# ELECTRODYNAMICAL PROPERTIES OF A VOLUME FREE ELECTRON LASER WITH A "GRID" RESONATOR

V.G. Baryshevsky, A.A.Gurinovich [*]

Research Institute for Nuclear Problems, 11 Bobryiskaya str., 220050, Minsk, Belarus.

## Abstract

The electrodynamic properties and lasing in Volume Free Electron Laser with a "grid" resonator ("grid" photonic crystal) with changing in space parameters are considered. The equations describing lasing of VFEL with such a resonator are obtained. It is shown that use of diffraction gratings (photonic crystal) with variable period increases radiation intensity and provide to create the dynamical wiggler with variable period. This makes possible to develop a double-cascaded FEL with variable parameters, which efficiency can be significantly higher then that of conventional system.

## INTRODUCTION

Diffraction radiation [1] in periodical structures is in the basis of operation of travelling wave tubes (TWT) [2, 3], backward wave oscillators (BWO) and such devices as Smith-Purcell lasers [4, 5, 6] and volume Free Electron Lasers [7, 8, 9] (see also [10]).

Volume Free Electron Laser (VFEL) is a radiation generator using non-one-dimensional distributed feedback, which is created with the aid of Bragg diffraction gratings or photonic crystals.

One of the VFEL types uses a "grid" volume resonator ("grid" photonic crystal) that is formed by a periodically strained either dielectric [11] or metallic threads [12, 13, 14, 15].

In the present paper the electrodynamic properties and lasing in Volume Free Electron Laser with a "grid" resonator ("grid" photonic crystal) with changing in space parameters are considered. The equations describing lasing of VFEL with such a resonator are obtained. It is shown that use of diffraction gratings (photonic crystal) with variable period provide to create the dynamical wiggler with variable period. This makes possible to develop a double-cascaded FEL with variable parameters changing, which efficiency can be significantly higher that of conventional system.

## THEORY OF LASING FOR VFEL WITH A "GRID" PHOTONIC CRYSTAL WITH VARIABLE PERIOD

To obtain equations, which describe VFEL lasing in the "grid" photonic crystal (see Fig.1), the Maxwell equations

and motion equations for a particle in an electromagnetic field should be considered:

$$\mathrm{rot}\vec{H} = \frac{1}{c}\frac{\partial\vec{D}}{\partial t} + \frac{4\pi}{c}\vec{j}, \; \mathrm{rot}\vec{E} = -\frac{1}{c}\frac{\partial\vec{H}}{\partial t},$$

$$\mathrm{div}\vec{D} = 4\pi\rho, \; \frac{\partial\rho}{\partial t} + \mathrm{div}\vec{j} = 0, \qquad (1)$$

here $\vec{E}$ and $\vec{H}$ are the electric and magnetic fields, $\vec{j}$ and $\rho$ are the current and charge densities, the electromagnetic induction $D_i(\vec{r}, t') = \int \varepsilon_{il}(\vec{r}, t-t')E_l(\vec{r}, t')dt'$ and, therefore, $D_i(\vec{r}, \omega) = \varepsilon_{il}(\vec{r}, \omega)E_l(\vec{r}, \omega)$, the indices $i, l = 1, 2, 3$ correspond to the axes $x, y, z$, respectively. The current and charge densities are respectively defined as:

$$\vec{j}(\vec{r}, t) = e\sum_{\alpha}\vec{v}_{\alpha}(t)\delta(\vec{r}-\vec{r}_{\alpha}(t)), \; \rho(\vec{r}, t) = e\sum_{\alpha}\delta(\vec{r}-\vec{r}_{\alpha}(t)),$$

where $e$ is the electron charge, $\vec{v}_{\alpha}$ is the velocity of the particle $\alpha$ ($\alpha$ numerates the beam particles),

$$\frac{d\vec{v}_{\alpha}}{dt} = \frac{e}{m\gamma_{\alpha}}\left\{\vec{E}(\vec{r}_{\alpha}, t) + \frac{1}{c}[\vec{v}_{\alpha}\times\vec{H}(\vec{r}_{\alpha}, t)] - \frac{\vec{v}_{\alpha}}{c^2}(\vec{v}_{\alpha}\vec{E}(\vec{r}_{\alpha}, t))\right\},$$

here $\gamma_{\alpha} = (1 - \frac{v_{\alpha}^2}{c^2})^{-\frac{1}{2}}$ is the Lorentz-factor, $\vec{E}(\vec{r}_{\alpha}, t)$ and $\vec{H}(\vec{r}_{\alpha}, t)$ are the electric and magnetic field in the point of location $\vec{r}_{\alpha} = \vec{r}_{\alpha}(t)$ of the particle $\alpha$.
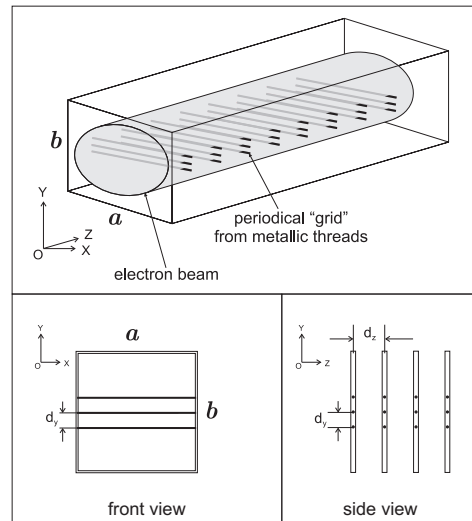


Figure 1: A "grid" photonic crystal.

The dielectric permittivity tensor can be expressed as $\hat{\varepsilon}(\vec{r}) = 1 + \hat{\chi}(\vec{r})$, where $\hat{\chi}(\vec{r})$ is the dielectric susceptibility.

[*] gur@inp.minsk.by

When $\hat{\chi} \ll 1$ the system (1) can be rewritten as:

$$\Delta \vec{E}(\vec{r}, t) - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \int \hat{\varepsilon}(\vec{r}, t - t') \vec{E}(\vec{r}, t') dt' = \qquad (2)$$

$$= 4\pi \left( \frac{1}{c^2} \frac{\partial \vec{j}(\vec{r}, t)}{\partial t} + \vec{\nabla} \rho(\vec{r}, t) \right).$$

When the grating is ideal $\hat{\chi}(\vec{r}) = \sum_\tau \hat{\chi}_\tau(\vec{r}) e^{i\vec{\tau}\vec{r}}$, where $\vec{\tau}$ is the reciprocal lattice vector [16, 17].

Let the diffraction grating (photonic crystal) period is smoothly varied with distance, which is much greater then the diffraction grating (ptotonic crystal lattice) period. It is convenient in this case to present the susceptibility $\hat{\chi}(\vec{r})$ in the form, typical for theory of X-ray diffraction in crystals with lattice distortion [18]:

$$\hat{\chi}(\vec{r}) = \sum_\tau e^{i\Phi_\tau(\vec{r})} \hat{\chi}_\tau(\vec{r}), \qquad (3)$$

where $\Phi_\tau(\vec{r}) = \int \vec{\tau}(\vec{r}') d\vec{l}'$, $\vec{\tau}(\vec{r}')$ is the reciprocal lattice vector in the vicinity of the point $\vec{r}'$. The expressions for $\hat{\chi}$ for the "grid" photonic crystal were obtained in [12, 14]:

$$\chi_{\|(\perp)} = \frac{4\pi}{\Omega_2 k^2} \frac{A_{0\|(\perp)}}{1 + i\pi A_{0\|(\perp)} - 2CA_{0\|(\perp)}}, \qquad (4)$$

the symbols $\|$ and $\perp$ indicate the waves with polarization parallel and perpendicular to the thread axis, respectively, $k = 2\pi/\lambda$ is the wave number, $R$ is the thread radius, $C = 0.5772$ is the Eiler constant, $\Omega_2 = d_y \cdot d_z$, where $d_y$ and $d_z$ are the photonic crystal periods along the axis $y$ and $z$, respectively. The values $A_{0(\|)}$ and $A_{0(\perp)}$ for the threads with finite conductivity are defined as[14]:

$$A_{0(\|)} = \frac{i}{\pi} \frac{J_0(k_t R) J_0'(kR) - \sqrt{\varepsilon_t} J_0'(k_t R) J_0(kR)}{J_0(k_t R) H_0^{(1)'}(kR) - \sqrt{\varepsilon_t} J_0'(k_t R) H_0^{(1)}(kR)},$$

$$A_{0(\perp)} = \frac{i}{\pi} \frac{J_0(k_t R) J_0'(kR) - \frac{1}{\sqrt{\varepsilon_t}} J_0'(k_t R) J_0(kR)}{J_0(k_t R) H_0^{(1)'}(kR) - \frac{1}{\sqrt{\varepsilon_t}} J_0'(k_t R) H_0^{(1)}(kR)},$$

where $\varepsilon_t$ is the dielectric permittivity of the thread material, $k_t = \sqrt{\varepsilon_t} k$, $H_0^{(1)}$ is the Hankel function of the zero order, $J_0$ and $J_0'$ are the Bessel functions and their derivatives, respectively.

In contrast to the theory of X-rays diffraction, in the case under consideration $\hat{\chi}_\tau$ depends on $\vec{r}$. It is to the fact that $\hat{\chi}_\tau$ depends on the volume of the lattice unit cell $\Omega_2$, which can be significantly varied for diffraction gratings (photonic crystals), as distinct from natural crystals.

It should be reminded that for an ideal crystal without lattice distortions, the wave, which propagates in crystal can be presented as a superposition of the plane waves:

$$\vec{E}(\vec{r}, t) = \sum_{\vec{\tau}=0}^{\infty} \vec{A}_{\vec{\tau}} e^{i(\vec{k}_\tau \vec{r} - \omega t)}, \qquad (5)$$

where $\vec{k}_\tau = \vec{k} + \vec{\tau}$.

In the case under consideration the solution of (2) can be written in the form (compare with [18]):

$$\vec{E}(\vec{r}, t) = \text{Re} \left\{ \sum_{\vec{\tau}=0}^{\infty} \vec{A}_{\vec{\tau}} e^{i(\phi_\tau(\vec{r}) - \omega t)} \right\}, \qquad (6)$$

where $\phi_\tau(\vec{r}) = \int_0^{\vec{r}} k(\vec{r}) d\vec{l} + \Phi_\tau(\vec{r})$ and $k(\vec{r})$ can be found as solution of the dispersion equation in the vicinity of the point with the coordinate vector $\vec{r}$, integration is done over the quasiclassical trajectory, which describes motion of the wavepacket in the photonic crystal with lattice distortion.

Let us consider now case when all the waves participating in the diffraction process lays in a plane (coupled wave diffraction, multiple-wave diffraction [17, 16]) i.e. all the reciprocal lattice vectors $\vec{\tau}$ lie in one plane. Suppose the wave polarization vector is orthogonal to the plane of diffraction.

Let us rewrite (6) in the form $\vec{E}(\vec{r}, t) = \vec{e} E(\vec{r}, t)$, where

$$E(\vec{r}, t) = \text{Re} \left\{ \vec{A}_1 e^{i(\phi_1(\vec{r}) - \omega t)} + \vec{A}_2 e^{i(\phi_2(\vec{r}) - \omega t)} + .. \right\}, (7)$$

$$\phi_1(\vec{r}) = \int_0^{\vec{r}} \vec{k}_1(\vec{r}') d\vec{l}, \qquad (8)$$

$$\phi_2(\vec{r}) = \int_0^{\vec{r}} \vec{k}_1(\vec{r}') d\vec{l} + \int_0^{\vec{r}} \vec{\tau}(\vec{r}') d\vec{l}. \qquad (9)$$

Then multiplying (2) by $\vec{e}$ one can get:

$$\Delta E(\vec{r}, t) - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \int \hat{\varepsilon}(\vec{r}, t - t') E(\vec{r}, t') dt' = \qquad (10)$$

$$= 4\pi \vec{e} \left( \frac{1}{c^2} \frac{\partial \vec{j}(\vec{r}, t)}{\partial t} + \vec{\nabla} \rho(\vec{r}, t) \right). \qquad (11)$$

Substitution of (7) to (11) gives the following system:

$$\frac{1}{2} e^{i(\phi_1(\vec{r}) - \omega t)} [2i\vec{k}_1(\vec{r}) \vec{\nabla} A_1 + i\vec{\nabla}\vec{k}_1(\vec{r}) A_1 - k_1^2(\vec{r}) A_1 +$$

$$+ \frac{\omega^2}{c^2} \varepsilon_0(\omega, \vec{r}) A_1 + i\frac{1}{c^2} \frac{\partial \omega^2 \varepsilon_0(\omega, \vec{r})}{\partial \omega} \frac{\partial A_1}{\partial t} + \frac{\omega}{c^2}{}^2 \varepsilon_{-\tau}(\omega, \vec{r}) A_2 +$$

$$+ i\frac{1}{c^2} \frac{\partial \omega^2 \varepsilon_{-\tau}(\omega, \vec{r})}{\partial \omega} \frac{\partial A_2}{\partial t}] + \text{conjugated terms} =$$

$$= 4\pi \vec{e} \left( \frac{1}{c^2} \frac{\partial \vec{j}(\vec{r}, t)}{\partial t} + \vec{\nabla} \rho(\vec{r}, t) \right), \qquad (12)$$

$$\frac{1}{2} e^{i(\phi_2(\vec{r}) - \omega t)} [2i\vec{k}_2(\vec{r}) \vec{\nabla} A_2 + i\vec{\nabla}\vec{k}_2(\vec{r}) A_2 - k_2^2(\vec{r}) A_2 +$$

$$+ \frac{\omega^2}{c^2} \varepsilon_0(\omega, \vec{r}) A_2 + i\frac{1}{c^2} \frac{\partial \omega^2 \varepsilon_0(\omega, \vec{r})}{\partial \omega} \frac{\partial A_2}{\partial t} + \frac{\omega^2}{c^2} \varepsilon_\tau(\omega, \vec{r}) A_1 +$$

$$+ i\frac{1}{c^2} \frac{\partial \omega^2 \varepsilon_\tau(\omega, \vec{r})}{\partial \omega} \frac{\partial A_1}{\partial t}] + \text{conjugated terms} =$$

$$= 4\pi \vec{e} \left( \frac{1}{c^2} \frac{\partial \vec{j}(\vec{r}, t)}{\partial t} + \vec{\nabla} \rho(\vec{r}, t) \right),$$

where the vector $\vec{k}_2(\vec{r}) = \vec{k}_1(\vec{r}) + \vec{\tau}$, $\varepsilon_0(\omega, \vec{r}) = 1 + \chi_0(\vec{r})$, here notation $\chi_0(\vec{r}) = \chi_{\tau=0}(\vec{r})$ is used, $\varepsilon_\tau(\omega, \vec{r}) = \chi_\tau(\vec{r})$. Note here that for numerical analysis of (12), if $\chi_0 \ll 0$, it is convenient to take the vector $\vec{k}_1(\vec{r})$ in the form $\vec{k}_1(\vec{r}) = \vec{n}\sqrt{k^2 + \frac{\omega^2}{c^2} \chi_0(\vec{r})}$.

For better understanding let us suppose that the diffraction grating (photonic crystal lattice) period changes along one direction and define this direction as axis $z$.

Considering the right part of (2) let us take into account that microscopic currents and densities are the sums of terms, containing delta-functions, therefore, the right part can be rewritten as:

$$e^{-i(\vec{k}_\perp \vec{r}_\perp + \phi_{1z}(z) - \omega t)} 4\pi \vec{e} \left( \frac{1}{c^2} \frac{\partial \vec{j}(\vec{r}, t)}{\partial t} + \vec{\nabla} \rho(\vec{r}, t) \right) =$$

$$= -\frac{4\pi i \omega e}{c^2} \vec{e} \sum_\alpha \vec{v}_\alpha(t) \delta(\vec{r} - \vec{r}_\alpha(t)) e^{-i(\vec{k}_\perp \vec{r}_\perp + \phi_{1z}(z) - \omega t)} \times$$

$$\times \theta(t - t_\alpha) \theta(T_\alpha - t), \tag{13}$$

here $t_\alpha$ is the time of entrance of particle $\alpha$ to the resonator, $T_\alpha$ is the time of particle leaving from the resonator, $\theta$−functions in (13) image the fact that for time moments preceding $t_\alpha$ and following $T_\alpha$ the particle $\alpha$ does not contribute in process.

Let us suppose now that a strong magnetic field is applied for beam guiding though the generation area. Thus, the problem appears one-dimensional (components $v_x$ and $v_y$ are suppressed). Averaging the right part of (13) over the particle positions inside the beam, points of particle entrance to the resonator $r_{\perp 0\alpha}$ and time of particle entrance to the resonator $t_\alpha$ we can obtain:

$$e^{-i(\vec{k}_\perp \vec{r}_\perp + \phi_{1z}(z) - \omega t)} 4\pi \vec{e} \left( \frac{1}{c^2} \frac{\partial \vec{j}(\vec{r}, t)}{\partial t} + \vec{\nabla} \rho(\vec{r}, t) \right) =$$

$$= -\frac{4\pi i \omega \rho \, \vartheta_1 \, u(t) \, e}{c^2} \frac{1}{S} \int d^2 \vec{r}_{\perp 0} \frac{1}{T} \times \tag{14}$$

$$\times \int_0^t e^{-i(\phi_1(\vec{r}, \vec{r}_\perp, t, t_0) + \vec{k}_\perp \vec{r}_{\perp 0} - \omega t)} dt_0 =$$

$$= -\frac{4\pi i \omega \rho \, \vartheta_1 \, u(t) \, e}{c^2} \langle\langle e^{-i(\phi_1(\vec{r}, \vec{r}_\perp, t, t_0) + \vec{k}_\perp \vec{r}_{\perp 0} - \omega t)} dt_0 \rangle\rangle,$$

where $\rho$ is the electron beam density , $u(t)$ is the mean electron beam velocity, which depends on time due to energy losses, $\vartheta_1 = \sqrt{1 - \frac{\omega^2}{\beta^2 k_1^2 c^2}}$, $\beta^2 = 1 - \frac{1}{\gamma^2}$, $\langle\langle \ \rangle\rangle$ indicates averaging over transversal coordinate of point of particle entrance to the resonator $r_{\perp 0\alpha}$ and time of particle entrance to the resonator $t_\alpha$.

According to [19] averaging procedure in (14) can be simplified, when consider that random phases, appearing due to random transversal coordinate and time of entrance, presents in (14) as differences. Therefore, double integration over $d^2 \vec{r}_{\perp 0} \, dt_0$ can be replaced by single integration [19].

The system (12) in this case converts to:

$$2ik_{1z}(z) \frac{\partial A_1}{\partial z} + i \frac{\partial k_{1z}(z)}{\partial z} A_1 - (k_\perp^2 + k_{1z}^2(z)) A_1 +$$

$$+\frac{\omega^2}{c^2} \varepsilon_0(\omega, z) A_1 + i \frac{1}{c^2} \frac{\partial \omega^2 \varepsilon_0(\omega, z)}{\partial \omega} \frac{\partial A_1}{\partial t} +$$

$$+\frac{\omega^2}{c^2} \varepsilon_{-\tau}(\omega, z) A_2 + i \frac{1}{c^2} \frac{\partial \omega^2 \varepsilon_{-\tau}(\omega, z)}{\partial \omega} \frac{\partial A_2}{\partial t} =$$

$$= i \frac{2\omega}{c^2} J_1(k_{1z}(z)), \tag{15}$$

$$2ik_{2z}(z) \frac{\partial A_2}{\partial z} + i \frac{\partial k_{2z}(z)}{\partial z} A_2 - (k_\perp^2 + k_{2z}^2(z)) A_2 +$$

$$+\frac{\omega^2}{c^2} \varepsilon_0(\omega, z) A_2 + i \frac{1}{c^2} \frac{\partial \omega^2 \varepsilon_0(\omega, z)}{\partial \omega} \frac{\partial A_2}{\partial t} +$$

$$+\frac{\omega^2}{c^2} \varepsilon_\tau(\omega, z) A_1 + i \frac{1}{c^2} \frac{\partial \omega^2 \varepsilon_\tau(\omega, z)}{\partial \omega} \frac{\partial A_1}{\partial t} =$$

$$= i \frac{2\omega}{c^2} J_2(k_{2z}(z)),$$

where the currents $J_1$, $J_2$ are determined by the expression

$$J_m = 2\pi j \vartheta_m \int_0^{2\pi} \frac{2\pi - p}{8\pi^2} (e^{-i\phi_m(t,z,p)} + e^{-i\phi_m(t,z,-p)}) dp,$$

$$\vartheta_m = \sqrt{1 - \frac{\omega^2}{\beta^2 k_m^2 c^2}}, \ m = 1, 2, \ \beta^2 = 1 - \frac{1}{\gamma^2} , \tag{16}$$

$j = e n_0 v$ is the current density, $A_1 \equiv A_{\tau=0}$, $A_2 \equiv A_\tau$, $\vec{k}_1 = \vec{k}_{\tau=0}$, $\vec{k}_2 = \vec{k}_1 + \vec{\tau}$. The expressions for $J_1$ for $k_1$ independent on $z$ was obtained in [19].

When more than two waves participate in diffraction process, the system (15) should be supplemented with equations for waves $A_m$, which are similar to those for $A_1$ and $A_2$.

Now we can find the equation for phase. From the expressions (8,9) it follows that

$$\frac{d^2\phi_m}{dz^2} + \frac{1}{v} \frac{dv}{dz} \frac{d\phi_m}{dz} = \frac{dk_m}{dz} + \frac{k_m}{v^2} \frac{d^2z}{dt^2}, \tag{17}$$

Let us introduce new function $C(z)$ az follows:

$$\frac{d\phi_m}{dz} = C_m(z) e^{-\int_0^z \frac{1}{v} \frac{dv}{dz'} dz'} = \frac{v_0}{v(z)} C_m(z), \tag{18}$$

$$\phi_m(z) = \phi_m(0) + \int_0^z \frac{v_0}{v(z')} C_m(z') dz'$$

Therefore,

$$\frac{dC_m(z)}{dz} = \frac{v(z)}{v_0} \left( \frac{dk_m}{dz} + \frac{k_m}{v^2} \frac{d^2z}{dt^2} \right). \tag{19}$$

In the one-dimensional case the motion equation can be written as:

$$\frac{d^2 z_\alpha}{dt^2} = \frac{e\vartheta}{m\gamma(z_\alpha, t, p)} \mathrm{Re} E(z_\alpha, t), \tag{20}$$

therefore,

$$\frac{dC_m(z)}{dz} = \frac{v(z)}{v_0} \frac{dk_m}{dz} + \tag{21}$$

$$+\frac{k_m}{v_0 v(z)} \frac{e\vartheta_m}{m\gamma^3(z, t(z), p)} \mathrm{Re}\{A_m(z, t(z)) e^{i\phi_m(z, t(z), p)}\},$$

$$\frac{d\phi_m(t, z, p)}{dz}\Big|_{z=0} = k_{mz} - \frac{\omega}{v}, \ \phi_m(t, z, p)|_{z=0} = p,$$

$$A_1|_{z=L} = E_1^0, \ A_2|_{z=L} = E_2^0, \ A_m|_{t=0} = 0, \ m = 1, 2,$$

$$t > 0, \ z \in [0, L], \ p \in [-2\pi, 2\pi],$$

$L$ is the length of the photonic crystal.

These equations should be supplied with the equations for $\gamma(z,p)$. It is well-known that

$$mc^2 \frac{d\gamma}{dt} = e\vec{v}\vec{E}. \tag{22}$$

Therefore,

$$\frac{d\gamma(z,t(z),p)}{dz} = \sum_l \frac{e\vartheta_l}{mc^2}\text{Re}\{\sum_l A_l(z,t(z))e^{i\phi_l(z,t(z),p)}\}.$$

The above obtained equations (15,18,21,22) provide to describe generation process in FEL with varied parameters of diffraction grating (photonic crystal). Analysis of the system (21) can be simplified by replacement of the $\gamma(z,t(z),p)$ with its averaged by the initial phase value

$$\langle\gamma(z,t(z))\rangle = \frac{1}{2\pi}\int_0^{2\pi}\gamma(z,t(z),p)\,dp.$$

Note that the law of parameters change can be both smooth and stepped.

Analysis of such a system shows that its efficiency significantly exceeds efficiency of a system with constant parameters. Use of photonic crystals provide to develop different VFEL arrangements (see Fig.2). It should be noted
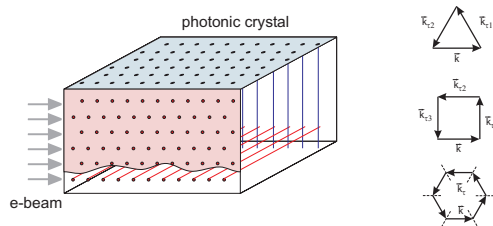


Figure 2: An example of photonic crystal with the thread arrangement providing multi-wave volume distributed feedback. Threads are arranged to couple several waves (three, four, six and so on), which appear due to diffraction in such a structure, in both the vertical and horizontal planes. The electronic beam takes the whole volume of photonic crystal.

that, for example, in the FEL (TWT,BWO) resonator with changing in space parameters of grating (photonic crystal) the electromagnetic wave with depending on $z$ spatial period is formed (see eq. (6)). This means that the dynamical undulator with depending on $z$ period appears along the whole resonator length i. e. tapering dynamical wiggler becomes settled. It is well known that tapering wiggler can significantly increase efficiency of the undulator FEL. The dynamical wiggler with varied period, which is proposed, can be used for development of double-cascaded FEL with parameters changing in space. The efficiency of such system can be significantly higher that of conventional system. Moreover, the period of dynamical wiggler can be done much shorter than that available for wigglers using static magnetic fields. It should be also noted that, due to dependence of the phase velocity of the electromagnetic wave on time, compression of the radiation pulse is possible in such a system.

## CONCLUSION

The electrodynamic properties and lasing in Volume Free Electron Laser with a "grid" resonator ("grid" photonic crystal) with changing in space parameters are considered. The equations describing lasing of VFEL with such a resonator are obtained. It is shown that use of diffraction gratings (photonic crystal) with variable period increases radiation intensity and provide to create the dynamical wiggler with variable period. This makes possible to develop a double-cascaded FEL with variable parameters, which efficiency can be significantly higher then that of conventional system.

## REFERENCES

[1] B.M.Bolotovskii and G.V.Voskresenskii, Usp. Fiz. Nauk. **88**, 209 (1966) (Sov. Phys. Usp. **9**, 73 (1966)).

[2] R.Kompfner, Wireless World **52**, 369 (1946).

[3] R.Pierce, Proc. IRE **35**, 111 (1947).

[4] S.J.Smith and E.M.Purcell, Phys. Rev. **92**, 1069 (1953).

[5] W.W.Salisbury, US Patent 2,634,372 (1953); J.Opt. Soc.Am. **60**, 1279 (1970).

[6] G.Doucas, J.H.Mulvey, M.Omori et al., Phys.Rev.Lett. **69**, 1761 (1992); John E. Walsh US Patent 5,790,585 (1996).

[7] V.G.Baryshevsky, NIM **445A**, 281 (2000); LANL e-print archive physics/9806039.

[8] V.G.Baryshevsky, K.G.Batrakov, A.A.Gurinovich et al., NIM **483A**, 21 (2002).

[9] V.G.Baryshevsky, K.G.Batrakov, A.A.Gurinovich et al., NIM **507A**, 137 (2003).

[10] V.L.Granatstein, R.K.Parker and C.M.Armstrong, Proceedings of the IEEE **87**, no.5 (1999).

[11] V.G.Baryshevsky, K.G.Batrakov, I.Ya.Dubovskaya, et al., NIM **393A**, 71 (1997).

[12] V.G.Baryshevsky, A.A.Gurinovich, LANL e-print archive: physics/0409107.

[13] V.G.Baryshevsky, K.G.Batrakov, N.A.Belous et al., LANL e-print archive: physics/0409125.

[14] V.G.Baryshevsky, A.A.Gurinovich, to be published in NIM B, Topical Issue RC2005.

[15] V.G. Baryshevsky, N.A. Belous, V.A. Evdokimov et al., LANL e-print arXiv: physics/0605122.

[16] R.W. James, *The Optical Principles of Diffraction of X-Rays* (Ox Bow Press, 1982).

[17] Shih-Lin Chang, *Multiple diffraction of x-rays in crystals* (Springer-Verlag, 1984).

[18] S.Takagi, Acta Crystall. **15**, 1311 (1962).

[19] K.G.Batrakov and S.N.Sytova, Computational Mathematics and Mathematical Physics **45**, No.4, 666 (2005).