Theoretical analysis of inelastic pion-nucleus scattering within the microscopic optical potential

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The basic microscopic potential for πA-scattering

- To study the π-nucleus inelastic scattering we utilize the microscopic optical potential (OP)[1] defined by both the nuclear density and πN-amplitude of scattering $F_{\pi N}(q)$.
- The πN-amplitude depends on three parameters: total cross section σ, the ratio $\alpha = \Re e\, F_{\pi N}(0)/\Im m\, F_{\pi N}(0)$, and the slope β_π. From scattering on a proton target one establishes the free πN amplitude, while the "in-medium"πN amplitude can be obtained from the data on scattering of pions from nuclei.
- Recently we had fitted experimental data on the pion-nucleus **elastic scattering** for a set of nuclei and obtained the "in-medium" parameters of the πN-amplitude. Now they are used for analysis of **inelastic scattering** data.

[1] V.Lukyanov *et al.* Phys.At.Nucl.,(2006)**69**240; Bull.RAS, Physics,**77**(2013)427

Basic idea for inelastic scattering calculations

- Thus by using the same method applied earlier to derive the elastic microscopic OP depended on the nuclear density distribution $\rho(r)$, we construct the microscopic transition OP (TOP) $U_{inel}(\mathbf{r}, \xi)$ depended on a transition density distributions $\rho_\lambda(r)$ and collective variables ξ of a nucleus.
- This TOP provides calculations of the pion-nucleus inelastic scattering with excitations of the quadruple 2^+ and octuple 3^- collective states of the nuclei ^{28}Si, ^{58}Ni, ^{40}Ca and ^{208}Pb studied earlier in elastic scattering of pions.
- This scheme does not contain free parameters except the static (or dynamic) deformations of nuclei β_λ ($\lambda = 2, 3$), that characterize their excited states.

Basic idea for inelastic scattering calculations

In principle a transition potential can be constructed in two forms:

- The ordinary approach is based on a phenomenological elastic scattering OP which is used to form the TOP being its derivative on a deformation admixture to the nuclear radius or to the relative motion radius-vector.
- **But we follow to the fully microscopic method when the derivative is used of the target nucleus density distribution function being under the folding integral of a microscopic OP.**

So we obtain elastic and transition optical potentials, and then can utilize the DWUCK4 program to get elastic and inelastic cross sections. However, to this end one should preliminary transform the relativistic wave equation inherent in a πA scattering to the standard non-relativistic form, and thus the relativistic and distortion effects in initial and final channels are accounted for automatically.

Basic equations for elastic scattering

The cross sections are calculated by solving the Klein-Gordon equation in its form at conditions $E \gg U$

$$\left(\Delta + k^2\right)\psi(\vec{r}) = 2\bar{\mu}U(r)\psi(\vec{r}), \quad U(r) = U^H(r) + U_C(r)$$

Here k is relativistic momentum of pions in c.m. system,

$$k = \frac{M_A k^{\text{lab}}}{\sqrt{(M_A + m_\pi)^2 + 2M_A T^{\text{lab}}}}, \quad k^{\text{lab}} = \sqrt{T^{\text{lab}}\left(T^{\text{lab}} + 2m_\pi\right)},$$

and $\bar{\mu} = \dfrac{EM_A}{E + M_A}$ – relativistic reduced mass, $E = \sqrt{k^2 + m_\pi^2}$ – total energy, m_π and M_A – the pion and nucleus masses.

Microscopic OP for elastic and inelastic scattering

The HEA-based microscopic OP is done in the folding form

$$U_{opt}(r) = -\frac{(\hbar c)\beta_c}{2(2\pi)^3}\sigma\,[i+\alpha]\cdot\int e^{-i\mathbf{qr}}\rho(\mathbf{q})\,f(q)\,q^2dq,$$

where $\beta_c = k/E$; $f(q)$ – formfactor of πN-amplitude; $\rho(q)$ – formfactor of a nuclear density distribution.

Here the charge-independent principle $f_{\pi^\pm p} = f_{\pi^\mp n}$ let to use only 3 parameters for πN-amplitude instead of 6 for πp and πn, separately,

$$\sigma = \frac{1}{2}[\sigma^{(p)}_{\pi^+} + \sigma^{(p)}_{\pi^-}], \quad \alpha = \frac{1}{2}[\alpha^{(p)}_{\pi^+} + \alpha^{(p)}_{\pi^-}], \quad \beta_\pi = \frac{1}{2}[\beta^{(p)}_{\pi^+} + \beta^{(p)}_{\pi^-}]$$

And so the πN-amplitude is

$$F_{\pi N}(q) = \frac{k}{4\pi}\sigma\,[i+\alpha]\cdot f(q), \qquad f(q) = e^{-\beta_\pi q^2/2}$$

Basic equations for elastic and inelastic scattering

To get the generalized microscopic OP for both the elastic scattering and the TOP for inelastic one, we transform the form factor of a spherically-symmetric density distribution function

$$\rho(\mathbf{q}) = \int e^{i\mathbf{qr}}\, \rho(\mathbf{r})\, d^3 r$$

by using the following standard prescription:

$$\mathbf{r} \Rightarrow r + \delta^{(\lambda)}(\mathbf{r}), \qquad \delta^{(\lambda)}(\mathbf{r}) = -\, r \left(\frac{r}{R}\right)^{\lambda-2} \sum\nolimits_{\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}(\hat{r}),$$

where $\alpha_{\lambda\mu}$ are variables of the nuclear collective motion for λ=2,3.

Basic equations for elastic and inelastic scattering

Substituting this one in the density and then in the initial optical potential, and terminating their expansions at linear terms in $\delta^{(\lambda)}(\mathbf{r})$ one obtains

$$\rho(\mathbf{r}) = \rho(r) + \rho_\lambda(r) \sum_\mu \alpha_{\lambda\mu} Y_{\lambda\mu}(\hat{r}), \quad \rho_\lambda(r) = -r \left(\frac{r}{R}\right)^{\lambda-2} \frac{d\rho(r)}{dr}$$

and then one gets their form factors

$$\begin{aligned}
\rho(\mathbf{q}) &= \rho(q) + \rho_\lambda(q)\, i^\lambda \sum_\mu \alpha_{\lambda\mu} Y_{\lambda\mu}(\hat{q}), \\
\rho(q) &= 4\pi \int j_0(qr)\, \rho(r)\, r^2 dr, \\
\rho_\lambda(q) &= 4\pi \int j_\lambda(qr)\, \rho_\lambda(r)\, r^2 dr.
\end{aligned}$$

Basic equations for elastic and inelastic scattering

Finally, we obtain potentials for elastic and inelastic scattering

$$U(\mathbf{r}) = U_{opt}(r) + U^{(\lambda)}(\mathbf{r}), \quad U^{(\lambda)}(\mathbf{r}) = U_\lambda(r) \sum_\mu \alpha_{\lambda\mu} Y_{\lambda\mu}(\hat{r}),$$

$$U_{opt}(r) = -\frac{\hbar v}{(2\pi)^2}\, \sigma\, (\alpha + i) \int j_0(qr)\, \rho(q) f(q)\, q^2 dq,$$

$$U_\lambda(r) = -\frac{\hbar v}{(2\pi)^2}\, \sigma\, (\alpha + i) \int j_\lambda(qr)\, \rho_\lambda(q) f(q)\, q^2 dq.$$

Here the spherically symmetric part $U_{opt}(r)$ provides elastic scattering calculations while the $U_\lambda(r)$ is the transition OP used for calculations of inelastic scattering cross sections with excitations of the 2^+ and 3^- collective states of nuclei.

Basic equations for elastic and inelastic scattering

The amplitude of inelastic scattering is constructed in the framework of the distorted wave Born approximation (DWBA).

The respective matrix element has a linear dependence on the transition potential $U^{\lambda}(\mathbf{r}) + U_{\lambda}^{(c)}(\mathbf{r})$ while the distorted waves in initial and final channels $\xi^{(\pm)}(\mathbf{r})$ are calculated using the $U_{opt}(r) + U^{(c)}(r)$ potential, and thus

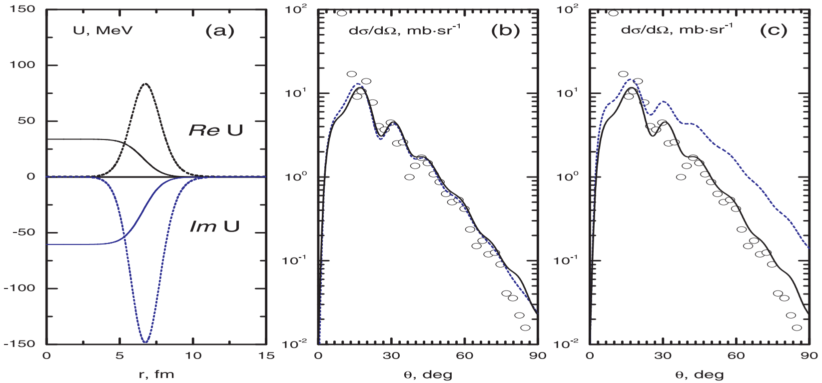
$$T_{BA} = \sum_{\mu} < B|\alpha_{\lambda\mu}|A > \int \chi^{(-)*}(\mathbf{r}_b)[U_{\lambda}(r)Y_{\lambda\mu}(\hat{r})]\chi^{(+)}(\mathbf{r}_a)d\mathbf{r}_a d\mathbf{r}_b.$$

In the case of the even-even nuclei the transition matrix element is

$$< B|\alpha_{\lambda\mu}|A >= (0\,\lambda\,0\,\mu|\lambda\,\mu) < \lambda||\alpha_{\lambda 0}||0 >= (1/\sqrt{2\lambda+1})\beta_{\lambda}$$

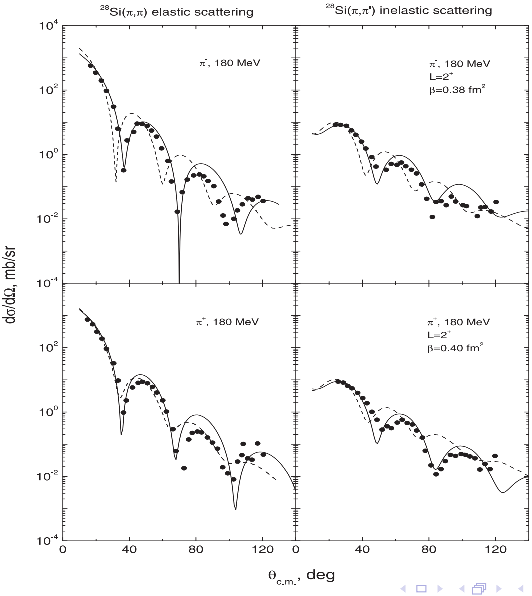
where β_{λ} ($\lambda = 2,3$) is a deformation parameter which is fitted in our study.

Methodical calculations for $\pi^+ + ^{208}$Pb at 291 MeV

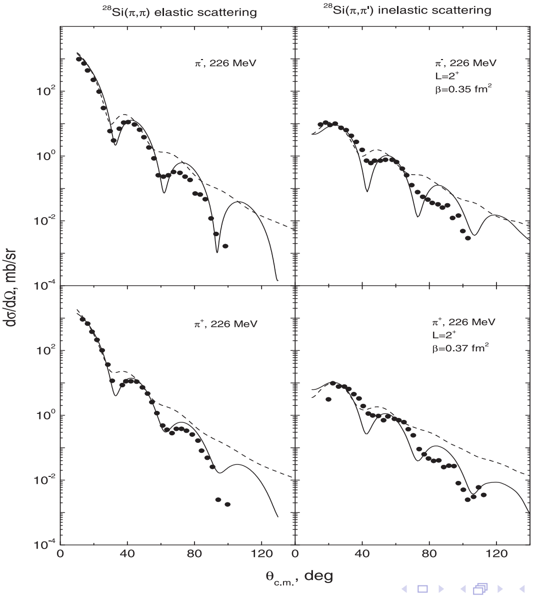


(a) calculated potentials (solid - OP, dashed TOP).
(b) role of the Coulomb (solid - with Coulomb TOP, dashed - without it).
(c) effect of "in-medium" parameters on cross sections (solid - with "in-medium" perameters, dashed - with free one).

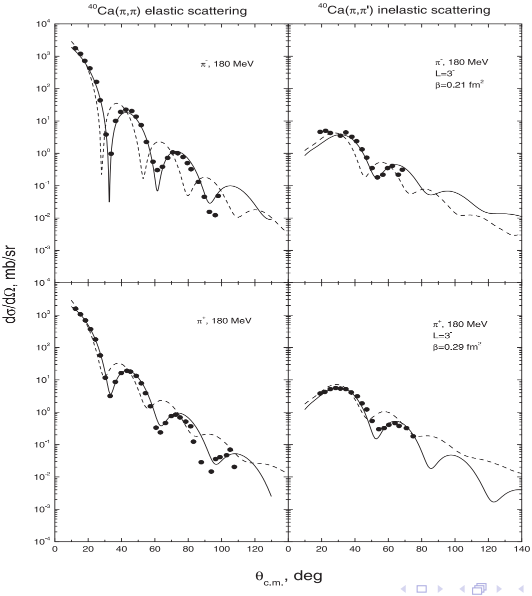
$\pi^{\pm}$-^{28}S scattering at T_{lab}= 180 Mev



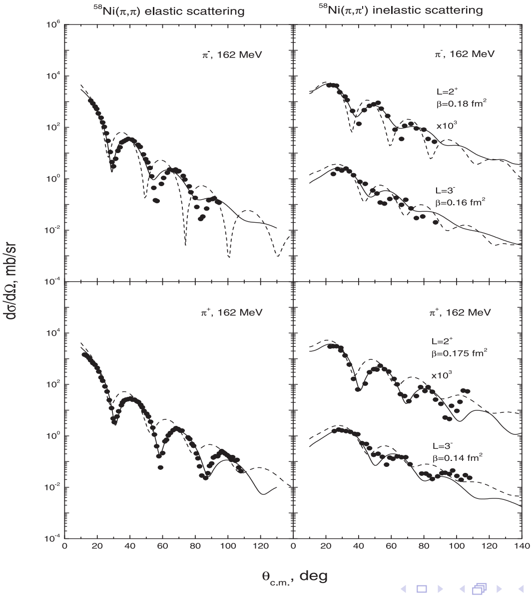
$\pi^{\pm}$-^{28}S scattering at T_{lab}= 226 Mev



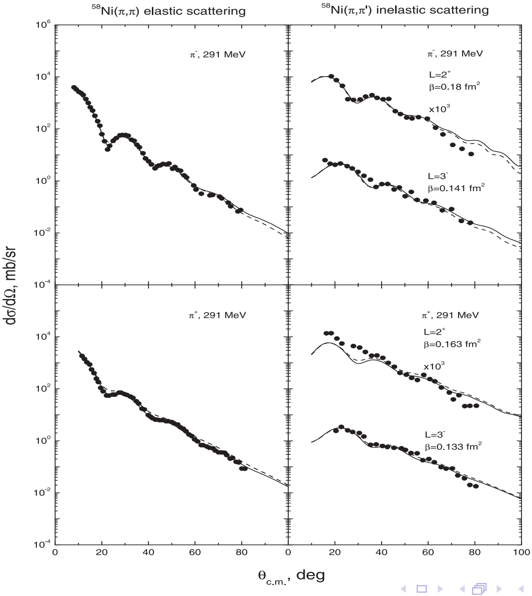
$\pi^{\pm}$-^{40}Ca scattering at T_{lab}= 180 Mev



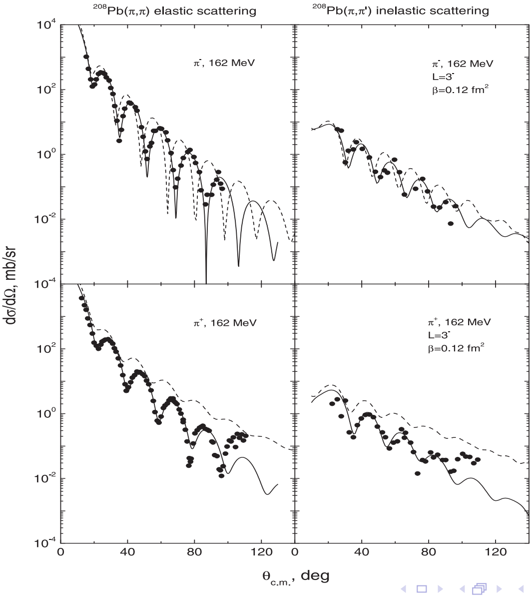
π^{pm}–^{58}Ni scattering at T_{lab}= 162 Mev



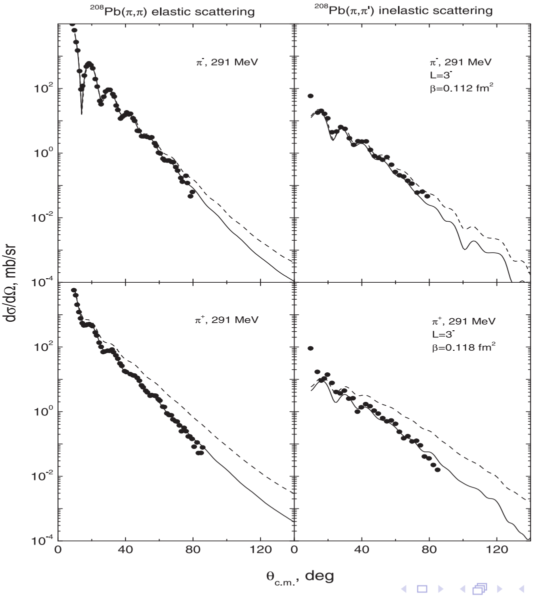
π^{pm}_^{58}Ni scattering at T_{lab}= 291 Mev



π^{pm}_^{208}Pb scattering at T_{lab}= 162, Mev



π^{pm}_^{208}Pb scattering at T_{lab}= 291, Mev

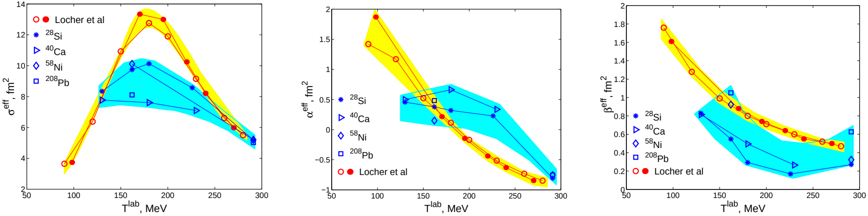


Fitting the πN amplitude parameters

Параметры "in-medium" πN-амплитуды для ряда реакций

reaction	T,MeV	σ,fm^2	α	β,fm^2	σ_f,fm^2	α_f	β_f,fm^2	β_λ
π^- ^{58}Ni	162	10.95	-0.16	1.10	12,14	0.37	1.07	β_2=0.18, β_3=0.16
π^+ ^{58}Ni		8.86	0.44	0.81				β_2=0.175, β_3=0.14
π^- ^{208}Pb		9.69	0.34	1.02				β_3=0.12
π^+ ^{208}Pb		6.18	0.58	1.24				β_3=0.12
π^- ^{28}Si	180	9.33	0.43	0.28	12.76	0.11	0.99	β_2=0.38
π^+ ^{28}Si		7.75	0.76	0.49				β_2=0.40
π^- ^{40}Ca		9.65	0.23	0.28				β_3=0.21
π^+ ^{40}Ca		5.75	1.09	0.69				β_3=0.29
π^- ^{28}Si	226	7.43	0.6	0.167	9.15	-0.52	0.69	β_2=0.35
π^+ ^{28}Si		5.87	1.08	0.42				β_2=0.37
π^- ^{58}Ni	291	4.79	-0.85	0.279	4.76	-0.95	0.44	β_2=0.18 β_3=0.141
π^+ ^{58}Ni		5.58	-0.66	0.354				β_2=0.163 β_3=0.133
π^- ^{208}Pb		4.47	-1.07	0.672				β_3=0.112
π^+ ^{208}Pb		5.52	-0.46	0.581				β_3=0.118

In-medium effect on πN scattering



- Yellow: "free" $\pi^{\pm}N$-scattering parameters [2].
- Blue: the best fit values $X^{eff} = (X_{\pi^+} + X_{\pi^-})/2$; $X = \sigma, \alpha, \beta$.
- Bell-like forms of σ^{free} and $\sigma^{eff}(T^{lab})$ have maximum at the same T^{lab}.
- "Blue" domain σ^{eff} is located below the "yellow" σ^{free} region.
- "In-medium" α^{eff} shows that refraction of pions in nuclei increases at $T^{lab} > T^{lab}_{(res)} \simeq 170$ MeV.
- "Blue" and "yellow" regions become closer at $T^{lab} > 250$ MeV.

[2] Locher *et al.*Nucl.Phys.**B27**(1971)593

Conclusions

- The proposed scheme for inelasic scattering differs from the usually used models in that it operates with the primary nature of a target nucleus, the density distribution function, while the other models use from the beginning the secondary description function - a derivative of an optical potential of scattering.
- If one so explains elastic scattering then the only structure parameter, the deformation of a target nucleus β_λ is neecessary to fit the data. In general they occur in coincidence in about 10% for scattering of π^-- and π^+-mesons on the same nucleus.The forms of theoretical curves are not distorted in this procedure.
- Comparisons of the calculated inelastic cross sections with the respective experimental data show a good agreement.

Thank you
for your attention!