativistic ^{10}B, ^{12}C, and ^{16}O nuclei and compared with the experimental data. It is shown that the suggested formation mechanism of this clustering during the fragmentation of relativistic nuclei is correct, as the calculations performed on its basis are experimentally corroborated.

INTRODUCTION

In recent years, the physics of nuclear clustering, i.e., the formation of clusters from a parent nucleus in nucleus–nucleus interactions at high energies, has received a great deal of attention from researchers [1–4]. This attention is due not only to the interest in the nucleosynthesis problem [5] and a desire to understand the general problems of many-particle systems [6] but also an interest in superstring theory [7] and the recent discovery of the Bose- and Fermi-particle condensate [8, 9].

It is clear that the nucleons in a nucleus are not free. Rather, they are quasiparticles surrounded by a cloud of other nucleons. A quasiparticle is a stream-type dynamic system (DS) [10] subjected to a sequence of spontaneous transformations in phase space. In DS theory (DST), this is a one-parameter group of transformations. In fact, DST is a section of modern mathematics whose conclusions we can use without going too deeply into the related proofs. From the viewpoint of physics, it is necessary for the quasiparticle lifetime to be large so that the description of a nucleus in terms of quasiparticles is adequate and also in agreement, to some extent, with experimental data. We will return to this problem below; however, at present, it is useful to note the correlation between DST and the naive parton model of multiple-particle production in hadron–hadron interactions at ultrahigh energies [11–13]. This correlation is of interest not only because the dominant subject matter of science is the search for correlations between phenomena but mainly because the technique and representations describing quark and hadron fragmentation, when applied to a description of the fragmentation of relativistic nuclei, makes it possible to calculate the quantitative characteristics of experimentally observed associations of nucleons, i.e., clusters, which are fragments of these nuclei.

The naive parton picture of the multiple-particle production or fragmentation of a relativistic nucleus is clear and straightforward. Each real particle (hadron, quark, nucleus, and nucleon) is always surrounded in its own rest frame with field quanta possessing strong interactions, i.e., point massless particles called partons. Each parton has a lifetime of about 10^{-24} s in its own rest frame. The parton cloud continuously changes its shape and composition without changing the quantum numbers of its initial state (a particle). In all parton transformations, the law of conservation of momentum is satisfied. However, the law of conservation of energy can be violated within the limits and for the time allowed by the uncertainty principle. All partons are correlated and coherent. A return at an arbitrary point of phase space is possible according to the Poincare recurrence theorem from DST.

This space–time description of ultrarelativistic nucleus–nucleus collisions is given both by Geiger [11] and Feynman [12]. The parton picture of the fragmentation of relativistic nuclei is based on the hypothesis that, after an interaction, we can experimentally observe fragments with the characteristics described above, which were already to be found in a nucleus before its interaction with another nucleus. This assumption enables us to predict not only the constants of the angular and momentum distributions of the fragments but also to calculate the probabilities of observing certain fragments [13–15]. This view of fragmentation is also a major subject of this review.

As it is a fragment of a relativistic nucleus, ^{8}Be is especially convenient for such investigations for a number of reasons. First, it is produced directly from a relativistic nucleus, whereas, for example, α particles or protons can also be emitted from so-called prefragments [16] (such as ^{5}Li, ^{5}He, or ^{8}Be). Second, as we will see below, this nucleus can be reliably identified in a photoemulsion. J.A. Wheeler experimentally established the existence of ^{8}Be nuclei in 1940. He studied α-particle scattering in helium [17] and showed that the two α particles observed in the final state arose from two metastable states with an even total angular momentum and a positive parity. The energies (0.125 and 2.9 MeV) and widths (about 100 eV and 0.8 MeV) of these states as determined by Wheeler differ little from modern data on the lowest states of ^{8}Be nuclei listed in [18].

The existence of ^{8}Be nuclei in the products of nuclear disintegration induced by high-energy particles was established in 1950 [19]. This stage of the investigation into the process is described in detail in [20]. α-particle pairs were detected in the events of the disintegration of a nucleus in emulsions irradiated with high-energy cosmic-ray particles. The fact that these pairs were a result of ^{8}Be decay into two α particles was established visually and was somewhat subjective. The angle between the tracks of the two α particles should be small (20–30 mrad) for an isotropic emission of these particles to occur. In addition, according to an operator estimate, the difference in the particle free paths should also be small.

At the same time, decays of an $^8\mathrm{Li} \longrightarrow {}^8\mathrm{Be} + e^- \longrightarrow 2\alpha + e^-$ type were also studied in the emulsions irradiated with cosmic-ray particles. In these events, the ^{8}Li nucleus, which is held in a photolayer, undergoes β decay in a time of about 0.84 s and, then, this ^{8}Be disintegrates into two α particles. A characteristic hammer-type event and an electron track [20] can then be seen.

Naturally, the production of relativistic ion beams stimulated interest in carrying out investigations of ^{8}Be-nuclei formation [21, 22]. The presence of a $^8\mathrm{Be} \longrightarrow 2\alpha$ channel was also established during the fragmentation of lead nuclei with energies of 160 GeV per nucleon [23]. In these studies, the events were identified by the spatial angle between the tracks of the particles and, in [23], specifically, by the angle between the track projections onto the emulsion plane during the fragmentation of lead with the energy of 160 A GeV. It was noted that the fraction of events that had a ^{8}Be fragment was large for light nuclei and decreased in the fragmentation of heavy nuclei. However, these observations were not accompanied with any kind of probability calculations for the formation of ^{8}Be for a particular given nucleus. Thus, we consider, in detail, exactly these facts as applied to the fragmentation of relativistic ^{10}B, ^{12}C, and ^{16}O nuclei. The clustering effects should be especially strong for these nuclei.

The review is organized as follows. In the next section, we consider the physics of clustering of nuclear matter in more detail. We obtain criteria that make it possible to identify the channel where $^{10}\mathrm{B} \longrightarrow {}^8\mathrm{Be}$ + all $\longrightarrow 2\alpha$ + all in a fragmentation of ^{10}B with an energy of 1 GeV per nucleon. Then, we describe a procedure for estimating the probability of the formation of ^{8}Be nuclei. These estimates are given for $^{12}\mathrm{C} \longrightarrow 3\alpha$ and $^{16}\mathrm{O} \longrightarrow 4\alpha$ fragmentation. Furthermore, the experimental procedure and the results obtained with a chamber irradiated by ^{10}B ions at the Laboratory of High Energies of JINR are considered in detail. Finally, we compare the data of this particular experiment with those obtained from calculations and predicted using the described picture of nuclear clustering.

1. PHYSICS OF NUCLEAR CLUSTERING

As is already clear from the previous section, the widespread view (see [24]) that the process of relativistic-nuclei fragmentation and the emission of the resulting fragments from a target nucleus irradiated by high-energy particles takes place at the second stage of nucleus excitation, in addition to the suggestion that this excitation occurs at the first stage of the fast-particle–nucleus interaction, is erroneous. There are no experimental data consistent with the concept that a certain time passes between a fast process of multiple-particle production at the first stage and either a slow process of evaporation of particles or fast decay of an excited nucleus at the second stage. Of course, such a view can post-factum give a description of the events observed in an experiment and fit a certain number of free parameters. However, though the theory has been in existence for many years, it has failed to give quantitative predictions of new phenomena.

Moreover, it would appear that, for example, the average number of slow particles produced in the disintegration of photoemulsion nuclei by high-energy particles, as well as their angular and momentum characteristics, is independent both of the energy and mass of the primary particles. This is valid for primary-particle energies ranging from 1 to 200 GeV with charges from 1 (proton) to 82 (lead). Whatever we used to act on a target nucleus, its excitation energy remained constant.

It is experimentally shown that, in the fragmentation of a relativistic ^{22}Ne nucleus with a momentum of 4.1 GeV/c per nucleon, the transverse-momentum distribution coincides for all the fragments with that distribution, which we would expect to be the case if each fragment were randomly composed from nucleons of the ^{22}Ne nucleus before its interaction with a photoemulsion nucleus [25].

A number of questions arise: Why do nucleons acquire the ability to conglomerate only at the second stage, and what prevents this event from occurring earlier, i.e., before the collision with a fast particle? However, there is a central question that must be addressed: What, in general, forces the nucleons to form quasiparticles? In answer, it is clear from [6] that it is the properties possessed by the constituents of many-body systems that cause the formation of quasiparticles. In our case, the many-body system consists of Fermi particles at zero temperature. In addition, this system of particles has an isotopic spin equal to zero and unity. Thus, only four nucleons with different quantum numbers can be in one phase-space cell $\hbar$, and it is precisely this strong formation from four nucleons that we define as an α cluster. This particular cluster lives much longer than, for example, a random formation in which a cluster is composed of four neutrons, or three neutrons and one proton, etc. The formation of clusters composed of two nucleons with opposite spins or isospins (deuteron) is also probable, but they also live for a shorter time than α clusters.

A completely new situation arises immediately after the formation of two α clusters in one nucleus: these clusters are now Bose quasiparticles, which, occupying the same energy level, form a state similar to a condensate. They start to efficiently interact and, as was shown in Wheeler's experiments [17], form a resonance (a ^{8}Be nucleus). Now, we investigate how frequently this event occurs in a certain nucleus. Because we can only use products of the $^8\text{Be} \longrightarrow 2\alpha$ decay, it is of importance to know the regularities governing α particles in general. For this problem, there are both reasonably strong theoretical and experimental bases.

In [26, 27], a study of the energy and angular distributions of fragments emitted from target nuclei irradiated by high-energy protons was performed. The experimental results were described according to the assumption that the fragments were emitted from an excited nucleus moving in the laboratory system with a certain velocity along the direction of the primary-proton momentum. A satisfactory description of the experimental data was obtained by the fitting of nine free parameters. For each experiment, there were different sets of these parameters, and their heuristic value was virtually equal to zero. However, it was impossible to predict the results of new experiments.

The use of the parton picture of nuclear fragmentation fundamentally changes this situation. When describing the angular and momentum distributions of relativistic nuclear fragments (and target-nucleus fragments), it is sufficient to have information about the ground state of the fragmenting nucleus before its interaction with another nucleus (or before the interaction of a fast proton with a target). It is necessary to know the properties of the distributions of the α-particle pairs emitted from a ^{10}B nucleus with an energy of 1 GeV per nucleon, which we then compare with the experimental data. Let us now demonstrate that these properties can be obtained without any free parameters but instead under certain assumptions, whose validity is checked in a comparison of the predicted properties with those found experimentally.

Thus, we assume that a nucleus is a system of Fermi particles, whose momentum space is a sphere of radius P_F, and that the momentum spread of the nucleons in the nucleus is equal to $\sigma_0^2 = P_F^2/5$. Here, P_F is the maximal Fermi momentum. Its value can be determined, for example, from the scattering of electrons on nuclei [28]. In conventional space, the nucleons of a nucleus with the mass number A fill a sphere with the radius $R = r_0A^{1/3}$. For various nuclei, the constant r_0 is also experimentally determined from the scattering of different particles on nuclei [29]. The phase volume Ω occupied by the A nucleons and equal to the product of the usual spherical volume on the momentum-space volume should be equal to $4A\hbar$. Then, we obtain

$$\sigma_0 r_0 = \frac{(9\pi)^{1/3}}{2\sqrt{5}}\hbar, \tag{1}$$

or $\sigma_0 r_0 = 134.422$ (MeV/c)fm. An experimental estimate in [25] gave the value $\sigma_0 = (102.5 \pm 2.5)$ MeV/c for the fragmentation of ^{22}Ne nuclei with a momentum of 4.1 GeV per nucleon, while the expected value of this quantity found from the Fermi momentum is equal to 105.1 MeV/c. Such confirmations of the expected value of σ_0 have been obtained in a number of experiments.

The most general ideas about the type of nucleon wave function in a nucleus lead to the conclusion that a nucleon-momentum projection onto an arbitrary direction in space should be normally distributed, with an average equal to zero and the variance σ_0^2. In [30], it is shown that, if, to randomly extract k nucleons from a nucleus with A nucleons for which the vector sum of their momenta is equal to zero and the variance of their distribution is equal to σ_0^2, the variance of the vector sum of these samples is equal to

$$\sigma_k^2 = \sigma_0^2\frac{k(A-k)}{A-1}. \tag{2}$$

In essence, this is pure combinatorics. Relation (2) is called the Goldhaber parabolic law. This law has undergone multiple experimental tests and has essentially been confirmed. For each fragment with the number of nucleons k in the fragmentation of an arbitrary relativistic nucleus with the mass number A, we can estimate the projection $P_\varphi = kP_0\tan\varphi$ of the transverse momentum $P_\perp$ onto an emulsion plane from the measured angle φ between the direction of the primary-nucleus momentum P_0 and the projection $P_\perp$ onto the emulsion plane. It is clear from this estimate that the random variable

$$P_Y = P_\varphi\sqrt{\frac{A-1}{k(A-k)}} \tag{3}$$

for this relativistic nucleus should be normally distributed with an average equal to zero and the variance σ_0^2. In [25], it is experimentally shown that, for more than 6000 fragments of a ^{22}Ne nucleus, the hypothesis regarding the normal distribution of values defined by Eq. (3) cannot be disregarded.

Of course, the distribution of the transverse momenta $x = P_\perp$ themselves for relativistic nuclear fragments or the emission angles $x = \theta$ for these fragments should follow the χ_2 or Rayleigh distribution, whose probability density is

$$f(x,\sigma) = \frac{x}{\sigma^2}\exp(-x^2/2\sigma^2), \quad (4)$$

and distribution function is

$$F(x,\sigma) = 1-\exp(-x^2/2\sigma^2). \quad (5)$$

In [28], the Fermi momentum was not measured for the ^{10}B nucleus, but, according to the data from [31], the value r_0 is equal to 1.54 fm. From above, it follows that the emission-angle distribution for the α particles produced in the fragmentation of ^{10}B nuclei with the momentum P_0 = 1696 MeV/c per nucleon should follow Eq. (4) with the constant σ_θ = 21.0 mrad. However, this is only an inclusive distribution of all the α particles in the $^{10}B \longrightarrow 2\alpha$ + all process. We are interested in the angles between the α particles both in this process and in the $^{8}Be \longrightarrow 2\alpha$ decay, in which a pair of such particles is produced.

If an excited ^{10}B nucleus of this form decayed via the $^{10}B \longrightarrow 2\alpha + d$ or $^{10}B \longrightarrow 2\alpha + p + n$ channel over phase space, it would be inevitable, for such a small number of particles in the final state, that there would be strong kinematic correlations between the transverse momenta. Thus, these correlations also exist for the angles of the emission of secondary particles. Then, the distribution of the angles between the α particles in question can be calculated by simulating the ^{10}B-nucleus decay over phase space at a certain excitation energy; i.e., it will be necessary to introduce a certain adjustable parameter on which the average angle between the pair of these particles depends.

However, if the real particles observed in the final state are only a small fraction of a parton cloud with an uncertain number of particles, for which the vector sum of all their transverse momenta is always equal to zero, there should be no correlations between the transverse momenta of the particles in the final state. In fact, in such a case, the particles would appear to escape independently from each other. One can verify both these statements experimentally. If α particles are emitted from a ^{10}B nucleus independently, the angle θ_{12} between them, as was shown in [16], should have the χ_2 distribution with the constant $\sigma(\theta_{12}) = \sqrt{2}\sigma_\theta$; i.e., $\sigma(\theta_{12})$ = 29.7 mrad. The average angle between the two α particles should be equal to $\langle\theta_{12}\rangle = \sqrt{\pi/2}\sigma(\theta_{12})$, or $\langle\theta_{12}\rangle$ = 37.2 mrad.

In photoemulsion experiments, the emission angles are usually determined from the two angles equal to the angles between the projections of the momentum onto two perpendicular planes—the emulsion plane (angle φ) and the plane perpendicular to the emulsion plane (angle α). If the two particles emerge independently of each other in each event, and both angles, φ and α, are random samples from the normal distribution with the same variance for each particle, the variance of the sum of four such angles in each event should be equal to four variances of the distribution for these angles and, therefore,

$$\sigma(\varphi_1+\varphi_2+\alpha_1+\alpha_2) = 2\sigma_\theta. \quad (6)$$

The basic characteristic of two-particle correlations between the particles in a transverse plane is the azimuthal asymmetry parameter A, which is determined as the ratio of the difference between the probabilities of observing the azimuthal-angle difference $\Delta\Psi$ at above 90° and below 90° for two particles to the sum of these two differences:

$$A = \frac{N(\Delta\Psi > 90°) - N(\Delta\Psi < 90°)}{N(\Delta\Psi > 90°) + N(\Delta\Psi < 90°)}. \quad (7)$$

For an independent emission of particles, this parameter should be equal to zero. The distribution over the angles $\Delta\Psi$ between the vectors of the transverse momenta of the two particles in the event should be uniform in this case. For the decay of an excited system into n particles over phase space when the vector sum of the transverse momenta of all the particles is equal to zero in each event, kinematic correlations inevitably arise in the transverse plane [32]. In this case, the azimuthal asymmetry parameter A should be equal to $1/(n-1)$. In the fragmentation of a ^{10}B nucleus, the total number n of particles cannot be so great that A does not differ from zero.

In the decay of ^{8}Be into two α particles, all the azimuthal-angle differences for the two particles should be less than 90° if ^{8}Be is emitted from ^{10}B. The azimuthal asymmetry parameter A for these events should be close to $k-1$. It is exactly this fact that we intend to check.

Now, we focus on what can occur if the events with two α particles in our experiment go through the $^{8}Be \longrightarrow 2\alpha$ decay. We assume that ^{8}Be escapes from a ^{10}B nucleus in the same way as it does in a typical fragmentation. Under such circumstances, the transverse momenta of the ^{8}Be nucleus follow the Rayleigh distribution, whose constant we can easily calculate by knowing the constant r_0 for the ^{10}B nucleus. The longitudinal momentum of a ^{8}Be nucleus is virtually invariable and equal to $8P_0$ = 13.6 GeV/c. As a result, we know the direction and momentum of a nucleus decaying in flight into two α particles. The kinetic energy

$$T^*_\alpha = (P^*_\alpha)^2/(2M_\alpha)$$

for every α particle in the center-of-mass system (CMS) of a ^{8}Be nucleus decaying from the 0^+ state is equal to 45.96 keV. Here, P^*_α is the momentum of an α particle in the ^{8}Be CMS, while M_α is the rest mass of an α particle. It is clear that

$$2T_\alpha^* = \sqrt{(E_1+E_2)^2 - (\mathbf{P_1}+\mathbf{P_2})^2} - 2M_\alpha \quad (8)$$

if E_1, E_2, $\mathbf{P_1}$, and $\mathbf{P_2}$ are the α-particle total energies and vectors of the momenta, respectively, in an arbitrary reference system.

If the angle between the vector $\mathbf{P}_\alpha^*$ and the vector of the ^{8}Be-nucleus momentum is $\theta^* = 90°$, then $P_\alpha^* = P_{\perp\alpha}$; i.e., the α-particle momentum in the ^{8}Be-nucleus CMS is equal to its transverse momentum in the laboratory system. In general,

$$P_{\perp\alpha}^* = P_{\perp\alpha} = 4P_0\sin(\theta_{12}/2).$$

This relation is only satisfied because the outgoing angle for a ^{8}Be nucleus is small. The transverse momenta of the α particles in the ^{8}Be CMS and ^{10}B CMS virtually coincide. Occasionally, this circumstance is considered as an opportunity to determine the energy levels of the ground 0^+ and excited 2^+ states of the ^{8}Be nuclei in a photoemulsion experiment by only measuring the transverse momenta of the α particles. However, in this procedure, it is possible to determine only the transverse part of invariant mass of the two α particles instead of its total value. It is evident that, even if the total invariant mass of the two particles is strictly fixed, an experimental estimation of the transverse part of invariant mass will have the same distribution as θ_{12}^2 (see Fig. 1). In comparison with the angle θ_{12}, this fact gives no new information.

We now continue our description of the procedure in which the distribution of the angles θ_{12} between the α particles is obtained for the $^8\text{Be} \longrightarrow 2\alpha$ decay. The angular distribution of the α particles in the CMS of a decaying ^{8}Be nucleus is assumed to be isotropic. Using the Monte Carlo simulation for the angle θ^* between the α particles in the nucleus rest frame, we obtain the α-particle momentum in the laboratory system and find the angle between the particles for each event. The distribution of the angles between the particles simulated in this way is shown in Fig. 1. The figure has a sharp peak corresponding to the probability of observing these angles at 5.45 mrad, due to the fact that the solid angle for the emission of two α particles under an angle of 90° in the decaying-nucleus CMS is much larger than that for their emission under a zero angle for the decaying-nucleus momentum. If the primary-particle energy increases, the form of this distribution remains unaltered; however, the limiting angle θ_{max} decreases. For the momentum 4.5 GeV/c per nucleon, the limiting angle between the α particles emitted in the ^{8}Be decay from the 0^+ state in an emulsion is about 2 mrad.

For the ^{8}Be decay, in the first excited state 2^+ for which $2T_\alpha^* \simeq 3$ MeV, the distribution of the angles θ_{12} is similar to that shown in Fig. 1 but the maximal angle between α particles is equal to ≃30 mrad. Furthermore, the momentum P_α^* is 5–6 times larger, while the transported momentum of the α particles remains unchanged ($4P_0 = 6.8$ GeV/c).

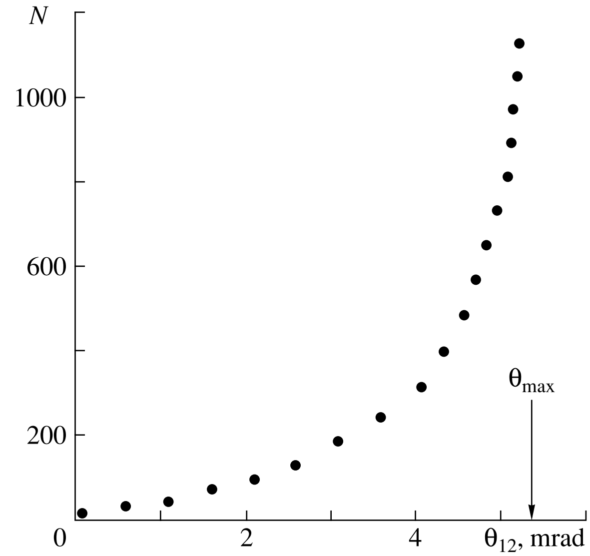

Fig. 1. Distribution of the angles θ_{12} between the α-particle tracks for the ^{8}Be decay from a ^{10}B nucleus with a momentum of 1.7 GeV/c for $\sum N = 2500$ events simulated using the Monte Carlo procedure. N is the number of events per unit interval $\Delta\theta_{12} = 0.5$ mrad.

In addition, this distribution is a mixture of distributions with different values of the maximal angle θ_{12}, which result from a width of the 2^+ level of about 0.8 MeV. As we have seen, the expected value is $\sigma(\theta_{12}) =$ 29.7 mrad for the $^{10}\text{B} \longrightarrow 2\alpha$ process. Therefore, it is rendered virtually impossible to single out the ^{8}Be decay from the 2^+ first excited state, either by the angle θ_{12} or by the difference between the invariant mass and the sum of rest masses of the two particles given by Eq. (8) in a photoemulsion experiment.

In circumstances in which the primary-nucleus energy is high and its mass allows the fragmentation of this nucleus into several α particles (more than two), new problems arise. Now, the distribution of the angles θ_{12} between all the pairs of particles depends on the number of particles in each event. The larger this number, the higher the probability of observing small angles θ_{12} in the primary-nucleus decay into α particles, even without the formation of an intermediate ^{8}Be state. The background becomes more intense. However, in the events with the number of α particles equal to two, the peak of the probability of observing the angles θ_{12} in these events, as we have seen, coincides with the maximal angle θ_{12} in the ^{8}Be decay from the 2^+ state. If there are a large number of events with two α particles, the peak will probably appear against the background of the distribution of the angles θ_{12} in the events with the number of α particles exceeding two in this mixture of distributions. It is possible to interpret this peak as the $^8\text{Be} \longrightarrow 2\alpha$ decay from the 2^+ state. Certainly, an experiment with a reasonably precise determination of the total α-particle momenta would be ideal in that it would be possible to determine the invariant mass of the α-particle pair with an accuracy of better than 1 MeV.

Thus, the distributions of the angles θ_{12} between two α particles independently emitting from a ^{10}B nucleus and from a ^{8}Be intermediate ground state are very different. It is exactly this fact that enables us to single out the events developing via the $^{10}B \longrightarrow {}^{8}Be \longrightarrow 2\alpha$ + all channel from those moving via the $^{10}B \longrightarrow 2\alpha$ + all channel. In the next section, we calculate the probability of observing the former channel.

2. THE EXPECTED YIELD OF ^{8}Be FROM ^{10}B, ^{12}C, AND ^{16}O NUCLEI

A simple estimate of the probability of observing a certain fragment is justified in the following way [14, 15]. We designate the fragmentation channel of a primary relativistic nucleus with the charge Z_0 and atomic number A_0 as its arbitrary random disintegration into k fragments with the mass numbers and charges A_i and Z_i, respectively, provided that $\sum_{i=1}^{i=k} Z_i = Z_0$ and $\sum_{i=1}^{i=k} A_i = A_0$. The maximal and minimal k values are $k = A_0$ and $k = 2$, respectively. Out of all the possible combinations of nucleons in a primary nucleus, we consider only the combinations in which all the k fragments have mass numbers and charges indicative of stable or radioactive isotopes (whose masses are listed in [18]) and that can be observed in the final state in the fragmentation of this primary nucleus.

This means that, for example, the channels involving fragments with $A_i = 3$ and $Z_i = 0$ are not considered. We assume that the lifetime of such quasiparticles is much shorter than that of virtual particles with charges of 2 or 1. Transitions of a $pn \longrightarrow nn\pi^+$ type, Λ^0 particles, or K^+K^-, etc. can also occur in the parton cloud. However, they are not considered either for the reason that they need the energy ΔE for the transition into a real state, which greatly exceeds the energy necessary for transformations of virtual particles into real fragments with stable and radioactive isotopes. This energy can be expressed as

$$\Delta E_k = \sum_{i=1}^{i=k} (M_i + \langle T \rangle_i) - M_0. \qquad (9)$$

It is equal to the sum of the masses M_i of the fragments plus the sum of their average kinetic energies in the primary-nucleus CMS

$$\langle T \rangle_i = \frac{3\sigma_i^2}{2M_i}\sqrt{\frac{\pi}{2}} \qquad (10)$$

minus the primary-nucleus mass. Here, σ_i^2 is the variance of the distribution of the momentum projection for the fragment with the mass M_i onto an arbitrary direction, which we find from Eq. (2). We can designate the sum of all the average kinetic energies of the fragments as the average excitation energy of the nucleus. Now, by definition, this quantity does not depend on the primary-nucleus energy.

For the final state after the primary-nucleus fragmentation, the law of conservation of energy, in particular, is also satisfied. In this case, two colliding nuclei spend a part of their kinetic energy on the transformation of partons into real particles in their common CMS. The Noether theorem relates the law of conservation of energy to the invariance with respect to a time shift. Thus, our parton dynamic system can be assumed to be invariant with respect to this shift.

In DST, it is demonstrated that an unambiguous, continuous, and finite function, which is called the system potential U, always exists for such systems. The potential difference $\Delta E_{12} = U_1 - U_2$ for two states is the energy of transition from one state to another. Therefore, in the set of states of such a system, the Gibbs invariant normalized measure of this set can be found, which defines the probability

$$W(\Delta E_k, T) = \frac{\exp(-\Delta E_k/T)}{\Xi} \qquad (11)$$

of a transition from a nuclear state with A_0 and Z_0 into a state formed by k fragments each possessing A_i and Z_i. Here, Ξ is the sum of $\exp(-\Delta E_k/T)$ over all the possible states of the k fragments, which is called the statistical sum. In our case, we can calculate this sum by identifying all the possible combinations of fragments, which can be obtained from the given primary nucleus. The value $T = \sigma_0^2/m_n$ is proportional to the average kinetic energy of the nucleons in the primary-nucleus CMS; i.e., it represents the temperature.

This approach differs from the conventional thermodynamic description of the fragmentation of excited nuclei [24] in which the temperature is proportional to the excitation energy. The temperature is established after the interaction and in the process of attaining thermal equilibrium, which is certainly necessary for applying the Gibbs distribution but requires a certain amount of time. Now, the equilibrium is established before the interaction of two colliding nuclei and during the interaction of partons with a strong-interaction field, i.e., with the QCD vacuum. At the instant of the interaction of the two partons, which destroys the coherence of parton clouds, equilibrium has already been attained.

This is the theoretical basis for calculating the absolute and relative probabilities for particular channels of the fragmentation of a primary nucleus. By summing the probabilities of observing channels containing a certain fragment, we can obtain the probability of the fragment's observation.

The realization of this procedure is now a matter of technique. The main technical problem consists in iden-

Table 1. Probabilities of the first two channels of the fragmentation for several light nuclei (up to oxygen inclusively)

N	Original nuclei	Number of channels	Channel	W, %	Channel	W, %
1	^{6}Li $\longrightarrow$	13	^{4}He + ^{2}H	39.3	p + ^{5}He	23.7
2	^{7}Li $\longrightarrow$	20	^{4}He + ^{3}H	37.8	n + ^{6}Li	18.7
3	^{7}Be $\longrightarrow$	18	^{4}He + ^{3}He	50.0	^{2}H + ^{5}Li	16.6
4	^{9}Be $\longrightarrow$	47	^{8}Be + n	30.8	^{4}He + ^{5}He	27.7
5	^{9}B $\longrightarrow$	44	^{4}He + ^{5}Li	40.4	p + 2^{4}He	22.2
6	^{10}B $\longrightarrow$	73	^{4}He + ^{6}Li	19.7	^{2}H + ^{8}Be	16.4
7	^{11}B $\longrightarrow$	105	^{4}He + ^{7}Li	14.3	p + ^{10}Be	10.9
8	^{12}C $\longrightarrow$	159	^{4}He + ^{8}Be	26.2	3^{4}He	17.1
9	^{14}N $\longrightarrow$	319	p + ^{13}C	11.0	^{2}H + ^{12}C	8.7
10	^{16}O $\longrightarrow$	>400	2^{8}Be	20.6	^{5}He + ^{11}C	7.3

tifying all the possible combinations of fragments for a given A_0 and Z_0. This problem is not new (see [33]). Here, we estimate the number of possible channels in a heavy-nuclei fragmentation from the formula

$$N_{\text{chan}} = \frac{1}{4\sqrt{3}A_0}\exp(\pi\sqrt{2A_0/3}). \tag{12}$$

For nuclei with A_0 = 100, the number of possible fragmentation channels is $\simeq 2 \times 10^8$. It is clear that identifying all these channels is impossible. In contrast, this task is very easy for the lightest nuclei. For the ^{6}Li, ^{10}B, ^{12}C, and ^{16}O nuclei, the number of all the possible channels is equal to 13, 73, 159 and 530, respectively.

Here, there are two circumstances facilitating the identification problem: First, it is possible to calculate the relative probability of various channels with respect to one particular channel (preferably, to the most probable one). Then, it is unnecessary to calculate Ξ, which means that the identification of all the possible channels is also unnecessary. Second, from the above formulas, it is clear that the channels with a large number of fragments, for example with $k = A_0$, are improbable. This is caused by the fact that ΔE_k depends not only on the difference $\sum_{i=1}^{i=k} M_i - M_0$ but also on $\sum_{i=1}^{i=k}\langle T\rangle_i$.

The average kinetic energies of singly and doubly charged fragments are about 12–15 MeV for almost all nuclei. As a result, we can assume that the most probable channels are the channels with a small number of fragments. Then, the statistical sum Ξ can be found only from these possible channels of fragmentation, which make the main contribution to it. As a result, despite the fact that we know only an approximate value of the sum, this approximation exerts no decisive influence on the estimate of the probability of the channels observed in the experiment.

It should be noted that, for the light nuclei under discussion, the probability of the disintegration of a nucleus into two nuclei with approximately equal masses should be suppressed by the circumstance that the binding energy per nucleon increases with its mass number in this mass-number region. Therefore, if a nucleus with A_0 passes into a state involving two nuclei with $A_0/2$, this transformation requires energy.

It should be recalled that the difference in energies per nucleon between a fissile nucleus and its fragments is, in contrast, released in this process. Fortunately, it is not difficult to identify all the possible combinations for the transition from a nucleus with A_0 and Z_0 into two fragments. In the all calculations in the remainder of this paper, the channels with $k = A_0$ and $k = 2$ are taken into account. The probabilities of the first two most probable channels of the fragmentation of nuclei from ^{6}Li to ^{16}O are given in Table 1.

An analysis of this table confirms the qualitative expectations of the fragmentation probabilities for light nuclei. Only 2 channels out of the 20 available for 10 nuclei have 3 fragments in their final state. These are ^{12}C $\longrightarrow 3\alpha$ and ^{9}B $\longrightarrow p + 2\alpha$. All the other most probable channels are two-particle states.

For all the nuclei, the channels with the number of fragments $k = A_0$ and $k = A_0/2$ (for even A_0) are the least probable. For example, the probabilities of the channels indicated in parentheses are $W(^6\text{Li} \longrightarrow 3d) = 3.3 \times 10^{-4}$ and $W(^{12}\text{C} \longrightarrow 6p + 6n) = 1.7 \times 10^{-7}$. The probability of the channels in which the formation of ^{8}Be occurs, in contrast, appears very high. For ^{16}O, there is also the ^{16}O $\longrightarrow$ ^{8}Be + 2α channel with a probability equal to 6.4% in addition to the first channel cited in Table 1 with two ^{8}Be fragments in the final state. In total, 25.8% of such events pass through the ^{8}Be state if the ^{16}O $\longrightarrow 4\alpha$ fragmentation is detected. The direct ^{16}O $\longrightarrow 4\alpha$ transitions amount only to 2% of all the channels of the ^{16}O fragmentation.

In the experiment conducted in [34], it was found that the ratio

$$\frac{W(^{16}\text{O} \longrightarrow {}^{12}\text{C} + {}^4\text{He})}{W(^{16}\text{O} \longrightarrow 4\alpha)} = (3.2 \pm 0.6),$$

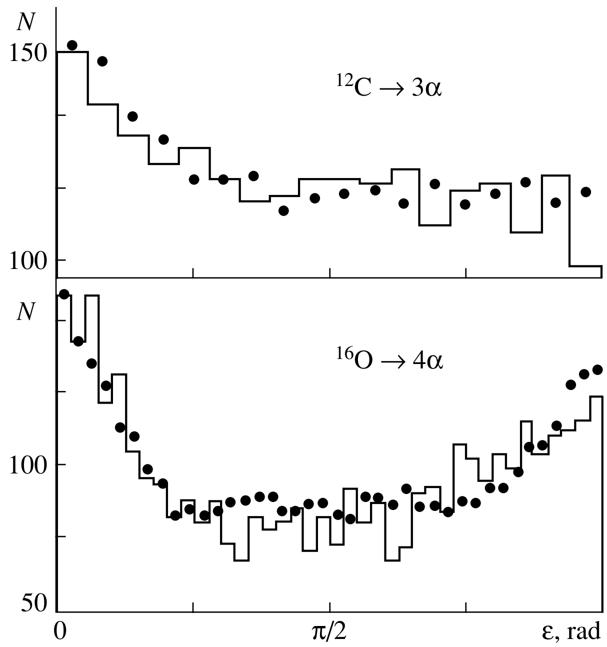

Fig. 2. Distribution of the pair azimuthal angles for the α particles in experiments with the fragmentation of carbon [21] and oxygen [35]. The histograms represent the experiment, and the dots (●) show the simulated distributions.

while the calculated value was equal to 3.04. The agreement between the calculated and experimental values can serve as an indication of the fact that we can also trust the calculations for the probabilities of the fragmentation channels of oxygen through ^{8}Be.

The fragmentation channels of ^{12}C with the production of ^{8}Be in an intermediate state also have a high probability (about 25%). In [21], the fraction of events with two and three α particles in the final state was (61 ± 4)% for a propane bubble chamber experiment, which involved the fragmentation of carbon nuclei, whereas the calculated value of this fraction was 58%. This fact shows that no distinction can be made between the mechanism of the fragmentation of relativistic nuclei and target nuclei.

A coherent $^{16}O \longrightarrow 4\alpha$ fragmentation was studied in [35]. However, neither in this study nor in those on the fragmentation of carbon were the individual events with ^{8}Be in the final state selected from the angle between the α-particle tracks. The fraction of such events was not determined. Therefore, the only and, indeed, indirect indication in these studies of the fact that the probability of observing channels with ^{8}Be in an intermediate state is very high is the experimental data about the azimuthal correlations of α particles. If the fraction of the $^{12}C \longrightarrow {}^8Be + {}^4He$ channel is large, there should be a peak in the distribution of the differences of the azimuthal angles $\varepsilon = \Delta\Psi_{i,j}$ between the α-particle pairs in the region of small angles ε.

The upper histogram in Fig. 2, which was obtained from the data in [21], shows that such a peak does in fact exist. The dots in this figure show the results from a simulation of the ^{12}C decay, with the fraction of the ^{8}Be states obtained via calculation. Both these distributions virtually coincide, which means that the peak relating to the probability of observing small ε values may be caused by the $^{12}C \longrightarrow {}^8Be + {}^4He$ channel with exactly the same probability, the value of which was obtained via calculation.

When studying the fragmentation of relativistic oxygen nuclei in [35], the distribution over the pair azimuthal angle has two peaks: one for small ε and another (lower) peak for the azimuthal-angle difference tending to 180°. Such a distribution of pair azimuthal angles is not observed for any of the fragmenting nuclei except oxygen. Indeed, only for oxygen is the probability of the production of two intermediate-state ^{8}Be nuclei in one $^{16}O \longrightarrow 4\alpha$ event high.

It is clear that the two ^{8}Be nuclei emerge in opposite directions in the CMS of a fragmenting ^{16}O nucleus. Then, each ^{8}Be nucleus decays into two α particles with a small angle between them in the laboratory system, which is due to the transported momentum of each α particle being much higher than its momentum P^*_α = 18.511 MeV/c in the ^{8}Be CMS. Between each α-particle pair belonging to one ^{8}Be nucleus, the differences in azimuthal angles are small, whereas they are large for the α-particle pairs belonging to different ^{8}Be nuclei. We can see exactly this picture on the lower histogram in Fig. 2, which was plotted using the data from [35]. The dots show the results of a simulation of the $^{16}O \longrightarrow 2{}^8Be \longrightarrow 4\alpha$, $^{16}O \longrightarrow {}^8Be + 2\alpha \longrightarrow 4\alpha$, and $^{16}O \longrightarrow 4\alpha$ processes and their calculated probabilities. It is evident that the highest probability (up to 30%) of ^{8}Be production in an intermediate state is in the $^{16}O \longrightarrow 4\alpha$ fragmentation.

According to the data in [22], this probability is constant and equal to (10 ± 1)% for nuclei with atomic numbers ranging from 20 to 60. In the experiment described in [36], it was found that the distribution over the pair azimuthal angle in a $^{22}Ne \longrightarrow n\alpha + X$ fragmentation with the momentum of the relativistic ^{22}Ne nuclei equal to 4.1 GeV/c per nucleon is similar to the same distribution for the fragmentation of carbon, i.e., as is shown in upper histogram of Fig. 2. No traces of the formation of two ^{8}Be nuclei in an intermediate state were found.

In [37], a virtually uniform distribution over the pair azimuthal angle was obtained in the fragmentation of relativistic ^{24}Mg nuclei for all the fragments with charges equal to two or higher. The distribution over the pair azimuthal angle for the α particles from a Pb + Pb $\longrightarrow n\alpha + X$ reaction was obtained when studying the fragmentation of relativistic lead nuclei with a momentum of 158 GeV/c per nucleon in their interaction with lead nuclei (histogram in Fig. 3) in [38]. Here, we see neither kinematic correlations, which should be excessive in events with large ε, nor correlations from

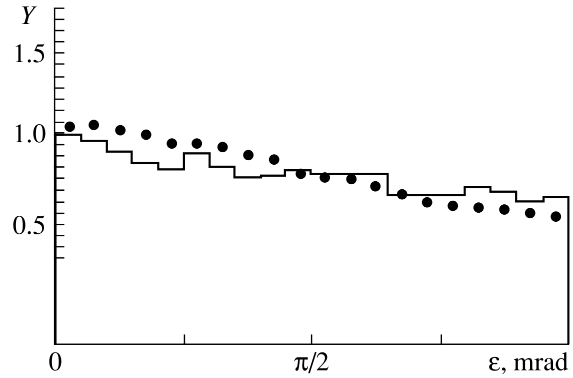

Fig. 3. Distribution of the pair azimuthal angles for the α particles in the Pb + Pb $\longrightarrow n\alpha + X$ reaction at a momentum of 158 A GeV/c. The value Y along the ordinate axis is the ratio between the number of events with a given pair angle in this angular interval and the average number of events in this interval at their uniform distribution. The histogram is experimental [38]. The experimental errors are approximately equal to one scale division in the ordinate axes. The calculated values (see the text) are shown by the dots (●).

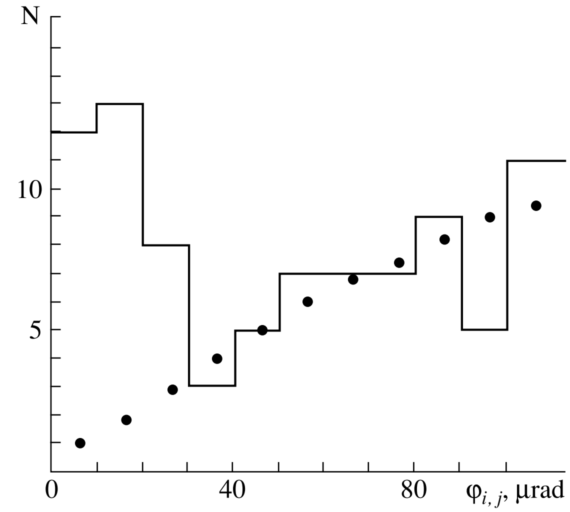

Fig. 4. Part of the distribution of the angles $\varphi_{i,j}$ between the projections of the α-particle tracks onto the emulsion plane in the region of their small values in the fragmentation of lead nuclei with the energy of 160 A GeV according to data from [23]. Along the ordinate axis, the number N of observed angles (in µrad) is plotted within the corresponding interval.

the ^{8}Be decays, for which more events with small ε should prevail. Instead, there is a steady rise in the probability of observing ε from 180° to 0°.

It is possible to show that an error in determining the primary-track direction, which is random in each event, could induce such a character of the azimuthal correlations. The dots in Fig. 3 show the results of a simulation of the distortions of the true azimuthal-angle values in the measuring procedure. During the simulation, a point, which was spaced from a point of the true primary-track direction by a certain distance whose distribution was a random sample of the Rayleigh distribution with a constant equal to 0.15 mrad, represented the primary-track direction in the transverse plane. It is clear that, for an arbitrary random deviation of the measured primary-track direction from its true one, all the differences between the azimuthal angles ε become smaller than their true values, and it was precisely this tendency that was observed in the experiment in [38].

In addition, the momentum transfer to the fragmenting nucleus as a whole could also induce the same effect in the distribution of pair azimuthal angles. Moreover, this momentum is redistributed between all the fragments proportionally to their masses. Both these causes lead to the same experimental result and are undistinguishable from each other. However, these azimuthal correlations probably have no relation to the ^{8}Be-nuclei emission.

Evidence for the possible existence of ^{8}Be nuclei in the fragmentation of lead nuclei with an energy of 160 GeV per nucleon in their interaction with photoemulsion nuclei was experimentally obtained in [23] only after developing a procedure for measuring the angles $\varphi_{i,j}$ between the tracks in the emulsion plane with an accuracy sufficient for this purpose. In Fig. 4, it can be seen that there is a peak in the region of small $\varphi_{i,j}$ angles. This peak is probably caused by the presence of ^{8}Be nuclei in the fragmentation of lead nuclei with an energy of 160 GeV per nucleon. Now, we consider the experiment regarding the estimation of the ^{8}Be-nuclei fraction in the fragmentation of ^{10}B nuclei in more detail.

First, we consider how large this fraction is. The calculation shows that the expected ^{8}Be-nuclei yield in the fragmentation of ^{10}B nuclei is equal to 19.7%, as there is also the ^{10}B $\longrightarrow n + p + {}^8$Be channel with a probability equal to 3.4% in addition to the ^{10}B $\longrightarrow d + {}^8$Be channel given in Table 1. Thus, in order to detect the two α particles and all the other charged fragments, $\simeq$20% of all the events of the ^{10}B-nuclei fragmentation should be realized through the intermediate ^{8}Be state. The fraction of events with the fragment-charge sum equal to 5 (Z_0 of a ^{10}B nucleus) is 10% according to [1]. This means that the α-particle pairs with the small angle θ_{12} between them should be found in $\simeq$2% of cases in all the inelastic interactions of the ^{10}B nuclei in a photoemulsion.

3. EXPERIMENTAL ESTIMATION OF THE ^{8}Be YIELD IN THE FRAGMENTATION OF ^{10}B NUCLEI

In the identification of ^{8}Be nuclei, the accuracy in estimating the angle θ_{12} between the tracks of particles is important. This accuracy depends not only on the quality of the treatment of the emulsion-chamber layers but also on the energy of the particles between which the angles are measured. At an early stage of the development of the photoemulsion method, the tracks of particles in an emulsion were assumed to be rectilinear. However, it was soon experimentally established that, in fact, the particles were subject to multiple Coulomb scattering, false scattering, and distortions [39]. The last two phenomena represent specific distortions of the tracks in an emulsion. Their nature is essentially unknown, but they lead to the fact that the trajectories of even high-energy particles are complex curves.

Owing to the granular structure of tracks, the accuracy in measuring the Y coordinate of a track in the emulsion plane appears to be about 0.2 μm [40]. The vertical Z coordinate in the emulsion is always measured much less accurately because the layer thickness during the process of measurement is different from that in the irradiation by a factor of 2. The vertical displacements of the lens or microscope stage are measured under conditions that are very different from those during the measurement of the spacings in the field of vision of the lens. Even if we strongly increase the base on which the measurements are performed, we do not achieve an unlimited increase in the accuracy of measuring the angle. Therefore, it is necessary to perform relative measurements and determine the angles from many points of a particle track.

In the experiment in [23] involving a chamber irradiated by lead ions with an energy of 160 GeV per nucleon, the error in measuring the angle $\varphi_{i,j}$ between the projections of the tracks onto the emulsion plane was found to be equal to 8×10^{-6} rad when recording the Y coordinates of 20 points through 1 mm. Naturally, the errors in measuring the angles θ_{12} in the space between α-particle tracks are much larger. In addition, the estimates of the small angles between tracks are always biased towards larger values [41]. It would appear that there are simply no unbiased estimates of small angles when using the coordinate method. In [42], before discussing the results of experimental estimates of the angles between particles, it was shown that the angle θ_{12} between the two α particles produced in the ^{8}Be-fragment decay from a ^{10}B nucleus can be measured with an accuracy of about 1.5 mrad. The maximal value of θ_{12} was equal to 5.45 mrad at an energy of 1 GeV per nucleon, and its regular bias at the smallest angles was equal to 1.5 mrad. For this reason, it was suggested that the angles θ_{12} smaller than 8.5 mrad are formed by the α-particle pairs produced in the ^{8}Be decay, while all the α-particle pairs with angles exceeding this value arise in the $^{10}\mathrm{B} \longrightarrow 2\alpha$ process. It is crucial to already know the expected characteristics of the distributions of the angles θ_{12} in these two processes. Then, it is only necessary to compare, as we did, these expected and experimental characteristics.

In the experiment described in [42], an emulsion chamber composed of layers of the emulsion BR-2 that were 10×20 cm^2 in area and 500 μm thick was irradiated along the layer in a ^{10}B-ion beam with the energy of 10 GeV in the Nuclotron at the Laboratory of High Energies (JINR). We then searched the events by scanning along a track. The total length of all the portions of the scanned primary tracks before detecting an inelastic interaction with the photoemulsion nuclei or their escape from the layer was equal to 243 m. We found 1823 inelastic interactions at this length. Thus, the mean free path before the interaction is equal to (13.3 ± 0.3) cm. In 217 events containing two doubly charged fragments of a ^{10}B nucleus, the X, Y, and Z coordinates were measured at 11 points with 100 μm intervals along the X axis on both tracks of the doubly charged fragments and on the primary-particle track.

If the average values of the coordinates x, y, and z are equal to $\langle x\rangle$ and $\langle a\rangle$, where $a = y, z$, the estimate of the tangent of the angle $\varepsilon = \varphi$ (for $a = y$) or the tangent of the angle $\varepsilon = \alpha$ (for $a = z$) is equal to

$$\tan\varepsilon = \frac{\langle xa\rangle - \langle x\rangle\langle a\rangle}{\langle x^2\rangle - \langle x\rangle^2}. \tag{13}$$

By calculating the angles φ and α for the given track, we obtain an estimate of the angle θ:

$$\tan\theta = \sqrt{\tan^2\varphi + \tan^2\alpha}.$$

In spite of the fact that the accuracy in measuring the coordinates along the axes Y and Z are different, the parameters of the distributions for the angles φ and α appeared to be virtually identical for these statistics. As was expected, both distributions agree with the hypothesis that they are samples of a normal distribution with the constant calculated using the ^{10}B-nucleus radius.

In Fig. 5, we show the normal-distribution function, with the average equal to zero and the standard deviation equal to 21 mrad, calculated from the value of the constant for the ^{10}B-nucleus radius (the smooth curve) and the empirical distribution functions for the angles φ and α. The sum of the squares of the differences along the vertical between the smooth curve and empirical distribution function gives the value of ω^2 (Kramers–Mises goodness-of-fit test), which can be used for checking the hypothesis regarding the goodness-of-fit between the empirical distribution function and the normal distribution [43]. According to our data, this hypothesis is accepted for both φ and α angles at a 1% CL.

This result is in complete agreement with that obtained in [1], where the experimental value of the average transverse momentum of deuterons is found to be equal to (140 ± 10) MeV/c. If we estimate this momentum from the value $r_0 = 1.54$ fm, it is equal to 145 MeV/c. Thus, as can be seen, there is quite good agreement between the two values.

The value $x = \varphi_1 + \varphi_2 + \alpha_1 + \alpha_2$ for this sample of events is normally distributed with the standard deviation $\sigma_x(\exp) = (39.7 \pm 2.7)$ mrad. Therefore, no angular correlations between the particles in the event are experimentally found. Then, it is quite natural that the distribution of the angles $\theta_{12} = x$ between the independent emissions the α-particle pairs should have distribution density (4) and distribution function (5).

When estimating the parameter σ of this distribution from experimental data, it is necessary to exclude the angles θ_{12} that are smaller than a certain value x_{min}. This exclusion is necessary because we are searching for a small excess over this distribution in the region of small angles θ_{12} arising due to the channels containing $^8Be \longrightarrow 2\alpha$. It is also necessary to exclude the angles θ_{12} exceeding a certain value x_{max} because they can correspond to rare events of an absolutely different nature, for example, a rescattering of particles in the final state.

Then, the likelihood function for the Rayleigh distribution, which is cut off on the left- and right-hand sides by the values x_{min} and x_{max}, has the form

$$L = \prod_{i=1}^{i=N} f(x_i, \sigma) F(x_{min}, \sigma)(1 - F(x_{max}, \sigma)). \quad (14)$$

In order to estimate the parameter σ, it is necessary to solve a nonlinear equation that can be derived by setting the derivative of the logarithm of the written likelihood function equal to zero. This problem is easily solved using the procedure from the MATHCAD-8 program [44].

The peak of L for the given sample is attained for $\sigma = (31.7 \pm 2.0)$ mrad. The azimuthal-asymmetry parameter A is equal to (0.05 ± 0.03) for all the events in the experiment, whereas it is equal to $-(0.96 \pm 0.04)$ for the events with $\theta_{12} < 8.5$ mrad. This result means that there are no correlations in the transverse-momenta directions for any of the events, whereas these correlations are strong for the events associated with the $^8Be \longrightarrow 2\alpha$ decay.

Finally, in the experiment, 33 events were observed with an angle $\theta_{12} < 8.5$ mrad (instead of the expected 36 events). This number indicates that the probability of observing ^{8}Be nuclei in the fragmentation of a ^{10}B nucleus in this experiment is equal to $(18 \pm 3)\%$ instead of the expected 19.7% yielded by the calculation.

If the events we observed with $\theta_{12} < 8.5$ mrad are indeed generated by the $^8Be \longrightarrow 2\alpha$ channel, the empirical distribution function for the angles θ_{12} in these 33 events should, in the limit, be the distribution function of these angles in this channel. The distribution density of such angles is shown in Fig. 1.

In order to check this hypothesis, we used two nonparametric goodness-of-fit criteria. The Kolmogorov goodness-of-fit criterion [43] requires that the maximal deviation D of the empirical distribution function from the assumed theoretical distribution function for the fit at a 1% significance level cannot exceed 1.63. In the experiment, $D = 0.32$.

The second criterion, which is stronger but less frequently used in experiments, is related to the sum

$$V = V^+ + V^-,$$

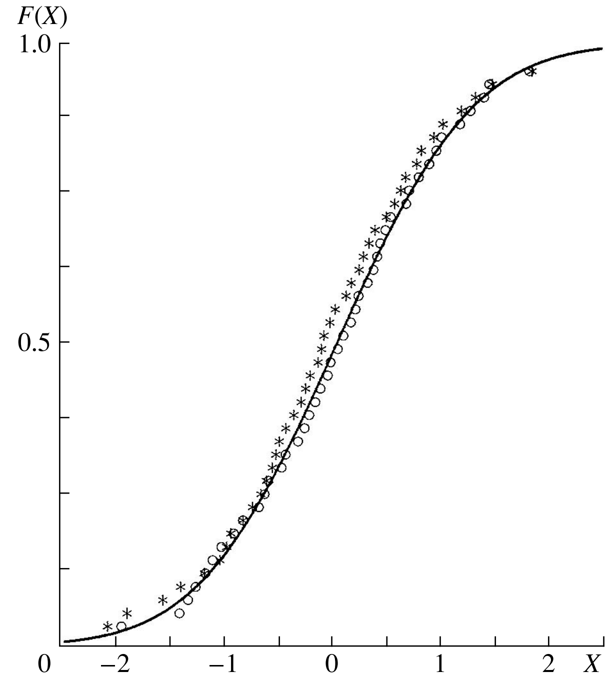

Fig. 5. The smooth curve represents an expected standard normal distribution. The empirical distribution function for the angles φ is shown by the asterisks. The empirical distribution function for the angles α is shown by the open circles.

which is equal to the sum of the deviations of the empirical-distribution-function values on both sides from the assumed distribution function. This is the Kuiper criterion [45]. Its critical value at the same confidence level is 2.0. In the experiment, $V = 0.88$. A similar result was also obtained using the previously mentioned Kramers–Mises goodness-of-fit criterion (see Table 2).

Therefore, according to all three goodness-of-fit criteria, the hypothesis that our sample of 33 angles $\theta_{12} < 8.5$ mrad has a distribution function for the θ_{12} angles between particles that corresponds to the $^8Be \longrightarrow 2\alpha$ process is not disregarded (as illustrated in Fig. 6).

CONCLUSIONS

The principal results of [42] are summarized in Table 2. All the predictions made before experiment have been confirmed. The ^{8}Be-fragment yield in the fragmentation of a relativistic ^{10}B nucleus at the energy of 10 GeV amounts to about 2% of all the events found by scanning the track in photoemulsion or about 20% from those events in which the sum of the charges of the secondary fragments is equal to the primary-nucleus charge.

In general, nuclear clustering is a special case of the general phenomena in an arbitrary system of many bodies. Therefore, at the first instant of the existence of the universe, when quark–gluon plasma expanded and cooled, this clustering took place, steady three-quark

Table 2. Calculated and experimental values of various quantities characterizing the ^{10}B-nucleus fragmentation

N	Quantities	Calculated	Experimental
1	$\langle P_\perp \rangle$ ^{2}H, MeV/c	145	140 ± 10
2	$\sigma(\varphi) = \sigma(\alpha)$ mrad	21.011	20.5 ± 0.7
3	$\sigma(\text{Rel}, \theta_{12})$ mrad	29.714	31.7 ± 2.0
4	$\langle \theta_{12} \rangle$ mrad	37.22	34.6 ± 2.2
5	$\sigma(\varphi_1 + \varphi_2 + \alpha_1 + \alpha_2)$ mrad	42.0	39.7 ± 2.7
6	$N_{\text{eu}}(\theta_{12} < 8.5$ mrad)	36	33
7	$W_{\text{obs}}(^8\text{Be} \longrightarrow 2\alpha)$ %	19.7	18 ± 3
8	A for all events	0	0.05 ± 0.03
9	A for ^{8}Be $\longrightarrow 2\alpha$	–1.0	-0.96 ± 0.04
10	$\langle \theta_{12} \rangle$ for $\theta_{12} < 8.5$ mrad	6.3	5.6 ± 1.0
11	D Kolm. crit.	1.63	0.32
12	V Kuip. crit.	2.0	0.88
13	ω^2 criterion	0.743	0.304

clusters appeared, and, thus, nucleons were formed. Then, the nucleons were combined into clusters (α particles) in stars during hydrogen burning.

The relatively long lifetime of the resonant state of two α particles in the form of a ^{8}Be nucleus favors overcoming a barrier in the form of nuclei with $A = 5$ during the synthesis of heavier nuclei as a result of the consecutive attachment of nucleons. In essence, all nuclei are the products of the clustering of the substance of stars.

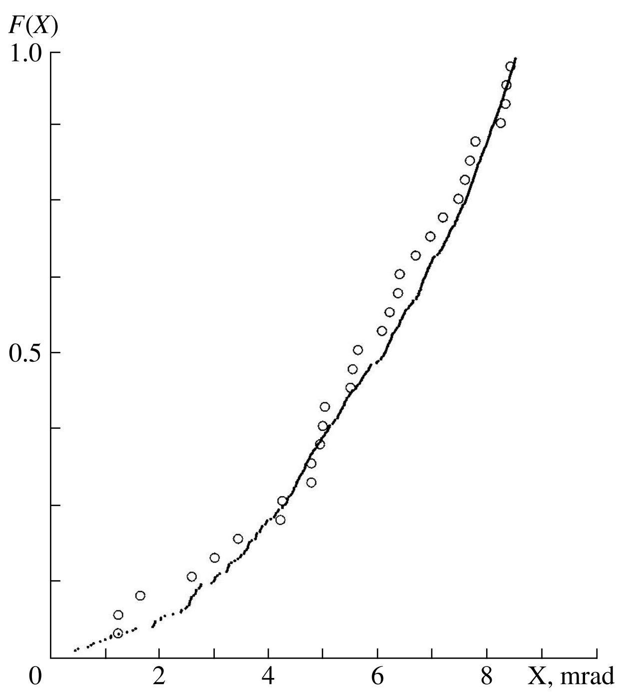

Fig. 6. The empirical distribution function $F(X) = F(\theta_{12} < X)$ for 33 angles (open circles). The dots are the assumed distribution function for the angles θ_{12} in the ^{8}Be $\longrightarrow 2\alpha$ process.

However, the stars themselves, as well as the galaxies, also form aggregates, i.e., clusters.

The probability of the formation of a cluster from A particles, whether they be nucleons or molecules, in the transition of a substance from one state into another appears to be proportional to $A^{-\tau}$, where $\tau \simeq 2.2$ [46, 47]. Therefore, Eq. (11) could be supplemented with this multiplier or, as in [15], with a multiplier taking into account the phase space of two or three partial states. However, it is necessary to have a wider experimental basis for these improvements.

The yields of clusters such as ^{5}Li and ^{5}He have been little studied. We hope that, in future, the yields of these isotopes will be investigated in the fragmentation of ^{10}B with the energy of 1 GeV per nucleon.

ACKNOWLEDGMENTS

We are grateful to the personnel of the Laboratory of High Energy Physics (JINR) for the irradiation and treatment of a large number of emulsion chambers and for providing the possibility to perform the cited works. We also thank the personnel of the nucleus–nucleus interaction group at the Department of High-Energy Physics (PINP, Russian Academy of Sciences) for decades of work in searching, measuring, and processing the events used in this review.

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